

PREDICTION OF DIAGNOSTIC SYMPTOM VALUES USING A SET OF MODELS $GM(1,1)$ AND A MOVING WINDOW METHOD

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Summary

The aim of this paper is to show methodology of forecasting with various horizon of prediction using grey system theory, basing on practical application to vibration condition monitoring problems. The method of forecasting was based on $GM(1,1)$ prognostic models with various window lengths for estimating model parameters. The model $GM(1,1)$ is very effective where we have only few data, incomplete, and with low accuracy. The moving window method applied to $GM(1,1)$ model enables to adapt to changes in data trend. However, selecting an inappropriate window length can result in excessive forecast errors. The applied algorithm is based on tracking the current prediction error for models having various window lengths, and then eliminating the models for which the error of prediction caused by the loss of adequacy of the model to the data increases excessively.

Keywords: condition monitoring, symptom forecasting, $GM(1,1)$ models

PROGNOZA WARTOŚCI SYMPTOMÓW DIAGNOSTYCZNYCH ZA POMOCĄ ZBIORU MODELI $GM(1,1)$ I METODY RUCHOMEGO OKNA

Streszczenie

Celem pracy jest przedstawienie metodyki prognozowania dla różnych horyzontów prognozy z wykorzystaniem teorii szarych systemów w odniesieniu do praktycznego problemu z zakresu drganiowej diagnostyki maszyn. Metoda prognozowania jest oparta o modele $GM(1,1)$ o różnej długości okna, w której wyznacza się parametry modelu. Model $GM(1,1)$ jest bardzo efektywny gdy istnieje niewiele danych, są one niekompletne i są niskiej jakości. Metoda ruchomego okna w zastosowaniu do modelu $GM(1,1)$ pozwala na budowanie modelu adaptacyjnego, który umożliwi dostosowanie się do zmian w trendzie krzywej życia. Niestety źle dobrana długość okna może powodować zwiększenie się błędu prognozy w porównaniu do warunków, w których długość okna byłaby optymalna. Zastosowany algorytm polega na śledzeniu aktualnego błędu prognozy modeli o różnej długości okna i eliminacji znaczenia tych modeli, które powodują nadmierny wzrost błędu prognozy związany z utratą adekwatności do danych.

Słowa kluczowe: diagnostyka techniczna, prognoza symptomu, modele $GM(1,1)$

1. INTRODUCTION

In machine condition monitoring it is very important to estimate the residual time to breakdown. To make such a prediction one can create an approximative model of a symptom and then extrapolate the values of the symptom outside the observation interval in order to compare them with limit values. Such an approach allows to determine the technical state of an object and to react to the approaching change of its state.

The predictive model itself may have various forms. From among many different prediction methods it is possible to use also grey system models $GM(1,1)$ [1,2,3,4,5], particularly when

measurements of symptom values are not numerous. With the approaching breakdown, the values of symptoms related to state characteristics change, sometimes suddenly, due to the beginning of a period of accelerated wear (turning point of the trend). This may lead to the situation, where the current model becomes inadequate and the predictions resulting from its use are subject to gross errors. Additionally, after the change of the trend, only few measurements important for creating correct prediction of time to breakdown are left. Hence, a good solution is to use $GM(1,1)$ models and a moving window technique [6], which enables the model to adapt to changes in data character. The problem lies in the fact that it is

difficult to predict which window length will turn out to be optimal. Hence the proposal to use a set of GM(1,1) models with various window lengths and making a prediction of symptom values based on many partial predictions [9].

The proposed algorithm is based on tracking the current prediction error for models having various window lengths, and then eliminating the models for which the error of prediction caused by the loss of adequacy of the model to the data increases excessively. As a result of previous tests it has been decided to estimate the prediction values based on weighted average of prediction determined from all the models in the set, and to make weights of the average dependent on the instantaneous value of the error measure *ex post* [9]. In this way, the models which are characterized by high values of prediction errors are eliminated gradually from the prediction process, because their influence on the final result decreases quickly. The symptom value for a given prediction step can be expressed by the following formula:

$$S'(T+t) = \frac{\sum w_i S_i'(T+t)}{\sum w_i} \quad (1)$$

where: $w_i = 1/A_i^2$, A_i - the *ex post* measure of the error, S_i' - local forecasts for model i .

2. PROGNOSIS BASED ON THE THEORY OF GREY SYSTEMS

Grey differential equation of p -th order and the m -th order of excitation GM(p,m), is defined as below [7]:

$$\sum_{i=0}^p a_i \frac{d^{n-i} \mathbf{x}^{(1)}}{dt^{n-i}} = \sum_{j=1}^{m-1} b_j \mathbf{y}_j^{(1)} \quad (2)$$

where: $x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i)$ - state variables of dynamical model, and \mathbf{y}_j independent excitation, a_i , b_j - model coefficients, $x^{(0)}(k)$, $k = 1, 2, \dots, n$ data sequence (symptom observations).

In most cases the first order model of grey system GM(1,1) is used. This is independent of the area of application such as market analysis [3] or diesel engine wear [2].

The GM(1,1) model can be expressed as follows:

$$\frac{d\mathbf{x}^{(1)}(t)}{dt} + a\mathbf{x}^{(1)}(t) = b \quad (3)$$

The solution of differential equation (3) with unit step of t variable can be described as below [7]:

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - b/a] \exp(-ak) + b/a \quad (4)$$

where $\hat{x}^{(1)}$ means the forecast of time series x element.

In order to calculate the unknown b and a coefficients we exchange differential equation (3) into a series of finite difference equations with unit step $t=1$. So we get [7]:

$$x^{(1)}(k+1) - x^{(1)}(k) = -\frac{a}{2} [x^{(1)}(k) + x^{(1)}(k+1)] + b \quad (5)$$

Model coefficients are calculated by solving the system of equations in least square sense [8]:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{Y}, \quad (6)$$

where:

$$z(k) = -\frac{1}{2} (x^{(1)}(k+1) + x^{(1)}(k))$$

$$\mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(n) \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} z(1) & 1 \\ z(2) & 1 \\ \dots & \dots \\ z(n-1) & 1 \end{bmatrix}$$

The final form of the forecasting model is as below [7]:

$$\hat{x}^{(0)}(k+1) = [x^{(0)}(1) - \hat{b}/\hat{a}] \{e^{-\hat{a}k} - e^{-\hat{a}(k-1)}\} \quad (7)$$

where: $k = 2, 3, \dots, n$.

It was found that the best effectiveness of GM(1,1) model one can obtain with so called rolling forecast (moving window), where the parameters of the model are calculated in short window ($n > 3$). The window is sliding (rolling) across the data we have [8][9].

3. FORECASTING USING METHODS BASED ON GM(1,1) MODELS WITH VARIOUS WINDOW LENGTHS

To evaluate the proposed method comparatively a set of models $\{GM(1,1)_0, GM(1,1)_w\}$ was selected, where w is the length from 4 to 8, and $GM(1,1)_0$ is a model, which parameters are estimated based on all available data. The range of variation of w was bounded from above by the number of available diagnostic data, and from below by the number of measurements, which enabled to estimate the parameters of the model GM(1,1). Additionally, the results were compared with the predictions obtained by means of the model GM(1,1) taking all the data into account (without any window - designation $GM(1,1)_0$). A measure of a mean square error was used to evaluate the quality of prediction obtained with the given model [9].

The described methods were applied to diagnostic data obtained from fan mills to pulverize the coal in a heat and power generating plant. These are huge and critical machinery units. The measurements concerned the root mean square value of vibration velocity in the frequency band from 5Hz to 1kHz. In case of the considered machines this value is a good diagnostic symptom, the technical state of an object can be determined based on.

In diagnostic applications crucial are predictions with a horizon enabling to prepare oneself to stop the machine. It was assumed that such a horizon will equal at least five steps ahead. In the work it has been assumed that range of prediction horizon is from 5 to 10 steps. This horizon corresponded to a significant machine exploitation time enabling preparation to the repair.

Due to various characters of trends of the considered life curves the final comparison of methods was made by averaging the obtained prediction errors for all the collected time series. Such a juxtaposition for five life curves and forecasting horizon 5 is shown in Fig. 1. As it can be seen therefrom, the arbitrary adoption of a model $GM(1,1)$ with the given window width or without a window determines the value of the obtained error. The proposed method (denoted WAM – weighted average of prediction method) prevents the excessive increase of the error value in case of inappropriately selected window length [9].

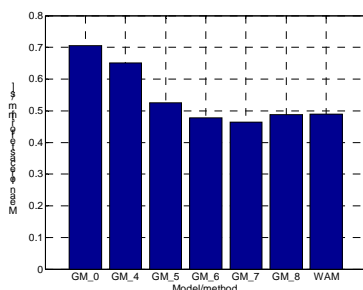


Fig. 1. Comparison of the averaged prediction error for all the considered life curves and prediction horizon 5. In the figure models $GM(1,1)_w$ are denoted briefly GM_w

Similar conclusions can be reached by observing the maximum error of the forecast averaged for all curves of life. This is illustrated in Figure 2.

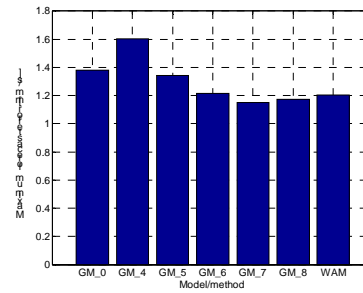


Fig. 2. Comparison of the averaged maximal prediction error for all the considered life curves and prediction horizon 5. In the figure models $GM(1,1)_w$ are denoted briefly GM_w

The next figure shows a comparison of the average forecast error for the horizon of 10 steps. In this case, in an average sense, the method enables to obtain lower errors than “the best” from the compared models.

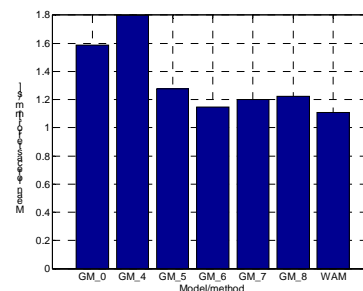


Fig. 3. Comparison of the averaged prediction error for all the considered life curves and prediction horizon 10. In the figure models $GM(1,1)_w$ are denoted briefly GM_w

The proposed method works properly for each forecast horizon, and in particular, shows an advantage over individual models for longer horizons. Summary of results for different forecast horizons and models is shown in Figure 4.

The figure also shows a strong advantage of adaptive models $GM(1,1)_w$ in comparison with $GM(1,1)_0$ model. It manifests itself particularly strongly for long prediction horizons. For the considered diagnostic data it was also found that too short window (length equal to 4) leads to significant forecast errors. This is due to very accurate approximation of small symptom fluctuations by the model.

The proposed method prevents an excessive increase in the prediction error but does not guarantee the minimum error. It seems, however, to sufficiently safeguard against the selection of an inappropriate model in case of automatically generated forecasting.

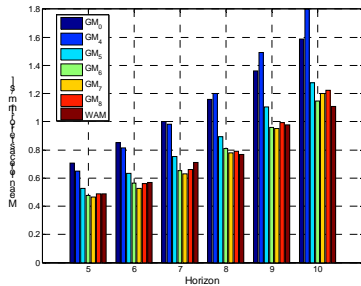


Fig. 4. The averaged prediction error for various horizon of forecasting

4. CONCLUSIONS

It is worth mentioning that in case of many considered life curves with the development of wear of the diagnosed object, and consequently with the higher values of a symptom, its variation increases [10]. As the failure time approaches, the symptom values change, sometimes rapidly, which is due to the onset of the rapid wear-out period. Thus, for purposes of modelling and forecasting values of the symptom adaptive models are needed. Such models are $GM(1,1)_w$, (model $GM(1,1)$ with a moving window of length w), which can be used for a small number of data. The forecast error depends on the chosen length of the window in which the parameters of the model are estimated. The proposed method of building forecasts based on the weighted average takes into account the estimates obtained from all considered models, but with various voting power [9]. This allows to reduce the contribution of the models, which are becoming inadequate for the data. It is of particular importance in case of forecasts with longer horizons.

The proposed, very simple method, gives good results (a little worse than described in [9]) preventing on one hand excessive increase of the prediction error, and on the other hand stabilizing the forecast.

5. ACKNOWLEDGEMENTS

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