An application of the fuzzy set theory and fuzzy logic to the problem of predicting the value of goods rests

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Abstract. Applying the fuzzy set theory and fuzzy logic, we construct a mathematical model for predicting the value of unrealized goods rests and demonstrate this model on an empirical example showing good correlation with the real data.
Key words: fuzzy set, fuzzy logic, linguistic variable, fuzzy knowledge base.

INTRODUCTION

Modern problems of mathematical modeling in economics are related with the presence of incomplete information and the necessity to make economical predictions in the situation of uncertainty. In such situation the apparatus of classical logic is inapplicable [3], [5], [8], [18]. In contrast, such problems can be effectively resolved with help of the fuzzy logic created by Lotfali A. Zadeh in 60-ies of XX century. In today's increasing complex and uncertain business environment, financial analysis is yet more critical to business managers who tackle the problems of an economic or business nature. Knowledge based on formal logic and even experience becomes less sufficient. This volume systematically sets out the basic elements on which to base financial analysis for business in the new century. In dealing with rapid and unpredictable changes in technological and business conditions, it postulates a growing reliance on the opinions of experts instead of past data or probabilistic forecasts, which is a radical change but may yield fruitful results. For this reason, much emphasis is devoted to the problem of aggregation of the opinion of experts in the financial field, with the object of limiting, wherever possible, the subjective component of the opinions and making sure that the decisions have the best guarantee of reaching the desired objectives [10].

Methods of fuzzy logic and fuzzy set theory are widely used in the modern mathematical economics (see [2], [6], [19], [20]). Fuzzy sets and fuzzy logic have been applied virtually in all branches of science, engineering, and socio-economic sciences [1]. The principal notion of the theory is that of a linguistic variable. As defined by L.Zadeh [16] "By a *linguistic variable* we mean a variable whose values are words or sentences in a natural or artificial language. For example, *Age* is a linguistic variable if its values are linguistic rather than numerical, i.e., *young, not young, very young, quite young, old, not very old and not very young*, etc., rather than 20, 21, 22, 23."

In this paper we shall apply the methods of fuzzy logic to predicting the value of rests of preparates for chemical defense of plants and will elaborate an algorithm for predicting the value based on the apriori knowledge of an expert.

PROBLEM STATEMENT

We shall assume that the value y of a goods rest depends on the values of certain variables $x_1, x_2, ..., x_n$, and the dependence will be described by a function:

$$y = f(x_1, x_2, ..., x_n). \qquad (1)$$

In order to apply methods of fuzzy logic, we shall fuzzily all the variables and turn them into the linguistic terms:

$$U_i = \left[\underline{u_i}, \overline{u_i} \right], \quad i = \overline{1, n}, \qquad (2)$$

$$Y = \left[\underline{y}, \overline{y} \right], \qquad (3)$$

where: $\underline{u_i}, \overline{u_i}$ denote the smallest and the largest value of the variable x_i and $\underline{y}, \overline{y}$ the minimal and maximal value of the output variable y.

For calculating the function f in (1) we shall consider the input parameters x_i, $i = \overline{1,n}$ and the output parameter y as linguistic variables defined on the universal sets (2), (3). Qualitative terms for evaluating the linguistic variable $x_1, x_2, ..., x_n$ will be taken from the term-set A_i and for the linguistic variable y from the term-set D. For constructing the term-sets one can apply the method suggested in [9].

For each term $a \in A_i$ from the term-set A_i we define a compatibility (membership) function $m_a : U_i \to [0;1]$ (trapezoid, triangle, etc.. [7, 11, 13] built on the base of expert knowledge. The function m_a describes the measure of compatibility of an element $x_i \in U_i$ with the term a. Analogously, to each term $d \in D$ we assign a compatibility function $m_d : Y \to [0;1]$ describing the measure of compatibility of $y \in Y$ with the term d.

Determining linguistic terms and the corresponding compatibility functions for evaluating input and output variables is the first step (called the fuzzification) in constructing a fuzzy model of the investigated object, [20].

The next step is the creation of a database of fuzzy expert knowledge [9], [17]. Let for the function (1) we know N rules describing the relation between the inputs and outputs.

According to the modeling principle from [15] we suppose that:

$$N < |A_1| \cdot |A_2| \cdot \mathbf{K} \cdot |A_n|,$$

i.e., the quantity of experimental data is less than the total number $|A_1| \cdot |A_2| \cdot \mathbf{K} \cdot |A_n|$ of all possible combinations of terms of input variables. This information can be collected in the *knowledge database,* which is a table containing N rows of length $(n+2)$ each. The k - th row of the table has form:

$$a_{k1}, a_{k2}, ..., a_{kn}, w_k, d_k,$$

where: $a_{ki} \in A_i$, $d_k \in D$ and $w_k \in [0;1]$ is a real number expressing the certainty of an expert about the correspondence of the output term d_k to the input terms $a_{k1}, a_{k2}, ..., a_{kn}$. A typical knowledge table looks as follows.

Based on the knowledge table, for any fixed values $x^* = (x_1^*, x_2^*, ..., x_n^*)$ of the variables $x = (x_1, x_2, ..., x_n)$, we can calculate a compatibility function $l_{x^*}(d) : D \to [0;1]$ by the formula:

$$l_{x^*}(d) = \max_{d_j = d} \left(w_j \cdot \min_{1 \le i \le n} m_{a_{ji}}(x_i^*) \right). \quad (4)$$

Table 1. A typical knowledge table

No	Input variables						Weight	Output variable
	x_1	x_2	…	x_i	…	x_n		
1	a_{11}	a_{12}	…	a_{1i}	…	a_{1n}	w_1	d_1
2	a_{21}	a_{22}	…	a_{2i}	…	a_{2n}	w_2	d_2
…	…	…	…	…	…	…	…	…
N	a_{N1}	a_{N2}	…	a_{Ni}	…	a_{Nn}	w_N	d_N

Here we assume that $\max \varnothing = 0$.

Next, we use the function $l_{x^*}(d)$ to mix the compatibility functions $m_d : Y \to [0;1]$, $d \in D$, to produce the compatibility function $m : Y \to [0;1]$ for the accumulative output variable z:

$$m(z) = \max_{d \in D} \min \left(l_{x^*}(d), m_d(z) \right). \quad (5)$$

To find the predicted value of the output variable y in the interval $[\underline{y}, \overline{y}]$, we can defuzzify the compatibility function $m : Y \to [0;1]$, e.g., by calculation its center of gravity :

$$y^* = \frac{\int_{\min}^{\max} z\, m(z)\, dz}{\int_{\min}^{\max} m(z)\, dz}, \quad (6)$$

where: min and max are the left and right ends of the interval $[\underline{y}, \overline{y}]$ of the support of the fuzzy set of the output variable y.

We apply this scheme to the problem of predicting the value of unrealized goods of certain type of some trade firm selling preparates of chemical defense of plants at the end of some trade season.

An expert established that essential factors that influence on the goods rest Y are: x_1 (*"rest"*) – the rest from the previous trade season (in U.S. dollars); x_2 (*"new purchase"*) – the cost of new purchases (in U.S. dollars); x_3 (*"margin"*) – the average value of the trade margin (in percent); x_4 (*"term"*) – length sale of the product (the presence of the product on the market, in years). Universal set for the described variables defined as follows: $U_1 = [0;600000]$; $U_2 = [20000;1500000]$; $U_3 = [0;50]$; $U_4 = [0;10]$. Universal set for the predicted value coincides, obviously, with $Y = U_1$.

For each input and output variables we built term-sets:

$A_1 = \{"small", "medium", "large", "critical"\} = \{S, M, L, C\}$,

$A_2 = \{"small", "medium", "large"\} = \{S, M, L\}$,

$A_3 = \{"small", "medium", "large"\} = \{S, M, L\}$,

$A_4 = \{"short", "medium", "long - term"\} = \{S, M, L\}$,

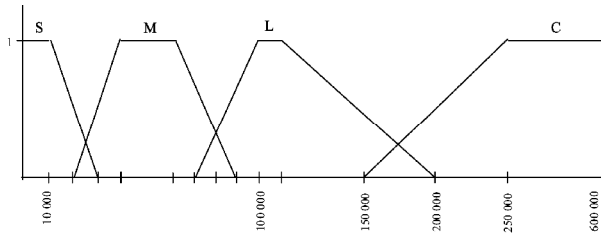$D = \{"small", "medium", "large", "critical"\} = \{S, M, L, C\}$.

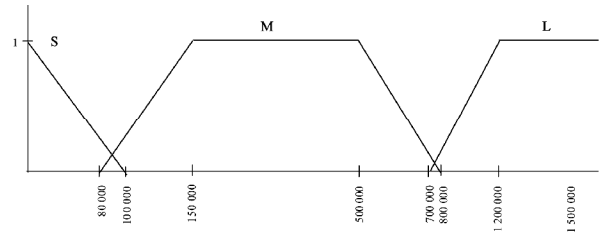**Fig. 1.** The compatibility function of the linguistic variable "rest"

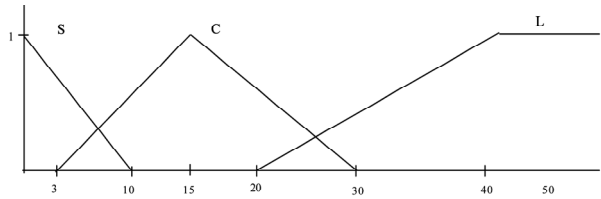**Fig. 3.** The compatibility function of the linguistic variable "new purchase"

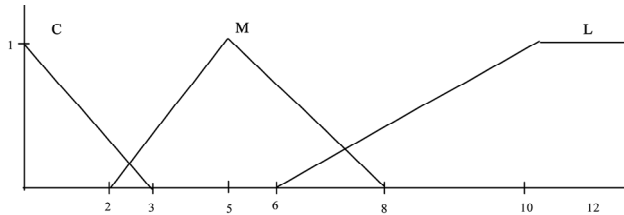**Fig. 2.** The compatibility function of the linguistic variable "margin"

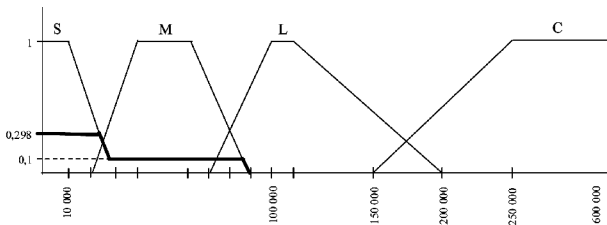**Fig. 4.** The compatibility function of the linguistic variable "term"

Fig. 5. The graph of the the compatibility function for the accumulative output variable

Table 1. The knowledge table

Number of input combinations (logical rules)	Input variable				weight	Output variable
	x_1	x_2	x_3	x_4	w	y
1	S	S	S	S	1	S
2	L	S	S	M	0,9	
3	L	S	S	L	1	
4	C	S	S	M	1	
5	S	S	L	L	0,7	
6	S	M	L	M	0,7	
7	M	S	M	M	0,8	
8	M	S	M	L	0,8	
9	M	M	M	M	0,7	
10	L	S	M	L	0,9	
11	L	S	L	L	0,5	
12	M	S	L	L	1	M
13	S	M	M	M	1	
14	S	L	L	M	0,8	
15	M	M	S	M	0,8	
16	M	L	S	L	0,7	
17	M	L	M	M	0,7	
18	L	M	M	M	0,7	L
19	L	M	M	L	0,5	
20	L	M	L	L	0,3	
21	C	M	L	L	0,8	
22	S	L	M	M	0,9	
23	S	L	M	S	1	
24	M	L	S	S	1	
25	M	L	M	L	0,8	
26	S	L	L	M	0,8	C
27	S	L	L	L	0,9	
28	M	L	L	M	0,9	
29	M	L	L	L	1	
30	L	L	L	L	1	

Based on the expert's opinion, we built the following compatibility functions for the terms of the input and output variables.

The next step is defining the knowledge table.

The following calculations are carried out for these commercial enterprises Season Sales 2013: $x_1^* = 80000$; $x_2^* = 36000$, $x_3^* = 22$, $x_4^* = 9$. In this case, the variable x_1^* refers to the terms "*medium*" (with compatibility function, $m(x) = 1 - \frac{1}{30000}(x - 60000)$, so $m(x_1^*) \approx 0,33$ or "*large*" (with compatibility function $m(x) = \frac{1}{30000}(x - 70000)$, so $m(x_1^*) \approx 0,33$; x_2^* refers to the term "small" (with compatibility function $m(x) = 1 - \frac{1}{100000}x$, so $m(x_2^*) = 0,64$); x_3^* - to the terms " *medium* " (with compatibility function $m(x) = 1 - \frac{1}{15}(x - 15)$, so $m(x_3^*) \approx 0,53$) or "*large*" (with compatibility function $m(x) = 1 - \frac{1}{20}(x - 20)$, so $m(x_3^*) \approx 0,1$); x_4^* – to the term "long-term" (with compatibility function $m(x) = \frac{1}{4}(x - 6)$, so $m(x_4^*) \approx 0,75$). Calculating $w_j \cdot \min_{1 \le i \le n} m_{a_{ji}}\left(x_i^*\right)$ one can see that nonzero values occur only in the rows numbered by 8,10,11 (which corresponding to term S) and 12 (corresponding to term M).

Calculating the values $l_{x^*}(d)$ for: $d \in D = \{S, M, L, C\}$ using (4),

$$l_{x^*}(S) = \max\{0,8\min\{0,33;0,64;0,53;0,75\},$$
$$0,9\min\{0,33;0,64;0,53;0,75\},$$
$$0,5\min\{0,33;0,64;0,1;0,75\}\} =$$
$$= \max\{0,264;0,3;0,05\} = 0,3,$$
$$l_{x^*}(M) = \max\{\min\{0,33;0,64;0,1;0,75\}\} = 0,1,$$
$$l_{x^*}(L) = 0; \quad l_{x^*}(C) = 0.$$

The compatibility function (5) :

$$m(z) = \max_{d \in D} \min\left(l_{x^*}(d), m_d(z)\right),$$

has the following form:

$$m(z) = \begin{cases} 0,3 , & \text{if } w \in (0;24000), \\ 1 - \frac{1}{20000}(z - 10000), & \text{if } z \in (24000;28000), \\ 0,1 , & \text{if } z \in (28000;87000), \\ 1 - \frac{1}{30000}(z - 60000), & \text{if } z \in (87000;90000). \end{cases}$$

The graph of the function $m(z)$ looks like (Fig. 5).

The precise value of the output variable y^* can be found as the result of defuzzification of $m(z)$ calculating its gravity center (6):

$$y^* = \frac{\int_{Min}^{Max} z\,m(z)dz}{\int_{Min}^{Max} m(z)dz},$$

$$\int_{Min}^{Max} z\,m(z)dz = \int_0^{24000} 0,3z\,dz +$$
$$+ \int_{24000}^{28000} z\left(1 - \frac{1}{20000}(z - 10000)\right)dz +$$
$$+ \int_{28000}^{87000} 0,1z\,dz + \int_{87000}^{90000} z\left(1 - \frac{1}{30000}(z - 60000)\right)dz =$$
$$= 353865180,$$

$$\int_{Min}^{Max} m(z)dz = \int_0^{24000} 0,3\,dz + \int_{24000}^{28000}\left(1 - \frac{1}{20000}(z - 10000)\right)dz +$$
$$+ \int_{28000}^{87000} 0,1\,dz + \int_{87000}^{90000}\left(1 - \frac{1}{30000}(z - 60000)\right)dz = 13588,5.$$

Finally, the predicted value of the goods rest equals $y^* = \frac{353865180}{13588,5} = 26041,5$, which is sufficiently close to the real rest of the goods, equal to 25200 dollars.

REMARK

The proposed method can be improved by modifying the weight coefficient in knowledge table (based on real observations) and adding new input variable and new dependences. However, for a large number of variables constructing the knowledge table become a difficult task because of the known psychological bound of human brain to keep at most 7 ± 2 notions simultaneously. In this case it is reasonable to organize the input variables into an embedded tree structure [13].

CONCLUSIONS

In the paper we suggested a mathematical model for predicting the value of goods rests based on fuzzy logic approach. Using the expert's knowledge table we constructed a compatibility function whose defuzzification yielded the value of the output parameters (equal to the predicted value of the goods rest)

This model is more open and clear then multifactor discriminant models because is based on the expressions in a natural language.

Using rules to make decisions in the fuzzy logic models allows to take into account expert's knowledge to avoid incorrect classification [4], [12],[16].

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