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THE IDENTIFICATION OF PARAMETERS OF A LINEAR AND A NON-LINEAR MODEL OF A KINEMATIC MEASUREMENT-CONTROL NETWORK

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Abstract. Engineering geodesy deals with a wide range of problems. There is also a part that deals with measuring displacements and deformations of engineering objects. Correct geodetic monitoring requires identifying the movement of points representing an engineering object in order to determine displacement values, taking into account the time function.
The paper presents the results of research on kinematic models of geodetic networks in the aspect of using them for describing the state of vertical displacements of engineering objects located on expansive soil. The paper presents two functional models of an observation system: one in the form of a second rank polynomial and the other in the form of an exponential function. The selected kinematic models of measurement-control geodetic networks were estimated with classic methods and neural networks.

Key words: kinematic model of a geodetic network, vertical displacements, neural networks

INTRODUCTION

Results of geodetic measurements play an especially important role in the analysis of the influence of expansive soils on engineering objects. Correct geodetic measurements provide data describing a behaviour of engineering objects undergoing uneven settlement. Geodetic monitoring consisting of measurements and their interpretation makes it possible to draw specific conclusions about a dynamism of changes occurring in engineering objects. The basic symptom of unfavourable phenomena in engineering objects located on expansive soil are vertical displacements of points of the measurement-control network representing a particular object. The determination of a geometric qualitative

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displacement model consists in identifying a set of mutually fixed points in which the reference system is defined [Prószyński and Kwaśniak 2006]. In recent years an importance of the use of kinematic models of geodetic networks for determining displacements has increased because of the development of measurement technologies and algorithms for processing experimental data.

This paper presents the results of research on kinematic models of geodetic networks used for describing the state of displacements of a building located on expansive soil. The measurements were taken by means of precise levelling in one year at one-month intervals, which comprised a complete vegetation cycle of the trees surrounding the building. Each periodical measurement provided information about the state of stability of the object in the form of 22 observations during which the structure of the measurement-control network was the same. Kinematic models in the form of a second rank polynomial and an exponential function were suggested as functional models of the observation system. Parameters of the kinematic models were estimated with classic methods and a circular neural network.

ESTIMATION OF PARAMETERS OF KINEMATIC NETWORK MODELS WITH CLASSIC METHODS

The functional model of the kinematic network used for the analysis of deformations of the object will be written in general form [Kadaj 1998]:

$$A\big(X(t)\big) = L(t) \tag{1}$$

where: $A\big(X(t)\big)$ – the vector function dependent on the network structure in the form of the matrix of the system of correction equations, $X(t) = [\alpha_i] \in R^n$ – the vector of parameters, $L(t) = \left[l_j(t)\right] \in R^m$ – the vector of observations, t – the real variable (time).

The best results in the analysis of displacements are usually obtained when the differencing method invented by Lazzarini is used [Lazzarini 1961]. This method eliminates considerably the influence of systematic errors because it uses differences between observation results, which in the case of a correctly defined reference system are transformed into displacement values. During the preparation of vertical kinematic networks changes between differences are replaced with time functions. For this reason a kinematic network is an expansion of the classic differencing method. The procedure of calculating this kind of network consists of two stages: the equalization of observations and the approximation of functions.

Having a discrete set of observations in the form of height differences, we used two functional models describing the movement of points:

$$H_1(t) = \alpha_1 + \alpha_2 t + \alpha_3 t^2 \tag{2}$$

$$H_2(t) = \alpha_1 \exp(-\alpha_0 t) + \alpha_2 \exp(-\alpha_3 t), \quad (\alpha_0 = 0). \tag{3}$$

The model of a kinematic network (2) in the form of a second rank algebraic polynomial is a linear model in relation to the parameters $\alpha_i\,(i=1,\ 2,3)$, and the model (3) is a non-linear model only in relation to the parameter α_3. However, if we assume that $\alpha_3 = \text{const}$, then the model (3) is transformed into a linear model in relation to the parameters α_1 and α_2. This course of action is justified in the numerical realisation of the process of estimation of model parameters (change of the dimension of the minimisation problem).

For a set points (i,j), for which the height differences $\Delta h_{ij}(t)$ have been determined, the components of the function $A(X(t))$ which express changes of the height differences between the points (i,j) in the time t, will be written in the form:

$$A_{ij}\left(\alpha_{ij},t\right) = h_j\left(\alpha_j,t\right) - h_i\left(\alpha_i,t\right), \tag{4}$$

and the satisfaction of the assumption of a minimum sum of the square of corrections to the observations will be written as:

$$V\left(X(t)\right)^T = \sum\left[a_{ij}\left(\alpha_{ij},t\right) - \Delta h_{ij}\right]^2. \tag{5}$$

The objective function (energy function) E is defined in the form:

$$E = V\left(X(t)\right)^T V\left(X(t)\right), \tag{6}$$

which can be minimised by equating its gradient to zero. In this way we obtain normal equations in vector form.

In the case of the model (2) of a kinematic network, which is a linear model in relation to the vector of parameters $X(t)$, the set of observation equations will be written in the form:

$$F(X(\alpha,t)) = \Delta \text{h}(\alpha,t) + V\left(X(t)\right). \tag{7}$$

Since the equations which belong to the system (7) are non-linear, it is necessary to linearize them. This consists in the expansion of the function F into the Taylor sequence in the close vicinity of the point $X_0(\alpha,t)$, excluding the expressions of the second rank and higher. Then, the linearized system (7) will be written:

$$F(X(\alpha,t)) = F(X_0(\alpha,t)) + A(X_0(\alpha,t))\Delta X_0(\alpha,t) = \Delta \text{h}(\alpha,t) + V\left(X(t)\right), \tag{8}$$

where:

$$A(X_0(\alpha,t)) = \frac{\partial F(X_0(\alpha,t))}{\partial X_0(\alpha,t)^T} \tag{9}$$

$$\Delta h(\alpha,\ t) = h(\alpha,\ t) - h_0$$

$$h_0 = F\left(X_0(\alpha,\ t)\right)$$

The linearized system of correction equations will be written in concise form as:

$$A\big(X(t)\big)\Delta X-\Delta \mathrm{h}(\alpha,\ t)=V\big(X(t)\big). \tag{10}$$

We define the criterion of the least squares by applying the condition to the vector of corrections in the form:

$$V\big(X(t)\big)^{T}V\big(X(t)\big)=\big[A\big(X(t)\big)\Delta X-\Delta \mathrm{h}(\alpha,\ t)\big]^{2}=\min, \tag{11}$$

and we search for a vector of parameters that minimises the abovementioned criterion which results from the solution of the system of normal equations:

$$A\big(X(t)\big)^{T}A\big(X(t)\big)\Delta X=A\big(X(t)\big)^{T}\Delta \mathrm{h}(\alpha,\ t). \tag{12}$$

THE LEAST SQUARE METHOD IN THE NON-LINEAR ASPECT

When parameters of the model of a kinematic network described by the dependence (2) are estimated with the use of the least square method in the non-linear aspect, the method of formulating observation equations remains unchanged (7), as well as the basic assumption of the least square method, i.e.

$$\big[F(X(\alpha,t))-\Delta \mathrm{h}(\alpha,t)\big]^{2}=V\big(X(t)\big)^{T}V\big(X(t)\big)=\min. \tag{13}$$

The definition of the objective function (6), which will be minimised by solving the normal equations written in the form $\nabla E=\frac{\partial E}{\partial X}=0$ also remains unchanged. The function ∇E can be minimised by means of one of the methods of linear algebra (the Jacoby method, the Gauss-Seidel method or Gaussian elimination). The basic algorithm of the non-linear least square method is the Cartesian Descent method, which is based on the algorithm of the generalised Seidel process [Adamczewski 2002].

As has been mentioned before, the model in the exponential form (3) is a non-linear model in relation to the parameter α_3. For this reason, the change of height ΔH of the i-th point can be written in the form of the general dependence:

$$\Delta H=\Delta H\big(X(t)\big). \tag{14}$$

At this point it is necessary to mention that the components of the vector α_1 and α_2 represent the values of the displacements of the point, and the component α_3 represents its relative speed.

Taking into account the time t, we will write the change of height of the i-th point as:

$$\Delta H\big(X\ (t)\big)=\alpha_1^i\varphi_1^i\big(\alpha_0^i,\ t\big)+\alpha_2^i\varphi_2^i\big(\alpha_3^i,\ t\big)_{,}\ \ (i=1,2,\ldots,n) \tag{15}$$

where: $\varphi_1^i\big(\alpha_0^i,\ t\big)=1,\ \big(\alpha_0=0\big),\ \ \varphi_2^i\big(\alpha_3^i,\ t\big)=\exp\big(-\alpha_3^i t\big)$. When the change of height differences is defined in this way, the component of the vector of the non-linear functions $A\big(X(t)\big)$, which expresses the height differences between the points $\big(i,\ j\big)$ in the time, t takes the form:

$$A^{ij}\left(\alpha_1^i,\,\alpha_2^i,\,\alpha_3^i,\,t,\,\alpha_1^j,\,\alpha_2^j,\,\alpha_3^j,\,t\right)=H^j\left(\alpha_1^j,\,\alpha_2^j,\,\alpha_3^j,\,t\right)-H^i\left(\alpha_1^i,\,\alpha_2^i,\,\alpha_3^i,\,t\right). \quad (16)$$

Assuming that $\alpha_3 = \text{const}$, we minimise the objective function (13) in relation to the parameters α_1 and α_2 with the linear algebra method, i.e. $\acute{a} = \alpha_{1,2}\left(\alpha_3\right)$. We designate the value of the local minimum obtained in this way as $\Omega\left(\alpha_3\right)$, then we will write the form of the functional as:

$$\Omega\left(\alpha_3\right)=F\left[\acute{a},\,\alpha_3\right]. \quad (17)$$

The non-linear estimator of the vector of parameters $\hat{\acute{a}}_3$ will be determined with the algorithm of the steepest descent, which is obtained by restricting the expansion of the function $\Omega\left(\alpha_3\right)=F\left(\acute{a},\,\alpha_3\right)$ to linear approximation in any close vicinity of the known solution $\left(\alpha_3 \in R^{2n}\right)$, and on condition that the iteration process is convergent.

ESTIMATION OF PARAMETERS OF KINEMATIC MODELS WITH ARTIFICIAL NEURAL NETWORKS

Because of their non-linear character, artificial neural networks represent a sophisticated modelling technique and are regarded as *Computational Intelligence* methods. The functions for which a network can be used make it possible to obtain good results in practical applications such as: approximation, interpolation, recognition and classification of standards, compression, prediction and many others. In these and other applications, the neural network is a universal approximation system, which can realise a multi-variable non-linear function in the form:

$$y = f(X) \quad (18)$$

where X denotes the input vector, and y the vector function realised in the whole indefiniteness set.

Solving systems of linear equations is the basic problem in a number of fields of science and technology. The model of the kinematic network under discussion is a linear model in relation to the vector of parameters $X(t)$ in the form:

$$A\left(X(t)\right)=L(t) \quad (19)$$

where: $A\left(X(t)\right)\in R^{m,n}$ – the matrix of coefficients in the configuration of the system of corrections $(m>n)$, $L(t)\in R^m$ – the vector of observations. Neural networks estimate model parameters by minimising the energy function E defined as [Gil 2006]:

$$E(X)=\sum_{i=1}^{m}\omega\left[v_i\left(X(t)\right)\right], \quad (20)$$

where the function $\omega\left[v_i\left(X(t)\right)\right]$ represents a convex function in relation to the vector of parameters $X(t)$ and is called the weight function. Its derivative in relation to the correction $v_i\left(X(t)\right)$ is called the activation function.

Gauss-Markov models (linear models) with a specific redundancy are estimated with the least square method with the assumption that observation errors are in normal distribution. In the case of over-determined systems of linear equations, we do not expect the equation (20) to be satisfied, but we want the square of the norm of the vector of corrections:

$$V\left(X(t)\right)^T V\left(X(t)\right) = \left(A\left(X\ (t)\right)X(t) - L\right)^T \left(A\left(X(t)\right)X(t) - L\right) \tag{21}$$

to satisfy the Gaussian stipulation (Cf. (13)).

It is possible to solve the problem of the minimisation of the objective function (21) with gradient methods. If we use the steepest descent method, the solution of the optimisation problem boils down to solving a system of differential equations, written in a matrix form as [Osowski 2006]:

$$\frac{dX}{dt} = -\eta \nabla E(X) = -\eta A^T (AX - L), \tag{22}$$

where η denotes the training coefficient of the neural network, and the gradient of the objective function is $\nabla E(X)$:

$$\nabla E(X) = \left[\frac{\partial E}{\partial X_1}, \frac{\partial E}{\partial X_2}, \ldots, \frac{\partial E}{\partial X_n}\right]^T = A^T (AX - L)\,. \tag{23}$$

The scalar form of the equation (22) is defined as follows:

$$\frac{dX_j}{dt} = -\sum_{p=1}^{n} \eta \left[\sum_{i=1}^{m} a_{ip} \left(\sum_{k=1}^{n} a_{ik} X_k - L_i\right)\right]. \tag{24}$$

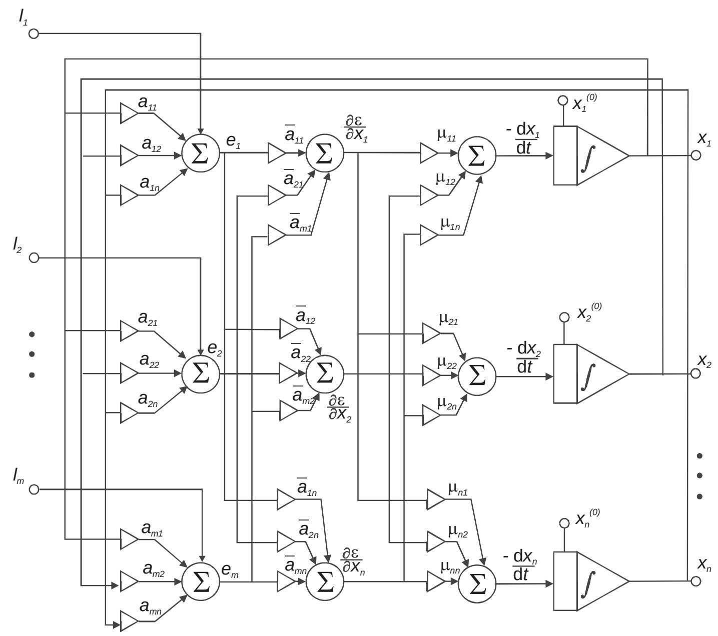


Fig. 1. The architecture of the network for solving systems of linear equations
Rys. 1. Architektura sieci do rozwiązywania układów równań liniowych

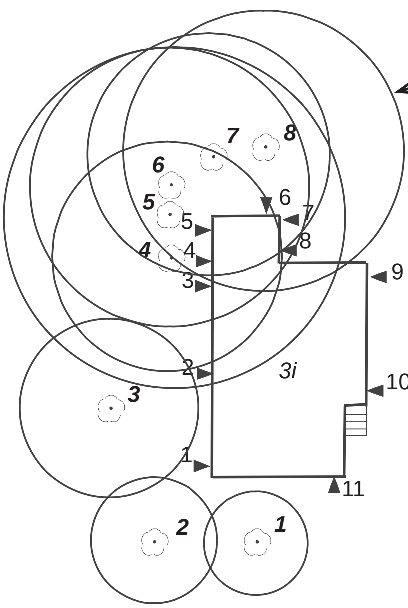
The use of the steepest descent method makes it possible to construct the simplest neural network structure (Fig. 1) that solves the system of linear equations. In theory the steepest descent method is convergent. However, in practice it is characterised by slow convergence, especially in any close vicinity of the estimator $\widehat{X}$ of the vector of parameters X.

A circular neural network was used for solving the systems of over-determined linear equations. Its architecture is shown in Figure 1.

NUMERICAL EXAMPLE

A dynamism of the phenomenon of uneven settlement of an engineering object, caused by changes in the hydrotechnical regime was observed on the basis of the values of changes of height differences between points of a measurement-control geodetic network. The scope of the measurements comprised a building located on expansive soil, which was represented by 11 controlled points fixed within the foundations of the building. 14 periodical measurements were taken at one-month intervals. The data obtained in this way were complemented with information about the range of the influence of the zone of tall trees (lindens, oaks), which cause changes in soil moisture in the process of transpiration. For the purpose of solving practical problems, it can be assumed that the zone of influence of a single tree on soil moisture looks like an upside-down cone with an almost circular base, and the radius of the circle is about 1.5 of the tree height [Collective work edited by Przystański 1991].

A schematic of the location of the measurement points on the building, and the location, species and height of the freely growing trees near the building, as well as their zone of influence are shown in Figure 2.



The number of trees Numer drzewa	Tree species Gatunek drzewa	Tree height [m] Wysokość drzewa
1	linden – lipa	6.50
2	linden – lipa	7.90
3	oak – dąb	11.20
4	linden – lipa	14.40
5	linden – lipa	21.40
6	linden – lipa	17.50
7	linden – lipa	15.20
8	oak – dąb	17.60

Fig. 2. A schematic of the location of points of the measurement-control network and the trees

Rys. 2. Szkic lokalizacji punktów sieci pomiarowo-kontrolnej wraz z zestawieniem drzew

In the research of displacements an important role is played by methods of identifying a reference system. The choice of methods can lead to quite different conclusions. If firm conclusions are to be drawn while solving practical tasks on the basis of a random sample with a small number of elements, technological correctness and accuracy are absolutely necessary. In practice, when the observation system is oversized, it is possible to evaluate the quality of the product, which is a geodetic network, on the basis of the parameter χ^2. This parameter represents a global measure of the quality of the geodetic network in the form of the sum of the squares of corrections to the observations, obtained by equalisation with minimum constraints on the degrees of freedom.

The displacement model of the controlled points was made on the basis of a defined, insignificantly elastic own reference system in a two-point set (points 10 and 11) beyond the range of the influence of changes in soil moisture. The reference system was defined in two stages. The first stage, regarded as preliminary, was carried out with the method of object adjacency, including the shortest distance procedure. In the second stage, the basic criterion for the determination of the reference system is the square form [*VV*], which cannot exceed the critical value for a set of points which are known to be mutually absolutely fixed. In a system defined with the method of the critical value of the increment of the square of the norm of the vector of corrections [Gil 1995], the occurrence of a type II error is less probable, so this kind of system is more reliable.

The final displacement values shown in Figure 3 were obtained by solving a system of linear equations using the least square operator with the assumption that the points are fixed. The displacement values in the form of the second rank polynomial (2), obtained from the model with the least square method, are shown in Figure 4.

In tasks of estimation of parameters which characterise the state of deformation of a geodetic network there is a linear and a non linear tendency in relation to time. They are written in the form of a mathematical model. Accuracy, precision and reliability are required from the calculation procedure. As results from the content of this paper, the estimation of model parameters as a concept for solving the kinematics of components of a geodetic network was carried out as follows:

a. linear models:
   - the least square method; the estimator obtained, called the Markov estimator, is an unbiased estimator with a minimum variance,
   - with the use of unidirectional recurrent neural networks; the square objective function is the Lapunov function, and the solution of the task of minimisation of the criterion function [*VV*] is asymptotically stable,
b. the non-linear method (the hybrid method [Gibowski 2009]):
   - alternate estimation of the decision variables which linearly (the linear algebra method) and non-linearly (the gradient method) influence the value of the estimator $\widehat{X}$ of the vector of parameters X.

The abovementioned methods of minimisation of the criterion function are not mutually competitive because the mean errors of single observations before and after equalisation (Table 1 – the influence of the bias of the estimators was disregarded) are not very different.

A typical representation of displacements of controlled points for a linear functional model of a geodetic network in the form of a second rank polynomial is shown in Figure 4. The representation of displacements obtained for the other functional network models was almost identical.

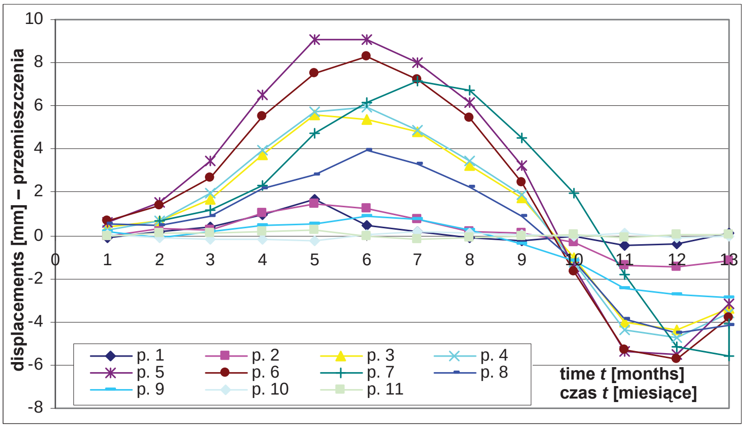


Fig. 3. Displacements of points from equalisation with the least square method with the assumption that the points are fixed

Rys. 3. Przemieszczenia punktów z wyrównania metodą najmniejszych kwadratów z warunkiem na układ odniesienia

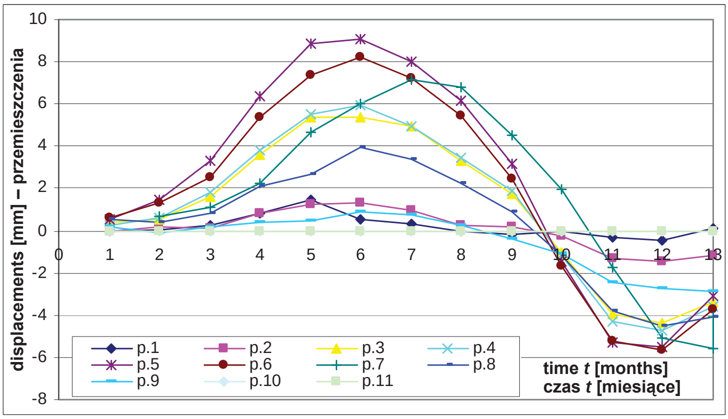


Fig. 4. A diagram of displacements of points for the model (2) – the least square method

Rys. 4. Wykres przemieszczeń punktów dla modelu (2) – metoda najmniejszych kwadratów

Table 1. The mean errors of a single observation dependent on the method used for preparing kinematic networks models

Tabela 1. Błędy średnie pojedynczego spostrzeżenia w zależności od metody opracowania modeli sieci kinematycznych

Network model Model sieci	Method used for preparing Metoda opracowania	The mean errors m_0 [mm] Błąd średni m_0
static network sieć statyczna	the least square method metoda najmniejszych kwadratów	0.21
second rank polynomial $H_1(t) = \alpha_1 + \alpha_2 t + \alpha_3 t^2$ wielomian drugiego stopnia	the least square method metoda najmniejszych kwadratów	0.21
	artificial neural networks sztuczna sieć neuronowa	0.22
exponential function $H_2(t) = \alpha_1 + \alpha_2 \exp(-\alpha_3 t)$ funkcja wykładnicza	the hybrid method – metoda hybrydowa	0.34
	artificial neural networks sztuczna sieć neuronowa	0.36

CONCLUSIONS

The article presents methods of calculating kinematic network models in the form of a second rank polynomial and an exponential function. The parameters of the models were estimated with the least square method, the hybrid method and a method based on artificial circular neural networks. It can be said that the displacements obtained from the kinematic network models in the form of a second rank polynomial with the least square method and the displacements obtained from a static network are identical. The displacements obtained from the models in the form of a second rank polynomial with a neural network and the displacements obtained with the least square method differ by 0.01–0.17 mm. In the case of the functional model of a kinematic network in the form of an exponential function, the differences between the results obtained with a neural network and the hybrid method are slightly greater and amount to 0.32 mm. In general, it can be said that the kinematic model in the form of a second rank polynomial very well illustrates the state of the displacements of points in the measurement-control network under analysis, which was placed on an object located on expansive soil.

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IDENTYFIKACJA PARAMETRÓW MODELU LINIOWEGO I NIELINIOWEGO SIECI KINEMATYCZNEJ POMIAROWO-KONTROLNEJ

Streszczenie. Rozpatrując szeroki zakres zagadnień związanych z geodezją inżynieryjną, można wyróżnić część zajmującą się pomiarami przemieszczeń i odkształceń obiektów budowlanych. Poprawnie prowadzony monitoring geodezyjny wymaga identyfikacji ruchu punktów reprezentujących badany obiekt budowlany, w celu określenia wartości przemieszczeń z uwzględnieniem funkcji czasu.
W artykule zostały przedstawione wyniki opracowań modeli kinematycznych sieci geodezyjnych w aspekcie ich zastosowania do opisu stanu przemieszczeń pionowych obiektu budowlanego posadowionego na gruntach ekspansywnych. W pracy zaprezentowano dwa modele funkcjonalne układu obserwacyjnego w postaci wielomianu drugiego stopnia oraz funkcji wykładniczej. Estymację wybranych modeli kinematycznych sieci geodezyjnych pomiarowo-kontrolnych wykonano z zastosowaniem metod klasycznych oraz z wykorzystaniem sieci neuronowych.

Słowa kluczowe: model kinematyczny sieci geodezyjnych, przemieszczenia pionowe, sieci neuronowe

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