Archives of
Transport System
Telematics

Volume 11

Issue 4

November 2018

Mathematical and Simulation Modelling of Normal Force for a Rail

J. KISILOWSKI[a], R. KOWALIK[b]
[a] UNIVERSITY OF TECHNOLOGY AND HUMANITES IN RADOM, Faculty of Transport, Malczewskiego 29, 26-600 Radom, Poland
[b] POLISH AIR FORCE ACADEMY, Faculty of Aviation Division, Dywizjonu 303 nr 35, 08-521 Deblin, Poland
EMAIL: jkisilow@kisilowscy.waw.pl

ABSTRACT
Description of dynamics occurring during the passage of a rail vehicle is a complex problem, and their mathematical relations are considered in several different aspects such as: interaction of normal force on a beam with fixed and variable cross-section, body, trolley and wheelset displacement and consideration of wheel-rail contact and other phenomena occurring in the contact surface, which include adhesion and creepage. The article presents the effects of work on the issues of normal force at the contact point of the wheel and rail. A mathematical model describing the dynamics of a rail vehicle was defined, determining the normal, transverse and longitudinal force appearing in the wheel-rail contact zone. Variations in the value of the normal force exerted by the wheelset on the rail are the quantity on which the parameters occurring in contact between the wheel and the rail depend. The obtained results from computer simulations show that the normal force is a variable parameter depending on the sign and magnitude of forces occurring between individual blocks of the rail vehicle during the movement. The results obtained will be used in further scientific work devoted to the contact problems.

KEYWORDS: rail vehicle dynamics, normal force, transport, railways

1. Introduction

Modeling of dynamic phenomena occurring during the motion of rail vehicle on the track is a very difficult task due to the complexity of mathematical equations describing the dynamics of a complete rail vehicle and the complicated shape of rolling profiles of wheels and rails should be taken into account. There are several packages that give you the ability to calculate forces occurring in the contact surface, non-linear contact stresses such as MSC MARC, ANSYS, MSC ADAMS and others. However, this program requires that during the construction of the model, give it initial conditions (in an unloaded state) for which the wheel and rail are in contact. Finding this point or several points of contact between the cross-sections of the wheel and rail is difficult due to the geometry of this system, if for new profiles this task can be done in CAD programs, this is a problem for the used ones [1, 2, 3].

The next problem in modeling is the task of forces acting on the system. In case of one load, this is a simple matter, it is modeled by setting the center of the rail at the point falling on the center of the wheel rolling circle, while in the case of two forces, i.e. vertical and lateral, the matter is more complicated, because depending on the lateral force acting on the wheel changes location of the contact zone so that it is impossible to include this phenomenon in CAD programs. The article presents the variation of normal force. In many papers, e.g. [1], it was a question of determining various quantities describing not only the movement of the whole vehicle, but also its components (body, bogies, wheelsets). These considerations are carried out using analytical methods of deriving the equations of motion for models with a large number of degrees of freedom using specialized computer programs. The dynamic phenomena are then simulated for the cooperation of the wheel (wheel set with rail).

Determining the working conditions that they are subject to while driving is a difficult issue, because during movement they adopt disturbances from the track, both in the vertical direction and in the transverse direction. Wheelsets are exposed to variable forces occurring in two planes: vertical and transversal. This is an important element due to the need to know the forces that cause contact between the wheel and the rail. In many works, normal force is assumed as a constant resulting from the weight of the wagon. In the process of operation, wheelsets and mainly their rolling profiles are subject to abrasive wear. The main cause of this wear is the stresses occurring in the contact zone of the wheel and rail and the accompanying plastic deformations. The Universal Mechanism (UM) program was used to investigate the dynamics of the wheelset of a wagon in this work. In the UM program for the locomotive model, normal forces acting on the wheel during the passage of the rail vehicle on the straight track were determined. Based on the proposed methodology, contact stresses and plastic deformations in the contact zone for the different position of the wheel relative to the rail and for the designated normal forces were determined [4, 5].

2. Mathematical model of a rail vehicle

In mechanical models describing the vehicle's interaction with the track, the vehicle elements are assumed as rigid solids: the body of a passenger or goods wagon, a bogie and a wheel set, which are characterized by masses and moments of inertia. All the elements mentioned above, considering the susceptible elements (connecting these solids), will reflect the real behavior of the rail vehicle moving on the railway track. Compliance between solids in a rail vehicle is generally accepted as linear elements.

Assumptions which should be followed when creating a mathematical model can be presented in a block diagram (Figure 1), and its implementation requires the following assumptions [16]:

- Normal force occurring on the bus will be a variable value and will be determined from the previous step of mathematical calculations carried out for specific train parameters (spacing of wheelsets and bogies).
- The railway track was modeled as the Euler-Bernoulli beam, on which the turning of the wheel with the speed v takes place (the motion on the track and turnout was considered).
- The contact between the rolling faces of the wheels and the heads of the rails is defined on the basis of the Kalkera linear theory (determining ellipsis with axes a and b).
- In the wheel-rail contact area, the Coulomb kinetic sliding friction is taken into account with constant coefficient of friction.
- In the dynamics of vehicle motion along the track, such phenomena as: adhesion, creepage and material wear of the wheel and rails were also taken into account.
- In the considered model, the possibility of occurrence of two contact ellipses occurring as a result of two-point rolling of the wheel on the rail was taken into account.
- The rail vehicle will consist of the following rigid solids: body, two bogies, four wheelsets (Figure 2).
- Variables appearing in the description of the dynamics of a moving object with the index *p* refer to the body, the variables of the bogie are marked with the index *w*, the letter *z* denotes the variables of the wheelset.
- Suspension elements of the first second stage were assumed as linear for all adopted coordinates [6, 8].

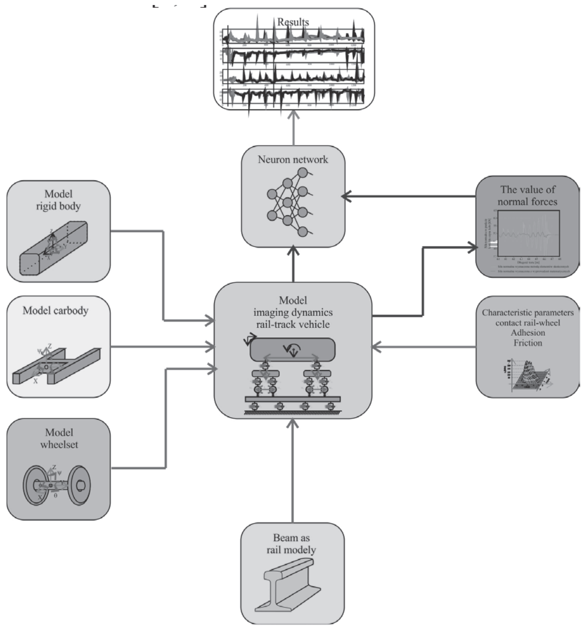

Fig. 1. Block diagram of generating a dynamic track vehicle-rail model [own study]

The body together with the bogie defines five degrees of freedom (DOF); displacements of the body and bogie in the direction of the y and z axes, marked as ; and the rotation of mentioned components around all three axes (x, y, z) is defined by [7]:

$$\begin{aligned} &\varphi_{w,x}, \varphi_{w,y}, \varphi_{w,z} \\ &\varphi_{p,x}, \varphi_{p,y}, \varphi_{p,z} \end{aligned} \qquad \textbf{(1)}$$

Each wheel set is described by three degrees of freedom, displacements in all three directions are marked by.

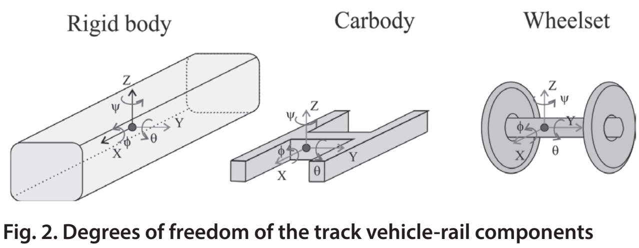

Fig. 2. Degrees of freedom of the track vehicle-rail components [own study]

Equations of constraints, which caused a decrease in the number of degrees of freedom, can be found in paper [1].

Based on the above assumptions, the nominal model has been developed shown in (Fig. 3,4,5).

© Copyright by PSTT , All rights reserved. 2018

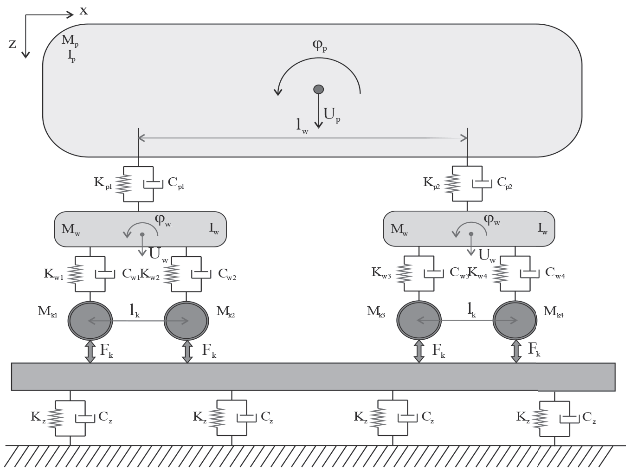

Fig. 3. Nominal model of track vehicle-rail (view in the longitudinal direction)

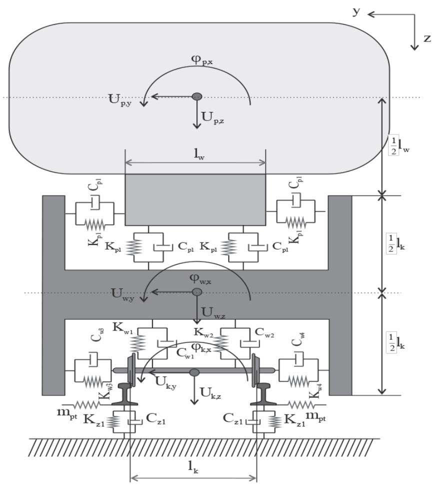

Fig. 4. Nominal model of track vehicle-rail (view in transverse direction)

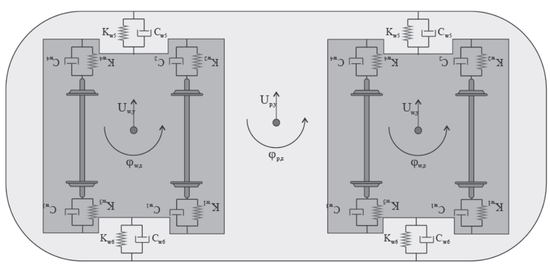

Fig. 5. Nominal model of track vehicle-rail (bottom view)

The equations of motion were derived using the Lagrange equations of the second type.

$$M_w \frac{\partial^2 u_w}{\partial t^2} + 2C_w \frac{\partial u_w}{\partial t} + 2K_w u(t) - C_w \frac{\partial u_{wz1}}{\partial t} - K_w u_{wz1}(t) - C_w \frac{\partial u_{wz2}}{\partial t} - K_w u_{wz2}(t) = 0 \quad \textbf{(2)}$$

$$J_w \frac{\partial^2 \varphi_w}{\partial t^2} + 2C_w L_c^2 \frac{\partial \varphi_w}{\partial t} + 2K_w L_c^2 \varphi_w(t) - C_w L_c \frac{\partial u_{wz1}}{\partial t} - K_w L_c u_{wz1}(t) + C_w L_c \frac{\partial u_{wz2}}{\partial t} - K_w L_c u_{wz2}(t) = 0 \quad \textbf{(3)}$$

$$M_{wz1} \frac{\partial^2 u_{wz1}}{\partial t} + \left(C_w + 2C_{pr}\right) \frac{\partial u_{wz1}}{\partial t} + \left(K_w + 2K_{pr}\right) u_{wz1} - C_w \left(\frac{\partial u_w}{\partial t} + L_c \frac{\partial \varphi_w}{\partial t}\right) - K_w \left(u_w + L_c u_{wz2}(t)\right) - C_{pr} \frac{\partial u_{w1}}{\partial t} - K_{pr} u_{w2} - C_{pr} \frac{\partial u_{w2}}{\partial t} - K_{pr} u_{w2} = 0 \quad \textbf{(4)}$$

$$J_{b1} \frac{\partial^2 \varphi_{b1}}{\partial t^2} + 2C_{pr} L_w^2 \frac{\partial \varphi_{b1}}{\partial t} + 2K_{pr} L_w^2 \varphi_{b1}(t) - C_{pr} L_w \frac{\partial u_{w1}}{\partial t} - K_{pr} L_w u_{w1}(t) + C_{pr} L_w \frac{\partial u_{w2}}{\partial t} + K_{pr} L_w u_{w2}(t) = 0 \quad \textbf{(5)}$$

$$M_{b2} \frac{\partial^2 u_{b2}}{\partial t} + \left(C_w + 2C_{pr}\right) \frac{\partial u_{b2}}{\partial t} + \left(K_w + 2K_{pr}\right) u_{b2} - C_w \left(\frac{\partial u_w}{\partial t} + L_c \frac{\partial \varphi_w}{\partial t}\right) - K_w \left(u_w + L_c u_{wz2}(t)\right) - C_{pr} \frac{\partial u_{w3}}{\partial t} - K_{pr} u_{w3} - C_{pr} \frac{\partial u_{w4}}{\partial t} - K_{pr} u_{w4} = 0 \quad \textbf{(6)}$$

$$J_{b2} \frac{\partial^2 \varphi_{b2}}{\partial t^2} + 2C_{pr} L_w^2 \frac{\partial \varphi_{b2}}{\partial t} + 2K_{pr} L_w^2 \varphi_{b2}(t) - C_{pr} L_w \frac{\partial u_{w3}}{\partial t} - K_{pr} L_w u_{w3}(t) + C_{pr} L_w \frac{\partial u_{w4}}{\partial t} + K_{pr} L_w u_{w4}(t) = 0 \quad \textbf{(7)}$$

$$M_w \frac{\partial^2 u_{w1}}{\partial t} + C_{pr} \frac{\partial u_{w1}}{\partial t} + K_{pr} u_{w1}(t) - C_{pr} \left(\frac{\partial u_{b1}}{\partial t} + L_w \frac{\partial \varphi_{b1}}{\partial t}\right) - K_{pr} \left(u_{b1} + L_w \varphi_{b1}(t)\right) + P_{WR1} = 0 \quad \textbf{(8)}$$

$$M_w \frac{\partial^2 u_{w2}}{\partial t} + C_{pr} \frac{\partial u_{w2}}{\partial t} + K_{pr} u_{w2}(t) - C_{pr} \left(\frac{\partial u_{b1}}{\partial t} + L_w \frac{\partial \varphi_{b1}}{\partial t}\right) - K_{pr} \left(u_{b1} + L_w \varphi_{b1}(t)\right) + P_{WR2} = 0 \quad \textbf{(9)}$$

$$M_w \frac{\partial^2 u_{w3}}{\partial t} + C_{pr} \frac{\partial u_{w3}}{\partial t} + K_{pr} u_{w3}(t) - C_{pr} \left(\frac{\partial u_{b2}}{\partial t} + L_w \frac{\partial \varphi_{b2}}{\partial t}\right) - K_{pr} \left(u_{b2} + L_w \varphi_{b2}(t)\right) + P_{WR3} = 0 \quad \textbf{(10)}$$

$$M_w \frac{\partial^2 u_{w4}}{\partial t} + C_{pr} \frac{\partial u_{w4}}{\partial t} + K_{pr} u_{w4}(t) - C_{pr} \left(\frac{\partial u_{b2}}{\partial t} + L_w \frac{\partial \varphi_{b2}}{\partial t}\right) - K_{pr} \left(u_{b2} + L_w \varphi_{b2}(t)\right) + P_{WR3} = 0 \quad \textbf{(11)}$$

Simulations of the determined mathematical model were carried out for various speeds of a moving rail vehicle (passenger train). To describe the phenomena in contact with the rail, a linear theory of Kalker was used. Simulations were carried out for the straight track and for traffic on a stock rail and closure rail.

Simulations were carried out on the part of the turnout where the switch point is located. The switch point was considered as a system with variable stiffness and variable moment of inertia and a radius of 1200 m. Details of these considerations can be found in paper [2].

3. Simulations

The simulations were performed using the parameters obtained from paper [7].

Tab 1. Parameters of a rail vehicle used in computer simulations [own study]

Parameters of the rail vehicle used in computer simulations		
Quantity	**Value**	**Denomination**
Body weight	42 400	[kg]
Torque of inertia of the body	7,06 e5	[kg·m2]
Moment of inertia occurring between body connectors	2,27e6	[kg·m2]
Torque of inertia of the body defined in the axis of rotation φ	2,08e6	[kg·m2]
Weight of the bogie	3 100	[kg]
Torque of inertia of the bogie	5 045	[kg·m2]
Moment of inertia occurring between the bogie connectors	2 806	[kg·m2]
Torque of inertia of the bogie determined in the axis of rotation φ	2,247	[kg·m2]
Weight of the wheelset	1 850	[kg]
Torque of inertia of the wheelset	717	[kg·m2]
Torque of inertia of the wheelset determined in the axis of rotation φ	717	[kg·m2]
Half of the longitudinal stiffness of the secondary suspension	1,45e5	[N/m]
Half of the transverse stiffness of the secondary suspension	2,05e5	[N/m]
Half of the vertical stiffness of the secondary suspension	1,48e5	[N/m]
Half of the longitudinal damping of the secondary suspension	3,43e5	[Ns/m]
Half of the lateral damping of the secondary suspension	2,45e4	[Ns/m]
Half of the vertical damping of the secondary suspension	3,16 e4	[Ns/m]
Half of the longitudinal stiffness of the primary suspension	2,80e7	[N/m]
Half of the transverse stiffness of the primary suspension	2,80e7	[N/m]
Half of the vertical stiffness of the primary suspension	2,80e7	[N/m]
Half of the lateral damping of the basic suspension	1,77e4	[Ns/m]
Vertical distance between the center of the body mass and the secondary suspension	1 100	[m]
The vertical distance between the secondary suspension and the center of gravity of the bogie	0,100	[m]
Vertical distance between the center of gravity of the bogie and the primary suspension	0,270	[m]
Half of the transverse distance between the primary suspension	0,813	[m]
Half of the transverse distance between the secondary suspension	0,978	[m]
Half of the distance between the axles of the bogie	1 350	[m]
Half of the distance between the centers of gravity of the body	8 750	[m]
Rated radius of the wheel rolling	0,430	[m]

The simulations were carried out in the Matlab and Uniwersal Mechanics environment.

To analyze the change in contact parameters, the change of normal forces for different conditions and speeds was determined.

Determining the values of normal forces and pressure distribution in the contact zone of the rail with the wheel is complicated primarily due to the non-linear surface of the rail head. The Finite Element Method was used to solve this problem. The essence of this method is the conversion of a continuous real object into a discrete model, in which a finite number of connectors is extracted, on the basis of which a finite number of finite elements is built. Connectors are assigned appropriate properties that uniquely determine their displacements and the features of the modeled object. The boundary and initial conditions are imposed on the connectors, which is the basis for solving the object traffic equation. Currently, Finite Element Method is an accepted standard in solving tasks in the field of continuum mechanics [9, 10, 11].

3.1. Determination of normal force for a rail vehicle passing through a straight track

The continuous geometric model depicting the wheel-rail contact model has been digitized with solid finite elements with a linear shape function. The discreet model of the test object is shown in Figure 5. The boundary-initial conditions were imposed on the discrete model: the rail foot was immobilized (translational and rotational degrees of freedom were removed), and a concentrated force was applied to the wheel axis corresponding to the weight of the transport cart together with the load per one wheel with a value: 12 [kN], 13 [kN], 17 [kN] and 22 [kN]. Dynamic analysis was also carried out for various rail vehicle speeds v = 100 [km/h], 180 [km/h] and 220 [km/h].

© Copyright by PSTT , All rights reserved. 2018

After performing numerical calculations and analytical calculations using the Kalker's formulas for a given case, the maximum contact pressure was determined. Based on the analyses carried out, conclusions were developed [13, 14, 15].

Fig. 6 and 7 show the change of normal forces of wheel transition from rail to turnout switch point 1200 [m] at speeds 230 [km/h] and 350 [km/h].

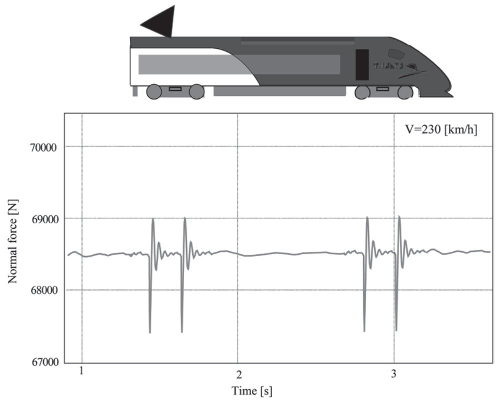

Fig. 6. The waveform of the normal wheel transition force from the rail to the switch point at the speed [v = 230 km / h] [own study]

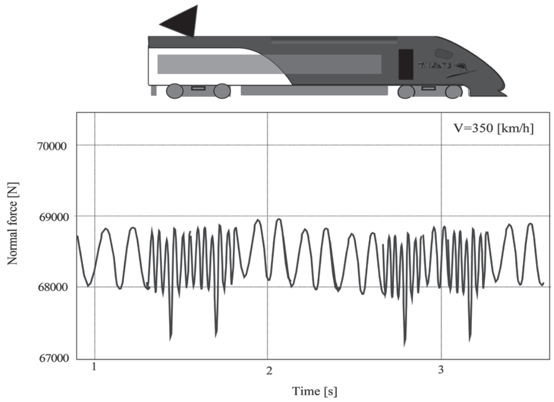

Fig. 7. The waveform of the normal wheel transition force from the rail to the switch point at the speed [v = 350 km / h] [own study]

Then, simulations of the passage through the switch point on the stock rail and closure rail for various speeds were performed. The quantities of these forces will be used to determine the contact areas between the wheel and the rail and the switch point.

In order to assess the impact of construction parameters of the wagon model on the forces acting on the wheel while in motion on the track with modeled continuous irregularities, the average value from five maximum values of force (vertical or transversal), read from the obtained graphs, was used for the analysis. The obtained results regarding the magnitude of transverse and vertical forces acting on the railway wheel were included in [1].

The magnitudes of forces acting on the wheel determined in this way allowed to estimate the real values of variation of normal force occurring in the wheel-rail contact surface for the case of passing the rail vehicle on the straight track and railway turnout. On the basis of assigned forces, using the Kalker linear theory, the contact areas were determined on the switch point and at the transition of the wheel from the rail to the switch point.

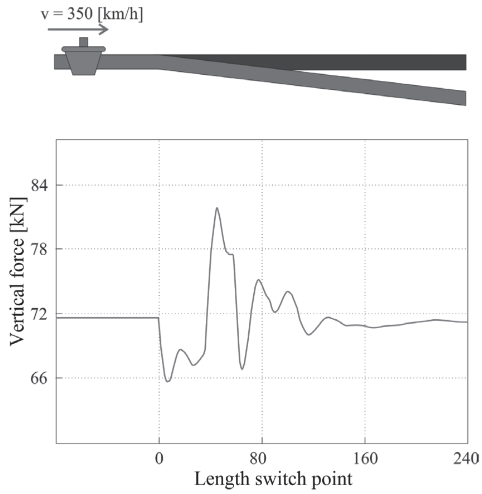

Fig. 8. The vertical force of the wheelset on the switch point in the closure rail [own study]

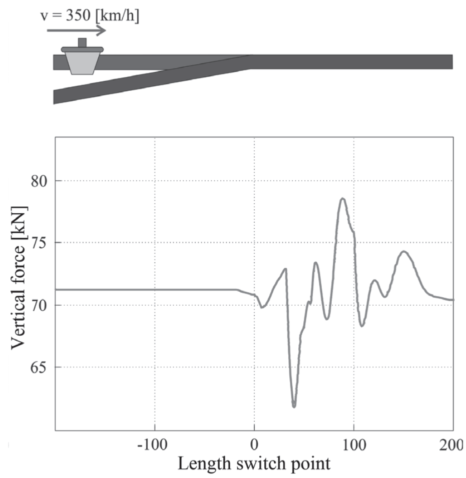

Fig. 9. The vertical force of the wheelset on the switch point in the stock rail [own study]

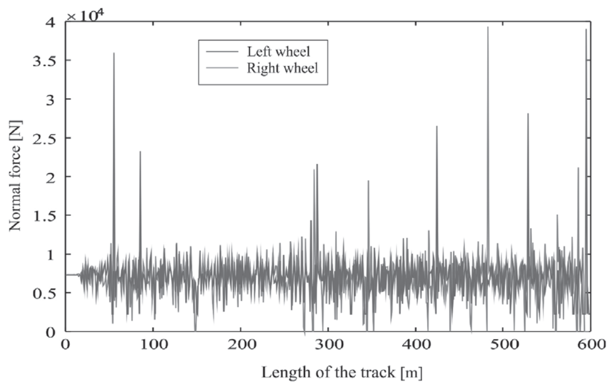

Fig. 10. The waveform of the normal force determined for the right and left wheel of the wheel set at the speed of 15 [m / s] [own study]

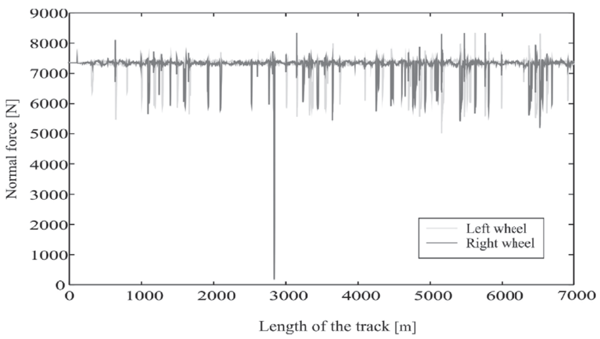

Fig. 11. The waveform of the normal force determined for right and left wheel of the wheel set at 20 [m / s] [own study]

3.2. Determination of normal force for a rail vehicle passing through the railway turnout

Model tests are used both during the search for new solutions in transport as well as in existing vehicles. Requirements for increasing the speed of travel, tonnage of transported cargo and improving the conditions of transport apply to existing vehicles. They enforce the necessity of resolving the problem of wheel cooperation control and maintaining vehicle stability in various working conditions. Simplified track models are no longer sufficient. The complexity of phenomena accompanying the wheel - rail contact induces the need to build complex track models, since the forces generated at the interface between these elements strongly affect the dynamic behavior of the entire rail vehicle. This fact is particularly important in the case of passing a rail vehicle through a specific section of track, which is a railway turnout. Simulation tests were also carried out for the mentioned railway infrastructure component. Due to the complicated procedure of defining the track surface in turnouts, to determine the contact forces in the turnout arch, it was decided to choose only an ordinary turnout, which is one of the common turnouts used in high-speed rail with a radius greater than 1000 [m]. In the first step, it was necessary to create a model that was supposed to map the actual track surface with the required accuracy. The geometry of the turnout was created in the Uniwersal Mechanics program. According to the developed methodology of determining the point of contact in the impact of the wheel - rail for the construction of the rail surface, it is necessary to define the track line and profile of the rail section. Dynamic phenomena occurring at the interface between the wheel and the rail in the case of passing the vehicle through the turnout are much more complicated than when driving on straight tracks or in arches. As a result of continuous changes in the profiles of railway rails and discontinuities in the area of the switch point and crossing, there is a multipoint contact generating impulsive forces with a wide range of constraints [5].

Fig. 14-17 shows the change of the contact surface depending on the change in nominal force and its disturbances. The presented results show that the contact surfaces change depending on the oscillation of the normal force. These quantities can be used to study the wear processes of both the wheel and rail, especially in the area of the switch point.

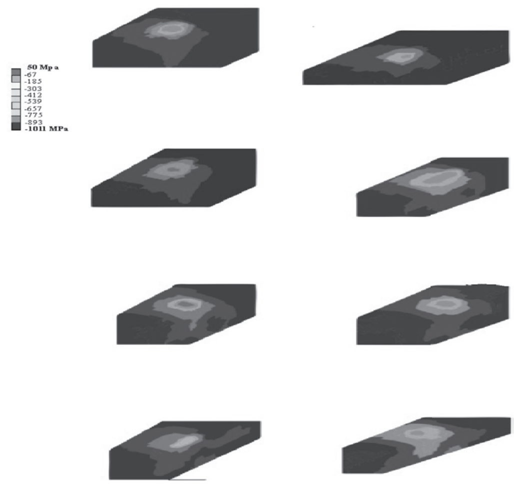

Fig. 12. Contact surfaces occurring on the switch point in case of normal force of 12 kN [own study]

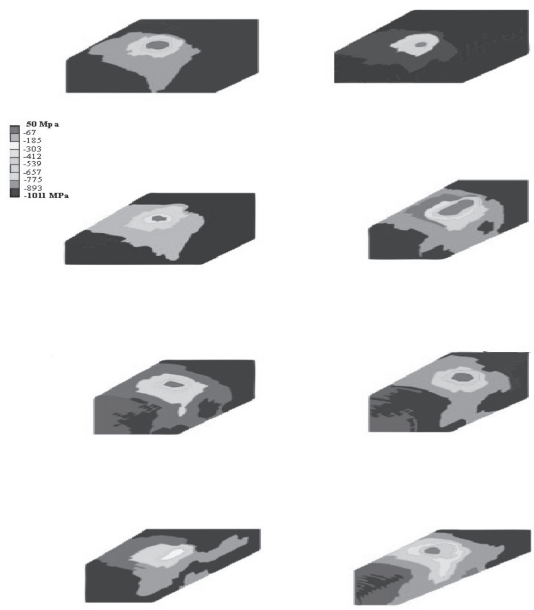

Fig. 13. Contact surfaces occurring on the switch point in case of normal force of 15 kN [own study]

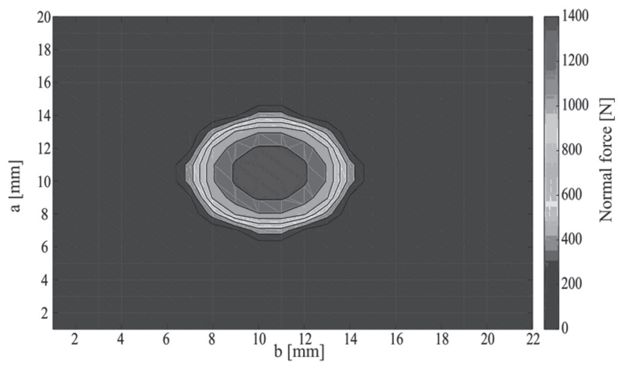

Fig. 14. Results from numerical computer simulations carried out in the Matlab environment; The variation waveform of the contact ellipse for normal force N = 12 [kN] [own study]

© Copyright by PSTT , All rights reserved. 2018

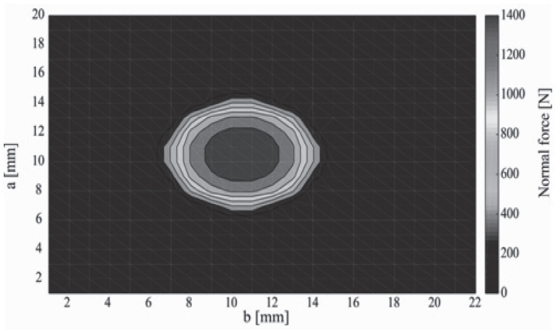

Fig. 15. Results from numerical computer simulations carried out in the Matlak environment; The variation waveform of the contact ellipse for normal force N = 13 [kN] [own study]

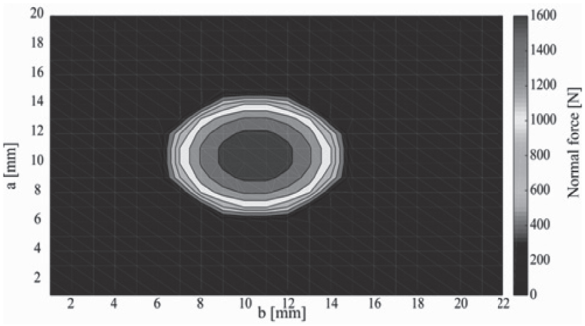

Fig. 16. Results from numerical computer simulations carried out in Matlab environment a) The variation waveform of contact ellipse variation for normal force N = 17 [kN] [own study]

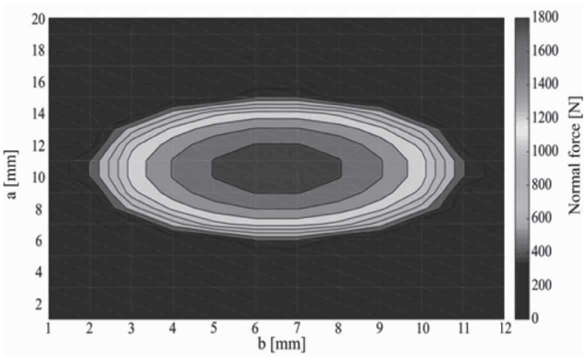

Fig. 17. Results from numerical computer simulations carried out in the Matlab environment; The variation waveform of the contact ellipse for normal force N = 22 [kN] [own study]

3. Conclusion

The paper presents model construction and simulation of selected dynamic phenomena occurring between the rail vehicle and the track, especially in the area of turnout.

The results of normal force simulation show that in the area of transition from track to turnout, the normal force increases, which results in a change in contact parameters between the wheel and the rail, both in qualitative and quantitative terms.

The presented results can be used to analyze wear processes, especially in areas where there is a large increase in normal force.

The presented methodology can be used for various types of turnouts. However, it is necessary to obtain geometrical parameters and moments of inertia for the switch points of turnouts with radii greater than 1200 m.

Simulations were also carried out for lower speeds, but normal force increments were low. After comparing the results of calculation of analytical contact pressure using the Finite Element Method, the following conclusions can be made:

- determination of contact pressure from Kalker's formulas is extremely simple, however, it is possible only for simple geometry ranges, when contact surfaces are curvilinear, pressure determination requires the use of another mathematical apparatus,
- determination of contact pressure using the Finite Element Method makes it possible to determine not only the maximum pressure values (as for the Kalker solution), but the entire distribution together with the contact area,
- for the analyzed case, the maximum values of contact pressure are greater for the numerical method of their determination, therefore the adoption of underestimated values (Kalker's solution) may lead to improper design of the wheels of transport bogies.

Bibliography

[1] ANDERSSON C., DAHLBERG T.: Load impacts at railway turnout crossing, Vehicle System Dynamics1999;33 (Suppl.), pp. 31–142

[2] KISILOWSKI J. (ed): Dynamics of the Mechanical System Track Vehicle-Rail, PWN, Warsaw 1991

[3] GRASSIE S.L.: Models of railway track and train-track interaction at high frequencies: results of benchmark test. Vehicle System Dynamics 1996, 25 (Suppl.), pp. 243–262

[4] FERMER M., NIELSEN J.C.O.: Wheel/rail contact forces for flexible versus solid wheels due to tread irregularities, Vehicle System Dynamics 1994; 23, pp. 142–157

[5] IWNICKI S.: Simulation of wheel–rail contact forces, Fatigue & Fracture of Engineering Materials & Structures 26(10), 2003, pp. 887 - 900

[6] KALKER J.J.: The computation of three dimensional rolling contact with dry friction. Int. J. Numer. Methods Eng.14, 1979, pp. 1293–1307

[7] KISILOWSKI J., KOWALIK R.: On some mechanical phenomena appearing on the turnout switch point with a radius greater than 1 000 m, IV International Scientific and Technical Conference entitled "Modern technologies in the design, construction and maintenance of railway turnouts", Gdańsk, 18 January 2018

[8] KISILOWSKI J., KOWALIK R., KWIECIEŃ K.: Analiza dynamiczna przejazdu pociągów szybkiej kolei przez rozjazd kolejowy, Logistyka, Zeszyt 6, 2014

[9] LEE M.L., CHIU W.K., KOSS L.L.: A numerical study into the reconstruction of impact forces on railway track-like structures, Structural Health Monitoring 4 (1), 2005, pp. 19–45

[10] NIELSEN J.C.O., IGELAND A.: Vertical dynamic interaction between train and track influence of wheel and track imperfections, Journal of Sound and Vibration; 187 (5), 1995, 825–839

[11] SHAHZAMANIAN SICHANI M.: Wheel-Rail Contact Modelling in Vehicle Dynamics Simulation, Licentiate Thesis Stockholm, Sweden 2013

[12] WARD CH.P., GOODALL R., DIXON R.: Contact Force Estimation in the Railway Vehicle Wheel-Rail Interface, Preprints of the 18th IFAC World Congress Milano (Italy) August 28 - September 2, 2011

[13] WICKENS A.: Fundamentals of Rail Vehicle Dynamics: Guidance and Stability, Swets and Zeitlinger, first edition 2003

[14] WU T.X., THOMPSON D.J.: The effects of track nonlinearity on wheel/rail impact. Proceedings of the Institution of Mechanical Engineers Part F: Journal of Rail and Rapid Transit, 218 (1), 2004, pp. 1–15

[15] XIA F., COLE C., WOLFS P.: Grey box-based inverse wagon model to predict wheel-rail contact forces from measured wagon body responses, Vehicle System Dynamics, 46 (Supplement), 2008, pp. 469–479

[16] ZHAI W., CAI Z.: Dynamic interaction between a lumped mass vehicle and a discretely supported continuous rail track. Computers and Structures; 63 (5), 1997, pp.987–997

© Copyright by PSTT , All rights reserved. 2018