# Understanding of overshooting systems

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## Abstract

It is important to build good intuitive knowledge of dynamic systems in order to make good decision. However, this intuitive knowledge is often lacking. This is indicated by learning problems at schools, laboratory experiments to test understanding, and observed mismanagement of real world systems. While mathematics is needed to untangle the complexities of dynamic systems, many find mathematics difficult and develop mental blockages. This paper discusses how insights from the field of System Dynamics could be used to help develop intuitive understanding of dynamic systems. The teaching method has four key elements. First one starts with a water analogy. For the dynamic systems that are dealt with in this paper, the analogy consists of a funnel and glass system. Second, a generic symbol language is used to describe system structure with the necessary cause and effect relationships. Third, simulation is used to study the behaviour over time that follows from the system structure. Simulation can both be used to see if the model structure is able to explain and replicate observed problems, and to test the effects of policy interventions. Fourth, stock and flow diagrams for simple systems are used as starting points when studying the dynamics of systems from different fields. Hence, the method enables interdisciplinary teaching, research, and collaboration. A common language is also crucial for effective transfer of knowledge from a known to an unknown territory.

## Keywords

teaching, System Dynamics, stock and flow diagram, analogies, simulation, transfer of knowledge, interdisciplinary

## Introduction

There is much evidence of misperceptions and mismanagement of dynamic systems. An historical example is Aristotle who did not have a correct understanding of the phenomenon of motion. While motion is observed every day, it may be surprising that this wise man got it wrong. On the other hand, it is also an indication that it is difficult to understand dynamic systems (Moxnes, 2004; Rouwette et al., 2004; Sterman, 2011).

Two thousand years after Aristotle, Newton and Leibniz developed the mathematics needed to study dynamic systems. Newton also formulated his two famous laws that since his time have been used to analyse motion (at speeds lower than the speed of light).

Today's students still make mistakes similar to that of Aristotle (diSessa, 1993). While mathematics could help students understand, limited knowledge of mathematics among both students and managers limits its usefulness for building intuition and making practical decisions. There is even a danger that mathematical models come to be viewed as theoretical and hence not related to practical decision making.

While also simulation models are mathematical models, the mathematics can be kept in the background while focusing on building intuition that is directly useful for strategy formulation and practical decision-making. In this paper, intuition is built in a four-step process. First, an analogy is used where water flows through a funnel and into a glass. Second, Jay W. Forrester's stock and flow diagram is used to give an overview of the system structure. Third, computer simulation is used to explore the dynamics, the model's behaviour over time. Fourth, the stock and flow diagram of the funnel and glass model is used as a starting point to understand more important and more challenging systems. In turn, models for more challenging systems can serve as analogies for further systems. Hence, as one learns about one system, one is also preparing to understand other and similar systems. Over time, this should make learning more effective.

Chapter 2 presents the funnel and glass analogy, its stock and flow diagram, a simulation of the water flow, and practical insights for better decision-making. In Chapter 3, the funnel and glass model is used to understand the dynamics of alcohol uptake in the body. Chapter 4 deals with market dynamics and commodity cycles. Chapter 4 concludes.

## 1. The funnel and glass analogy

A funnel and glass system shows Fig. 1. The illustration is copied from an animation where students can control the inflow of water to the funnel[1]. Water flows from the faucet into the funnel. From the funnel it flows into the glass. The funnel and the glass represent the system stocks (or states). The stocks are changed by flows, and by flows only. The flows are controlled by feedback. This is true both for the physical part of the system, the outflow from the funnel, and for the human control of the faucet inflow.

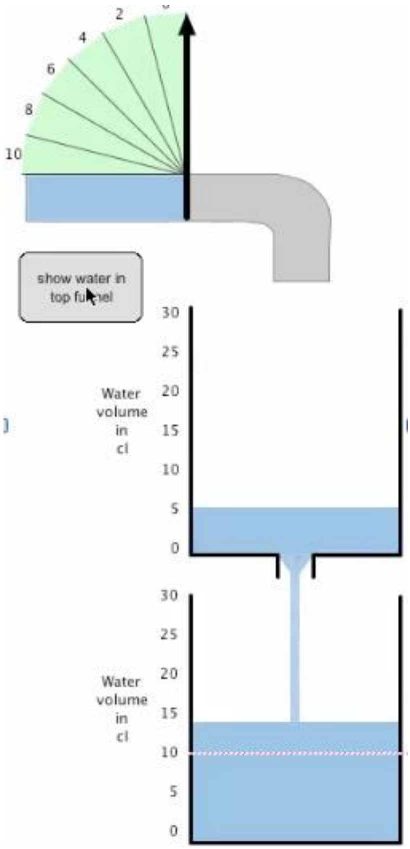


**Fig 1.** Illustration of a funnel and glass system

Source: own.

[1] The animation is used in an interactive, online course in Natural Resources Management (http://www.uib.no/rg/dynamics).

The outflow from the funnel is well described by Torricelli's law, which in this case says that the outflow is proportional to the square root of the amount of water in the funnel. In other words, the outflow is a non-linear function of the amount of water in the funnel stock.

Normally, a human's control of the faucet can also be described as feedback control. To simplify, assume that the person controlling the faucet cannot see how much water is in the funnel (in the animation one can cover the funnel by clicking the grey button). Then a simple, however, natural strategy is to control the faucet according to the gap between the goal for water in the glass (illustrated by a dotted line at 10 cl) and the actual amount of water in the glass. When the amount of water has reached the goal, the faucet is closed. As can be seen from the illustration in Fig. 1, such a strategy will make the water in the glass overshoot the goal. After the faucet has been closed, there is still water in the funnel, and this water will continue to flow into the glass, causing the overshoot.

In a transparent system of the type shown in Fig. 1, the simple feedback strategy is a bit stupid. Still, people at times experience overshoots when using funnels. Since the system is transparent, people tend to learn quickly from repeated experiences. In general however, one cannot rule out the possibility that humans make repeatedly use of non-optimal feedback policies. One reason for this is that simple feedback policies do not require much costly analysis and that they often lead to satisfactory results. However, in complex and opaque systems, simple feedback strategies can lead to problematic mismanagement.

Before we go on, note that in dynamic systems there are two different types of cause and effect relationships. In the funnel, the outflow changes immediately when the amount of water in the funnel changes. The cause and effect relationship between the funnel stock and its outflow is *instantaneous*. The full effect of a change in the stock is realized immediately (in practise almost immediately). This is a very different type of cause and effect relationship than that from flows to stocks. If for instance the inflow to the glass suddenly increases, there will be no immediate change in the amount of water in the glass. Over time, the amount of water will increase faster than before. No matter what the inflow is, it takes time before the effect to take place. The full effect is not realized immediately. The cause and effect relationship is said to be *accumulating*. Thus, when describing the structure of systems, it is very important to distinguish between stocks and flows.

Differential equations can be used to describe the funnel and glass system. To illustrate how complicated the mathematics can appear, the following equation describes the amount of water in the glass, G:

$$\frac{d^2G}{dt^2}\frac{dG}{dt} + \frac{k^2}{2}\frac{dG}{dt} - \frac{k^2}{2}(G^* - G)/T = 0 \tag{1}$$

In this equation the faucet flow is assumed to be given by the gap between desired and actual amount of water in the funnel, $G^*$-$G$, divided by an adjustment time $T$. The adjustment time determines the aggressiveness of the feedback policy, how quickly the gap should be closed. The outflow from the funnel is described by Torricelli's law and equals $k\sqrt{F}$, where $F$ is the amount of water in the funnel.

This equation is very complicated and it may not have an analytical solution. The main reason for this is the non-linearity introduced by Torricelli's law. The equation is not likely to be of any use to decision-makers. It will most likely be viewed as theoretical and only interesting to a small group of mathematicians. If the models is presented as a set of coupled differential equations,

$$\begin{aligned} \frac{dF}{dt} &= (G^* - G)/T - k\sqrt{F} \\ \frac{dG}{dt} &= k\sqrt{F} \end{aligned} \tag{2}$$

it is easier to understand the structure of the system. The derivatives denote the net flows for the two stocks. The funnel stock is increased by the faucet inflow and is reduced by the funnel outflow. The glass stock is increased by the funnel outflow. This set of equations can be used in simulation models. However, the equations by themselves are still beyond the reach of most people. So how could the equations be presented in a more intuitive way? For the funnel and glass system, Fig. 1 is a natural alternative. However, it is desirable to use symbols that are generic and that can be reused in all dynamic systems. Such as system of symbols was invented by Forrester (1961), nowadays referred to as *stock and flow diagrams*.

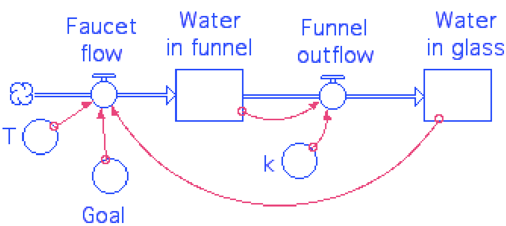


**Fig. 2.** Stock and flow diagram of funnel and glass system, here with the layout produced by the simulation program iThink

Source: own.

Stock and flow diagrams have four basic symbols. Rectangles denote stocks and double arrows denote flows. Together these two symbols represent accumulating cause and effect relationships. Thin arrows show instantaneous cause and effect relationships and circles contain the exact functional forms of these instantaneous relationships. For instance, inside the circle for Funnel outflow is a function representing Torricelli's law, a constant times the square root of the amount of water in the funnel, $k\sqrt{F}$. Hence while the stock and flow diagram gives an accurate description of system structure in terms of cause and effect relationships, it does not illustrate the exact functions. They must be described by additional words, graphs, or mathematical functions. However, in many cases, main insights about why problems occur and how they can be reduced follow from the stock and flow diagrams.

A simulation of the model (Fig. 3) described by Equation 2 and illustrated in Fig. 1 and 2. To begin with the difference between the goal and the water in the glass is 10 cl and the faucet flow is 10 cl/s. As the gap between the goal and the water in the glass closes, the faucet flow decreases. When the goal is reached, the faucet flow becomes zero. However, at this time there is still much water in the funnel and the funnel outflow causes the water in the glass to overshoot the goal.

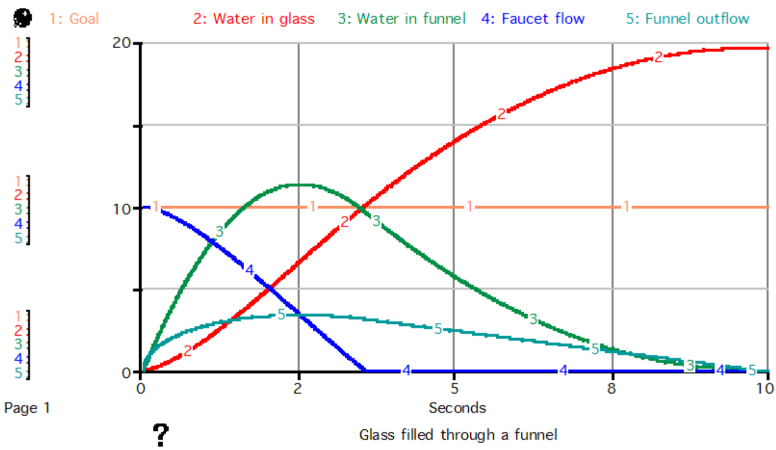


**Fig. 3.** Simulation of simple feedback strategy (k=1, T=1)

Source: own.

The funnel and glass model serves as an analogy for a whole class of overshoot problems. While most people control this system much better than what the simulation shows, other cases in this class of systems turn out to be more difficult to control. One possible complication could be that managers do not have information about the „funnel stock" or that they are not even aware of the existence of such a stock. Another possibility is that the overshoots develop over years rather than seconds. Hence, managers may not be able to benefit from repeated experiences and may not even have accurate records of how stocks and flows have developed over many years.

Finally, in general, what strategies can be used to avoid overshoots? Optimization theory says that, all stocks should influence decisions in a system. In the funnel and glass case that leads to a quite simple policy. The faucet should be closed down when the sum of water in the funnel and in the glass equals the goal. Other cases require more complex analysis that most people are not capable of.

In case the funnel cannot be observed, a more practical approach is to keep the faucet open until 10 cl has flowed into the funnel. However, this requires that one knows how fast the water flows out of the faucet. If this information is lacking, one may learn from repeated trials. In the first trials one should be careful and turn off the faucet long before the goal is reached. With more experience one could learn how many seconds the faucet must be open to reach the goal. Hence, with experience the strategy could be to count seconds.

These strategies are likely to be useful also in other cases where overshoots could occur. Knowledge about the „funnel and glass structure" of the systems one wants to control motivates consideration of such strategies in the first place is. If one is not aware of the „funnel and glass structure" and the potential for overshoots, it is easy and hence tempting to control these systems with an overly simplified feedback strategy of the type demonstrated in Fig. 3.

## 2. The case of alcohol

It is popular to drink alcohol among young people and those who drink normally expects to obtain desirable effects of the alcohol. However, quite frequently it happens that those who drink become more intoxicated than they intend. Undesired consequences vary from embarrassments, vomiting, and hangovers to less frequent injuries and even deaths from alcohol related fights, drowning, fires, traffic accidents, unsafe sex, rape, suicide attempts, poisoning etc. In addition to the individual costs, societal costs in terms of health care, social and police work are considerable.

Could the funnel and glass analogy be helpful in explaining why many juveniles overshoot their desired level of intoxication? A stock and flow diagram for the uptake of alcohol in the body shows Fig. 4. The stomach serves as a funnel stock where alcohol accumulates before it diffuses into the body. To simplify, the diagram ignores the slow elimination of alcohol from the body (mostly) through the liver.

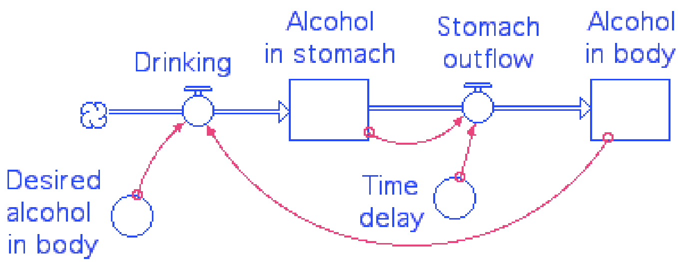


**Fig. 4.** Stock and flow diagram for alcohol passing through stomach and into body

Source: own.

The case of alcohol is more complicated than the funnel and glass system because those who drink cannot see how much alcohol is in their stomachs. Most drinkers do not even reflect on how much alcohol they may have in their stomach because they have no idea that the stomach serves as a funnel and that this funnel could create overshoots. Rather, drinkers are likely to base their drinking on the discrepancy between the desired feeling of alcohol in the body and the actual feeling. Also, they are likely to think of their drinking strategy as rational. When the desired feeling is reached, they stop drinking. However, after they stop drinking, the amount of alcohol in the body will continue to increase fro some time. When this happens, they are likely to be puzzled and seek other explanations than the stomach's funnel effect.

To test high school student understanding of the uptake of alcohol in the body, Moxnes and Jensen (2009) carried out a laboratory experiment. Students were asked to make drinking decisions every 15 minutes to reach a goal of a blood alcohol concentration (BAC) of 0.8 g/l after one hour of drinking. They were not given real alcohol; rather they made decisions in a computer simulator. Their payment increased with the closeness to the goal. One group used a simulator where the stomach on average delayed the uptake of alcohol by 22 minutes. The second group used a simulator where the delay time was only 4 minutes. Figure 5 shows that the long delay led to an average overshoot in the BAC of 86%, while the shorter delay led to almost no overshoot.

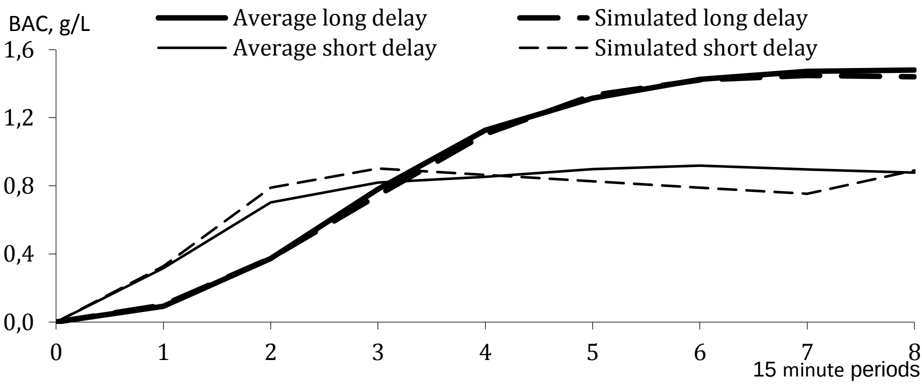


**Fig. 5.** Results of alcohol experiment

Source: (Moxnes and Jensen, 2009)

The result of a simulation where drinking is assumed to be given by the difference between the goal and the actual amount of alcohol in the body shows Fig. 5[2]. One and the same drinking strategy explains very well the average behaviour for both groups. In other words, it seems that the high school students used an overly simplified drinking strategy where only information about the glass stock was in use. With a funnel and glass analogy in mind, it is easy to understand that the simplified strategy will lead to overshoots. Drinking decisions are also influenced by many other factors such as mood at parties and intake of food. Important to note, the physical delay of alcohol uptake from the stomach could lead to overshoots in many different situations.

Going back to the strategies that were discussed for controlling the funnel and glass system, what strategies could be used to avoid becoming more intoxicated than desired? Since the content of alcohol in the stomach cannot be observed, inexperienced juveniles should drink slowly in order to learn about their own tolerance. With experience, a simple strategy is to count drinks, and stop drinking when the acceptable number of drinks is reached.

[2] The exact formula for number of bottles of beer to drink each 15 minutes was: Bottles=Integer((Goal-BAC)/0.4+0.95), where the adjustment time of 0.4 denotes the aggressiveness of the drinking strategy.

## 3. The case of commodity price fluctuations

Many commodity markets are troubled by product prices that fluctuate or cycle over time. These fluctuations can cause financial problems for both consumers and suppliers, may influence employment, and even destabilize nations. Fluctuations consist of alternating over- and undershoots. Can these over- and undershoots be explained by the funnel and glass analogy? The central stock in most markets is the production capacity. Capacity is a stock because it accumulates investments over time. Furthermore, it may take years to build new capacity. Thus, there is a funnel delay in terms of capacity on order. For instance, when expanding apple production, after ordering new seedlings, it takes several years before the new apple trees start to bear fruit and to be productive.

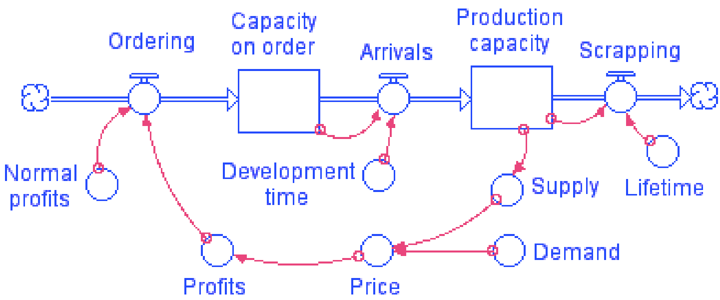


**Fig. 6.** Stock and flow diagram of commodity market

Source: own.

A stock and flow diagram for a commodity market shows Fig. 6. Production capacity represents total capacity in the market and capacity on order represents all capacity on order. Individual producers do not have precise information about how much capacity has been ordered by all the other producers in the market. Neither do they have precise information about the current total production capacity and about total demand. However, the market still works because the price adjusts to reflect the balance between capacity and demand. If demand is higher than capacity, prices tend to rise above production costs. This means that profits will be higher than normal profits in society, and suppliers find it attractive to order more capacity. This feedback from capacity to price and to ordering is what Adam Smith denoted the "invisible hand". The price serves to equilibrate supply and demand without the need for public planning and extensive information gathering. However, because of the funnel stock, the equilibrating process is not perfect; it could lead to overshoots. Scrapping of old capacity provides an outflow from the capacity stock. Hence, after an overshoot in capacity, and a period with low prices and little ordering, capacity could also undershoot the demand level. Alternating over- and undershoots show up as price fluctuations or cycles.

Similar to the laboratory experiment for alcohol uptake, a market experiment by Arango and Moxnes (2012) shows that the occurrence of fluctuations depends on the length of the construction period for new capacity. In real markets it is also likely that different types of market expectations may influence cyclical tendencies. Similarly, cash-flows and funds, which are influenced by product prices, seem important for investment and price development.

What strategies should commodity producers follow? Ideally, producers should invest counter-cyclically. This implies that investments should be made when product prices are below the trend development for the price. At this point other producers invest little, and the investment goods such as apple tree seedlings tend to be cheaper than normal. This may also be a good time to buy existing capacity from other producers. However, it is not trivial to invest counter-cyclically. A first step is to consider long time series for the product price, much longer than the data shown in most financial publications. Then the period length of possible cycles in the data should be seen in light of the construction time for new capacity. Roughly, period lengths should be close to two or three times the normal construction time. Ideally, one should also consider complete simulation models of commodity markets to see if there is a correspondence between observed price cycles and the market structure.

## Conclusions

The funnel and glass analogy is a simple starting point for developing intuitive understanding of overshooting phenomena. Laboratory experiments show that most people have great difficulties in understanding and managing dynamic systems. One reason for this is that it is difficult to describe system structure. Frequently people come to ignore or underestimate the importance of funnel stocks. They think of accumulating cause and effect relationships as instantaneous ones. And, they ignore the importance of feedback, see e.g. Sterman (2011). This should come as no surprise since it took 2000 years before Newton was able to correct Aristotle's erroneous understanding of motion. Furthermore, even if one should have a perfect system description, it is also difficult to predict the behaviour of dynamic systems. The commodity market exemplifies; it is difficult to predict cycle periods and dampening.

The three examples dealt with in this paper suggest that learning could be accelerated by use of stock and flow diagrams. These diagrams help learners see similarities and differences when transferring knowledge from analogies to systems of interest. To indicate this potential, think of a cycling spring where the funnel stock represents velocity and the glass stock represents distance. Another example is renewable resources management where the funnel stock represents extraction capacity and the glass stock represents the renewable resources, see e.g. Moxnes (1998). There is much historical evidence where capacity has overshot and resources have undershot desirable levels. A great challenge nowadays is climate change where an overshoot in capacity to emit greenhouse gases can lead to an overshoot in greenhouse gases in the atmosphere and hence to overshooting climate change, see e.g. Sterman (2008) and Moxnes and Saysel (2009). Stock and flow diagrams are particularly useful for knowledge transfer because they provide a common language for different fields of study. Hence, they provide a unique opportunity for interdisciplinary research and teaching.

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# Modelowanie systemowe i jego granice

## Streszczenie

Posiadanie dobrej wiedzy intuicyjnej w modelowaniu systemów dynamicznych jest ważne w celu podejmowania lepszych (optymalnych) decyzji. W rzeczywistości występuje brak wiedzy intuicyjnej. Potwierdzają to problemy spotykane w trakcie uczenia w szkole, eksperymenty laboratoryjne testujące wiedzę i obserwowane niegospodarności systemów rzeczywistych. Z pomocą przychodzi matematyka, aby rozwikłać zawiłości układów dynamicznych. Jednak dla wielu podejście matematyczne jest trudne do zrozumienia i blokują się psychicznie. W artykule omówiono, jak spostrzeżenia z dziedziny dynamiki systemów mogą być wykorzystane, aby pomóc rozwijać intuicyjne zrozumienie systemów dynamicznych (złożonych). Metoda nauczania ma cztery kluczowe elementy. Pierwszy rozpoczyna się od przykładu przepływu wody. Dla dynamicznych systemów, które zostały przewidziane w tym artykule, analogia składa się z systemu i lejka szklanego. Po drugie, zastosowano symbole, które są używane do opisu struktury systemu zależności między przyczynami i efektami. Po trzecie, zastosowano symulację do badanie zachowania przepływów w czasie, który wynika z konstrukcji danego systemu. Symulacja może być stosowane zarówno do sprawdzenia, czy struktura modelu jest w stanie wyjaśnić i replikować stwierdzone problemy jak i do testowania efektów interwencji w ramach polityki. Po czwarte, schematy przepływu i zasobów zbudowane na podstawie prostych modeli zależności systemowych stanowią pomoc i punkt wyjścia w badaniu systemów z różnych dziedzin. Metoda umożliwia interdyscyplinarne nauczanie, badania naukowe i współpracę. Wspólny język ma kluczowe znaczenie w skutecznym transferze wiedzy z obszarów gdzie jest ona znana na obszary gdzie wcześniej nie wystąpiła.

## Słowa kluczowe

nauczanie, systemy dynamiczne, diagramy przepływu i zasoby , analogie, symulacja, transfer wiedzy, interdyscyplinarność