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DETERMINATION OF THERMAL DIFFUSIVITY VALUES BASED ON THE INVERSE PROBLEM OF HEAT CONDUCTION – NUMERICAL ANALYSIS

Adam ADAMOWICZ*

*Faculty of Mechanical Engineering, Bialystok University of Technology, 45C Wiejska Street, 15-351 Białystok, Poland

a.adamowicz@pb.edu.pl

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Abstract: This paper presents a discussion on the accuracy of the method of determining the thermal diffusivity of solids using the solution of the inverse heat conduction equation. A new measurement data processing procedure was proposed to improve the effectiveness of the method. Using the numerical model, an analysis of the sensitivity of the method of thermal diffusivity determination to changes in operational and environmental parameters of the test was carried out. The obtained results showed that the method was insensitive to the parameters of the thermal excitation impulse, the thickness of the tested sample, and the significant influence of convection cooling on its accuracy. The work was completed with the formulation of general conclusions concerning the conditions for determining the thermal diffusivity of materials with the use of the described method.

Key words: thermal diffusivity, inverse heat conduction problem, impulse heating, finite element analysis

1. INTRODUCTION

Thermal diffusivity or thermal diffusion coefficient is a parameter describing the heat flow, and in fact, the movement of the isothermal surface in the material, occurring, inter alia, in the differential equation of the Fourier heat conduction:

$$\frac{\partial T}{\partial t} = k\nabla^2 T, \tag{1}$$

where: k - thermal diffusivity, T - temperature, t – time, ∇^2 – Laplacian. The thermal diffusion coefficient combines other thermal material properties, i.e. heat conduction coefficient λ, specific heat c_p and density ρ :

$$k = \frac{\lambda}{c_p \rho} \tag{2}$$

In simulation tests carried out with the use of analytical or numerical models, the key issue is the correct determination of the properties of materials. Even the most perfect calculation model will not allow for obtaining reliable results without correct input data, including material properties. The importance of this issue is evidenced by the multitude of methods for experimentally determining the properties of materials, their improvement and the search for new ones. In the case of thermal diffusivity, its value for a given material can be determined using the Angström method [1], which links the value of the thermal diffusion coefficient with electrical conductivity, several impulse methods. The first paper [2] presents a method of heating one surface of an isolated plane-parallel sample with a pulse of light. Based on the time $t_{1/2}$, reaching half of the maximum value of the temperature on the second surface of the sample, the value of the thermal diffusion coefficient was determined using a simple empirical dependence. In the work [3] an analytical solution to the problem of heat conduction for a cylindrical sample after being forced by a heat impulse was proposed. These results were used in the work [4] to increase the accuracy of the method of determining thermal diffusivity during heating with laser radiation. This method was modified and developed in the works [5, 6, 7] and to this day this subject is the focus of researchers [8, 9].

This paper presents an analysis of the accuracy of one of the methods of determining thermal diffusivity [10]. This method is based on the solution to the problem of inverse heat conduction by impulse heating of a flat sample, which is contrary to the previously described method, where it does not require special equipment, complicated sample preparation or test conditions. The simplicity of the method is counterbalanced by some simplifications that may cause inaccuracies. This study aims to investigate the potential causes of inaccuracies in the thermal diffusivity determination procedure and their impact on the obtained results.

2. THEORY OF THE METHOD OF DETERMINING THERMAL DIFFUSIVITY

Consider an infinite plate (Fig. 1) made of homogeneous, isotropic material of constant thickness g subjected to impulse heating on the wall $x = 0$. It is assumed that the total energy of the pulse with the surface flux density q is absorbed by the plate and there is no heat exchange with the environment.

The heat conduction equation for this one-dimensional problem can be written as follows:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \frac{1}{\rho c_p} q, \tag{3}$$

The initial boundary conditions for a given case are the initial temperature T_0 , equal to the ambient temperature T_a , the thermal pulse $q = q(t)$ with a rectangular time course and duration t_f :

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$$T = T_0: \quad t = 0,$$
$$K \frac{\partial T}{\partial x}\bigg|_{x=0} = \begin{cases} q(t): & 0 \le t \le t_f, \\ 0: & t > t_f, \end{cases} \tag{4}$$
$$\frac{\partial T}{\partial x}\bigg|_{x=0} = \frac{\partial T}{\partial x}\bigg|_{x=g} = 0: \quad t > 0.$$

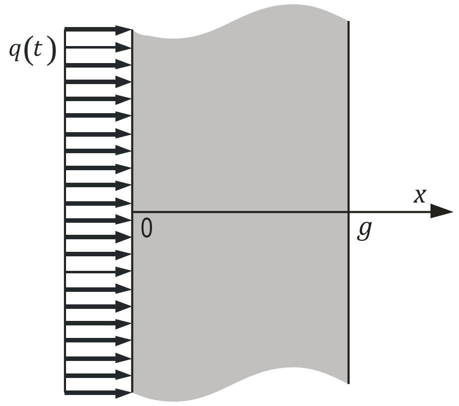

Fig. 1. Diagram of an infinite plate of thickness g

The solution to the boundary-initial problem of the heat conduction equation (3), (4) for the surface $x = g$ with the assumptions made regarding the shape of the heating function $q(t)$ and the short excitation time t_f and taking into account only one segment of the asymptotic series, it takes the form [10]:

$$T(t, x = g) = T_\infty - 2(T_\infty - T_0)\exp\left(-\frac{\pi^2 k}{g^2} t\right), \tag{5}$$

where: T_∞ - maximum surface temperature $x = g$:

$$T_\infty = T(t \to \infty). \tag{6}$$

When we logarithm the Eq. (5) we get the linear function of time:

$$\ln[T_\infty - T(t)] = -At + \ln 2(T_\infty - T_0), \tag{7}$$

with the directional coefficient: $A = \frac{\pi^2 k}{g^2}$. (8)

Eq. (5) shows a linear function of the temperature reached on the surface $x = g$ of the plate subjected to impulse heating on the surface $x = 0$. It is worth noting that, with the assumptions made, it does not depend on the conditions of thermal excitation $q(t)$ and knowing the value of the directional coefficient A of the Eq. (7) it is easy to determine the value of the required thermal diffusion coefficient:

$$k = \frac{g^2 A}{\pi^2}. \tag{9}$$

Experimental determination of the value of the thermal diffusion coefficient of the sample material based on solving the problem of inverse heat conduction consists of the following procedures:

– measuring the surface temperature of the plate $x = g$ during impulse heating of the surface $x = 0$ until reaching the steady state; the results of the measurements are the temperature values for discrete-time values, i.e. $T(t)$ (Fig. 2);
– determination of the temperature T_∞ and calculation of the value of $\ln[T_\infty - T(t)]$ (Fig. 3); under real conditions, i.e. under conditions of convection, take:

$$T_\infty = \max\{T(t): \quad t > 0\}. \tag{10}$$

– determination of the directional coefficient A of the Eq. (7) by a linear approximation of the dependence $\ln[T_\infty - T(t)]$;
– determination of the value of the thermal diffusion coefficient k based on the dependence (9).

The above-mentioned results of temperature measurements during impulse heating (Fig. 2) and an illustration of the linear approximation process (Fig. 3) were made for a sample made of composite brake material. A flash lamp with a maximum flash energy of 6,000 J and a flash duration of 0.2 s was used as a heat source. A Cedip Titanium 560 M thermal imaging camera was used to measure the surface temperature of the plate. Visible in Fig. 2 the temperature peak corresponds to the moment when the flash was triggered ($t = 0$). The geometrical features of the sample, material properties and test conditions are presented in Tab. 1.

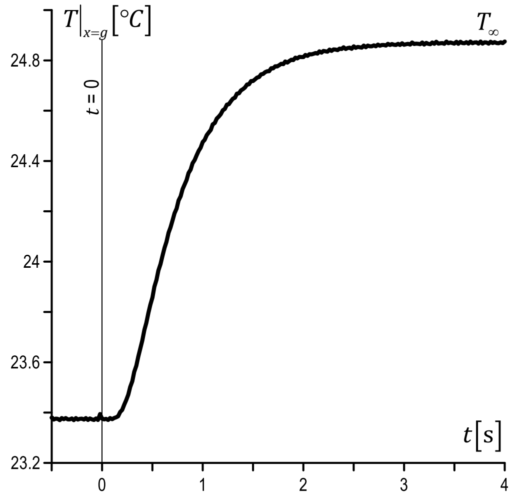

Fig. 2. Temperature increase on the back of the sample during impulse heating

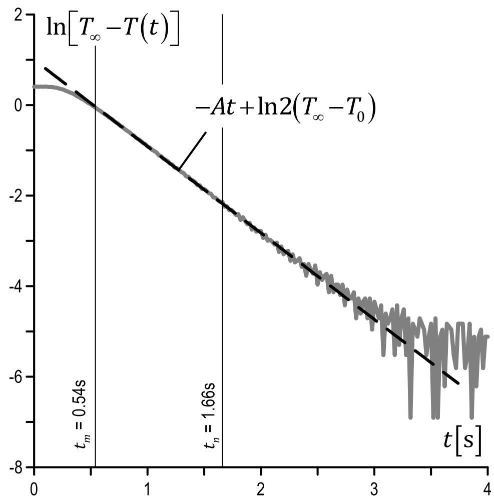

Fig. 3. Illustration of the linear approximation procedure to determine the coefficient A and the thermal diffusivity value k

Tab. 1. Geometric features and material properties of a material sample

Characteristic	Value
Plate thickness g	1.5 mm
Plate material density ρ	1,930 kg/m^3
Specific heat capacity of the plate c_p	870 J/(kg*K)
Start/ambient temperature T_0	23,375°C.
Maximum temperature T_∞	24,875°C.
Duration of the heat pulse t_f	0.2 s

The accuracy of determining the value of the thermal diffusion coefficient based on the above-described procedure is influenced by several factors related to the assumed model of the heat conduction problem (e.g. assumption of no heat transfer), simplifications of the solution to the inverse problem (one segment of the solution series has been taken into account), the conditions for carrying out measurements (other than the rectangular shape of the heating function) and the method of processing measurement data. Therefore, several questions arise regarding the model and procedure of the experimental determination of the k coefficient based on a solution to the inverse problem of heat conduction:

1. In Fig. 3 it can be seen that the nature of the dependence ln[T_∞ - T(t)] in its initial and final interval is far from linear. Using the entire available range of measurement data for linear approximation will distort the results obtained. Relying on a subjective evaluation of what range of data to include in a linear approximation can be unreliable. Therefore, the question arises, what range of measurement data $\Delta t = t_n - t_m$ should be used in the procedure to ensure the best quality of linear approximation and accuracy of k coefficient determination?
2. The issue of the shape of the impulse heating function $q(t)$ is considered extremely important in the literature. However, the dependencies presented above show that the parameters of the thermal excitation pulse, apart from the requirement of short duration, do not affect the determined value of the thermal diffusion coefficient. It should be investigated whether the method of exciting a heat wave in the material affects the results of the thermal diffusivity determination method.
3. The next element of the model that requires analysis in terms of the influence on the accuracy of the method is the sample thickness g.
4. The assumption of the lack of heat exchange between the sample of the tested material and the environment after the completion of the impulse heating seems to be the greatest source of method inaccuracy. Therefore, it would be necessary to investigate how serious the errors are and how to minimize them.

In the next chapter, an algorithm will be presented that ensures optimal conditions for linear approximation for the accuracy of thermal diffusivity determination. The analysis of the remaining issues, specified in points 2–4, will be carried out using the numerical model of the problem, using the finite element method.

3. LINEAR APPROXIMATION OF MEASUREMENT DATA AND CALCULATION RESULTS

The first issue to be considered is to ensure reproducible conditions for the processing of measurement data or the results of numerical calculations. Fig. 3 shows that to find the value of the directional coefficient of the line Eq. (7) for the linear approximation, only the interval of dependence ln[T_∞ - T(t)] with linear characteristics should be used. Part of the data corresponding to the beginning and end of the impulse heating process of the tested material sample should be eliminated from the approximation task.

This task will be performed with the procedure looking for such a time interval <t_m, t_n>, for which the quality parameter of the linear approximation of the dependence ln[T_∞ - T(t)] reaches its maximum value. The Pearson linear correlation coefficient will be used as a parameter for assessing the quality of the linear approximation:

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}, \tag{11}$$

where x_i, y_i are elements of correlated sets. For the case in question, these sets are the measured data:

$$x_i = \ln[T_\infty - T(t_i)] \tag{12}$$

and the values of the approximation function for the same values of time t_{and} :

$$y_i = -A\cdot t_i + \ln 2(T_\infty - T_0) \tag{13}$$

$\bar{x}$and$\bar{y}$ are the mean values of the correlated sets:

$$\bar{x} = \frac{1}{n-m}\sum_{i=m}^{n}\ln[T_\infty - T(t_i)], \bar{y} = \frac{1}{n-m}\sum_{i=m}^{n}(A\cdot t_i + B). \tag{14}$$

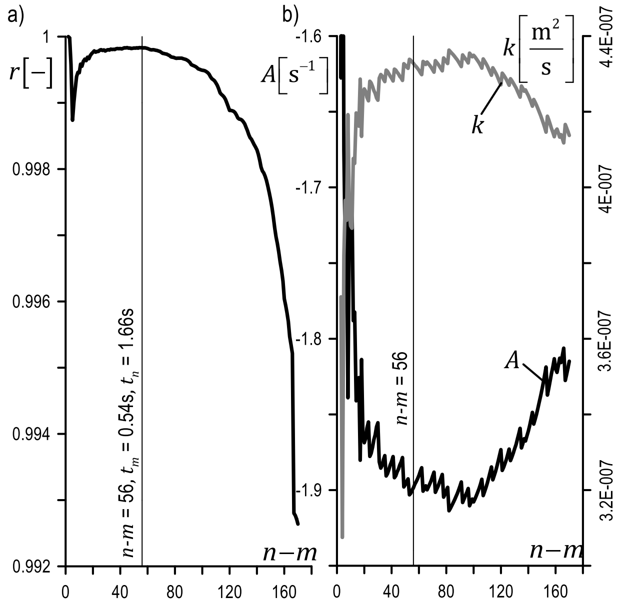

Fig. 4. The maximum value of the linear correlation coefficient r (a) and the value of the directional coefficient A and the value of the thermal diffusion coefficient k (b) depending on the number of measurement data points used in the approximation

An example of searching for optimal approximation conditions is presented in Fig. 4. The highest quality of linear approximation (r = 1) is achieved by definition for each pair of data points (n - m = 2, Fig. 4a). Taking into account more points, the value of the correlation coefficient r decreases sharply and then increases again reaching its maximum at n - m = 56 (r_{max} = 0.998) for the time interval from t_m = 0.54s to t_n = 1.66s (Fig. 3).

Picture Fig. 4b shows the value of the coefficient A of the linear approximating function and the value of the thermal diffusion

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coefficient k determined based on the approximation. Taking into account the data range optimal for the quality of the approximation, the obtained value was A = 1.8984 s^{-1} and $k = 4.32793 \cdot 10^{-7}\ m^2\ s^{-1}$. Taking into account the material properties c_p and ρ (Tab. 1) and taking into account the dependency (2), the value of the thermal conductivity coefficient for the tested material is K = 0.7267 W/(m · K).

The procedure presented above was used to develop the experimental data (Fig. 3) and in the further part of the work to analyse the results of numerical calculations for the simulation of impulse heating.

4. NUMERICAL MODEL OF THE IMPULSE HEATING ISSUE

The numerical computational model, based on the finite element method, simulates the conditions of impulse heating of a plane parallel plate of material to create a synthetic environment for the sensitivity analysis of the procedure for determining the value of the thermal diffusion coefficient, described in Chapter 0. Using the obtained results of numerical calculations, i.e. time-varying temperature distributions, a linear approximation of the relationship $\ln(T_\infty - T)$ to determine the value of the coefficient A will be carried out, and finally thermal diffusion k is determined. By recreating the procedure of thermal diffusivity determination based on the results of the numerical model, it is possible to estimate the influence of assumptions simplifying the solution of the inverse problem.

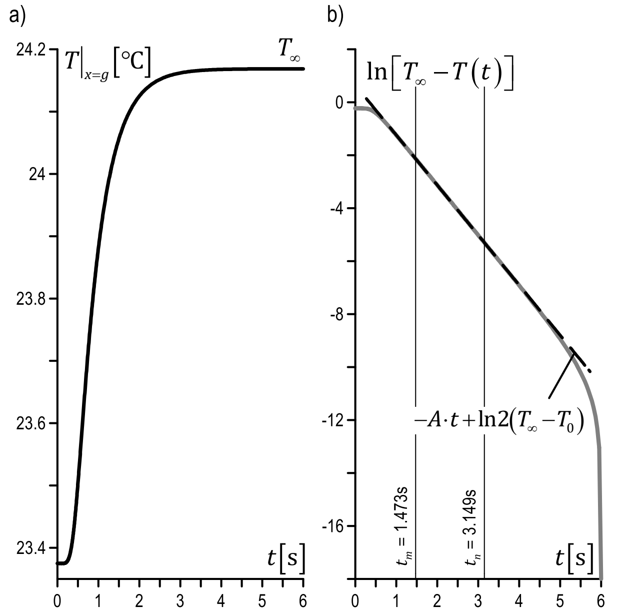

Fig. 5. Simulation of the temperature rise on the back of the sample during impulse heating (a); illustration of the linear approximation procedure to determine the coefficient A and the value of thermal diffusivity k (b)

To solve the one-dimensional problem of heat conduction, described by the Eq. (3) and with the application of boundary conditions (4), the finite element method was used. Using the material properties of Tab. 1, a rectangular shape of the heating function $q(t)$ was assumed with the maximum value of the surface density of the heat flux q = 10 kW/m^2 and the previously determined value of the heat conduction coefficient K = 0.7267 W/(m · K). The computational model consists of 300 one-dimensional second-order elements (square shape function) with linear length distribution and compaction on the side of the impulse-heated surface. The ratio of the size of the largest to the smallest element is 5, which gives the values l_{min} = 0.00166 mm and l_{max} = 0.00833 mm. The calculations used a time-dependent solution with a time step determined based on the backward differentiation method The Backward Differentiation Formula (BDF) with an initial step of 1e–9 s. The preliminary calculations showed that the temperature on the surface $x = g$ reaches the state of settling after 6 s and this time was taken as the end of the analysis.

The results of the calculations and the graphical interpretation of the procedure for determining the value of the thermal diffusion coefficient based on these results are shown in Fig. 5 . The lower temperature rise (Fig. 5a) than was observed in experimental studies (Fig. 2) is noteworthy. The difference (0.79°C against 1.50°C) results from an underestimation of the assumed value of the surface density of the heat flux q in the calculations. The issue of the influence of thermal excitation impulse parameters on the accuracy of the method will be analysed later in the paper.

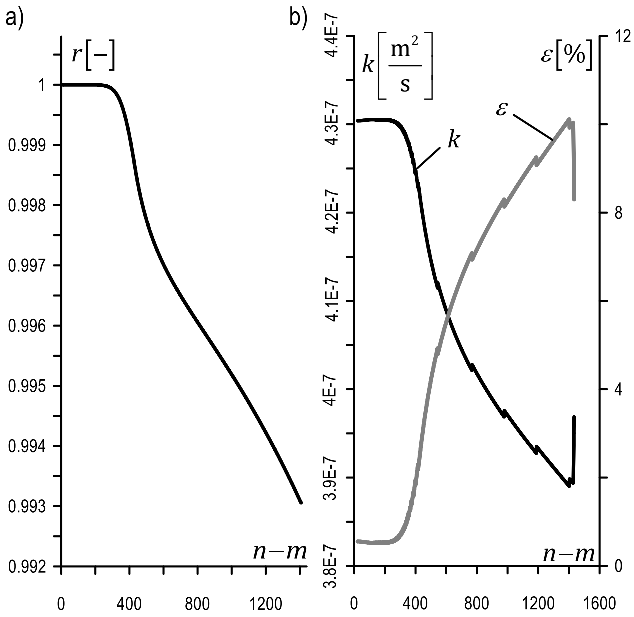

Fig. 6. The maximum value of the linear correlation coefficient r (a) and the value of the thermal diffusion coefficient k and the relative error of the method of its determination ε depending on the number of points included in the linear approximation $n – m$

When comparing the results of numerical calculations and experimental tests, it should be emphasized that there is no measurement noise and a much larger amount of data, which facilitates the linear approximation of the data. The value of the regression coefficient r remains very high even for a large number of points (Fig. 6a). Assuming the use of $n – m$ = 100 points of calculation results, which corresponds to the time interval 1.676 s, the quality of linear approximation of the dependence $\ln[T_\infty - T(t)]$, expressed by the linear regression parameter, is reached at the level r = 0.999999993 (Fig. 6b). The values of the determined thermal diffusion coefficient k and the values of the accuracy parameter of the ε method depending on the conditions of linear approximation are shown in Fig. 6b. The method error is expressed as the relative difference between the obtained result and the reference value k_{ref} of the thermal diffusion coefficient, adopted in the calculations:

$$\varepsilon = \frac{|k-k_{ref}|}{k_{ref}} 100\% \quad (15)$$

For the indicated number of points and the range of data, it gives an accuracy of ε = 0.524%. The result should be assessed as more than satisfactory, especially since the obtained error ε accumulates the inaccuracy of the k coefficient determination method and the inaccuracy of solving the problem using the finite element method.

5. SENSITIVITY STUDY OF THE METHOD FOR DETERMINING THERMAL DIFFUSIVITY

The first parameter, the influence of which on the accuracy of determining the value of the thermal diffusion coefficient was investigated, was the time course of the heat flux $q(t)$, hereinafter referred to as the heating function. Several variants of the heating process were compared while maintaining the same amount of energy supplied to the system during heating, i.e.:

$$\int_0^{t_f} q(t)\mathrm{d}t = const. \quad (16)$$

The above analysed rectangular heating function (1, Fig. 7) with the duration t_f = 0.2 s and the pulse size q = 10 kW/m^2, corresponds to the surface energy of 2 kJ/m^2. Impulse parameters with the course of a parabola (2, Fig. 7) and three variants of triangular waveforms (3–5, Fig. 7) were chosen so that the energy of the heat excitation was identical. The obtained results, in the form of a temperature value change on the plate surface $x = g$, are shown in Fig. 8a. In all cases, the final temperature was T_∞ = 24.17°C, with slight differences in reaching it. Note that for a square (1), parabolic (2) and symmetrical triangle (3) waveform, the differences are negligible.

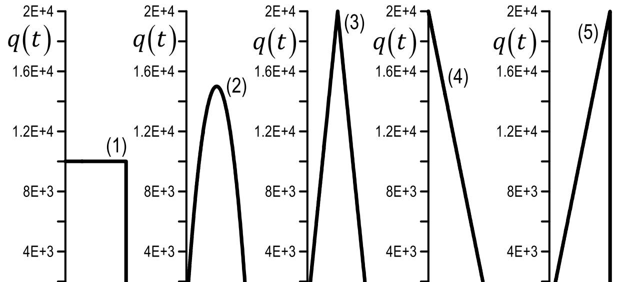

Fig. 7. Time courses of the impulse heating function $q(t)$

By linear approximation of the dependence ln(T_∞ - T) (Fig. 8b) the directional coefficients A and the values of the thermal diffusion coefficient k were determined. The results, including the values of linear regression for r, are provided in Tab. 2. It is worth emphasizing that in the case of variants of the heating function (2–4), the relative error of the method decreased to the value of ε = 0.045%. This should be explained by the improvement of the conditions of the numerical solution of the problem of the finite element method. For the square waveform (1) and the variant (5) of the triangle waveform, there is a sudden jump in the value of the heat flux $q(t)$ from the maximum value to zero, which is a great difficulty for the BDF solver.

Based on the results of the calculations, it can be assumed that the shape of the heating function does not affect the accuracy of the procedure for determining the value of the thermal diffusion coefficient.

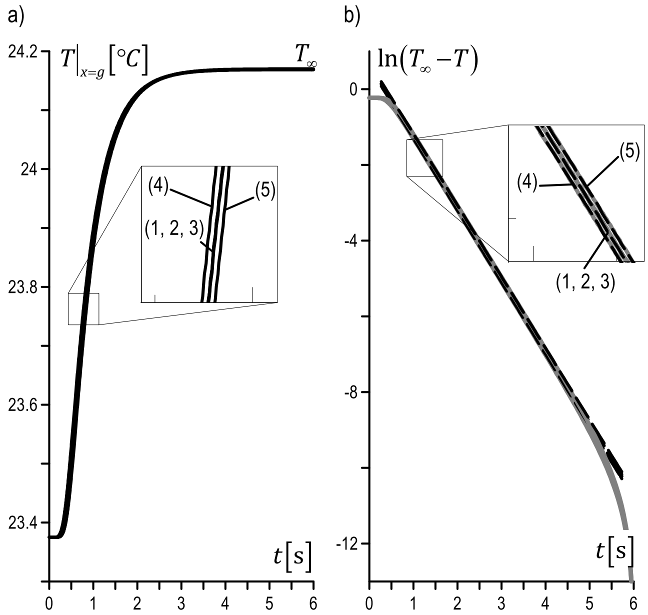

Fig. 8. Temperature increase on the back of the sample during impulse heating (a); illustration of the linear approximation procedure to determine the coefficient A and the thermal diffusivity value k (b) for different time courses of the heating function $q(t)$ (1–5)

Tab. 2. Summary of the results of calculations of the thermal diffusion coefficient k for various courses of the heating function $q(t)$

Case	A (s^{-1})	r (-)	k (m^2/s)	ε (%)
(1)	1.88807	0.999999999	4.304e-07	0.546
(2)	1.8993	0.999999989	4.33e-07	0.046
(3)	1.89931	0.999999989	4.33e-07	0.046
(4)	1.89929	0.99999999	4.33e-07	0.045
(5)	1.88806	0.999999999	4.304e-07	0.546

The next analysed parameter of the thermal excitation is its size assuming a constant duration t_f = 0.2s and a rectangular course of the heating function $q(t)$. The surface density of the heat flux varies in the range of $q = 10 \div 10^6$ W/m^2. In this calculation variant, the amount of energy supplied to the tested material sample is different, which means that the temperature increase on the sample surface $x = g$ after the completion of heating changes in the range of 0.00079÷79.19°C (Fig. 9a). It is also worth noting that the time of settling the temperature after the end of the thermal pulse is the same regardless of the size of the pulse.

An obvious, but very important conclusion is that the magnitude of the thermal excitation impulse should be selected to obtain the optimal conditions for recording temperature changes on the opposite surface for the applied measurement system. An attempt to determine the value of thermal diffusivity with too small a temperature difference after thermal excitation may be burdened with a significant measurement error.

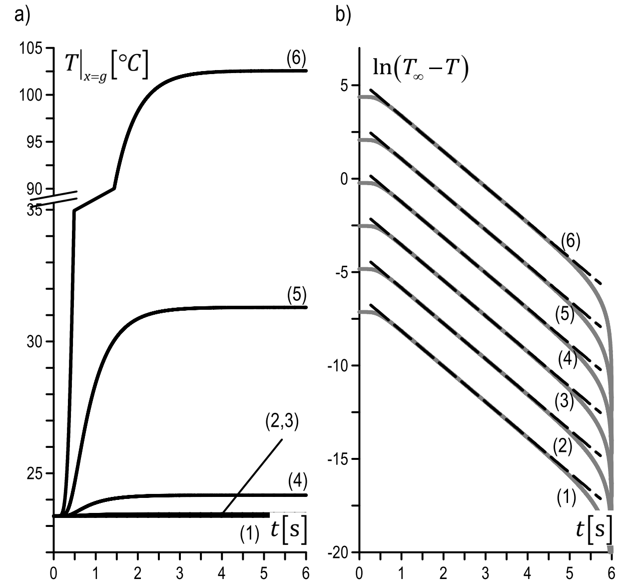

Fig. 9. Temperature increase on the back of the sample during impulse heating (a); illustration of the linear approximation procedure to determine the coefficient *A* and the thermal diffusivity value *k* (b) for different values of the heat flux surface density *q* = 10 (1), 100 (2), 1000 (3), 10000 (4), 2.1544 (5), 1 · 10^6 (6) W/m^2

Despite the very different end values of T_∞ achieved, in the process of linear approximation of the dependence ln(T_∞ - T) the parallel lines (Fig. 9b) were obtained, which showed no effect of the heat impulse size on the accuracy of the procedure for determining the value of the thermal diffusion coefficient. In Tab. 3 the obtained results are presented and, as can be seen in the conditions of the numerical experiment, the method was shown to be insensitive to the heat pulse size.

Tab. 3. Summary of the results of calculations of the thermal diffusion coefficient *k* for different values of the surface density of the heat flux *q*

Case	*q* (W/m^2)	*A* (s^{-1})	*r* (-)	*k* (m^2/s)	*ε* (%)
(1)	10	1.89937	0.999999996	4.33005e-07	0.049
(2)	100	1.89937	0.999999996	4.33005e-07	0.049
(3)	1,000	1.89937	0.999999996	4.33005e-07	0.049
(4)	10,000	1.89937	0.999999996	4.33005e-07	0.049
(5)	100,000	1.89937	0.999999996	4.33005e-07	0.049
(6)	1,000,000	1.89937	0.999999996	4.33005e-07	0.049

Another parameter that may affect the accuracy of determining the value of the thermal diffusion coefficient is the duration of the thermal input pulse t_f . Fig. 10a shows the results of the simulation of impulse heating of the sample with the excitation duration t_f variable in the range from 0.1 s to 10 s and the same values of the surface density of the heat flux *q*. As in the previous calculations, the amount of heat supplied to the test sample is different. However, in this case, the time to reach the steady-state conditions, i.e. reaching the temperature T_∞ is different. The parameters for the calculations and the results are presented in Tab. 4. The plot of the dependence of ln(T_∞ - T) in time (Fig. 10b) indicates that the length of the interval with a linear characteristic decreases with the increase of the pulse length t_f. This can be seen especially clearly in Fig. 11, showing the regression values *r* versus the number of points used in the linear approximation (*n* - *m*). For subsequent calculation variants, the range of data for which a high value of the approximation quality parameter is achieved is reduced. For the thermal impulse excitation test with an impulse of length t_f = 10 s, the relative error of the thermal diffusivity determination method was 3.66%.

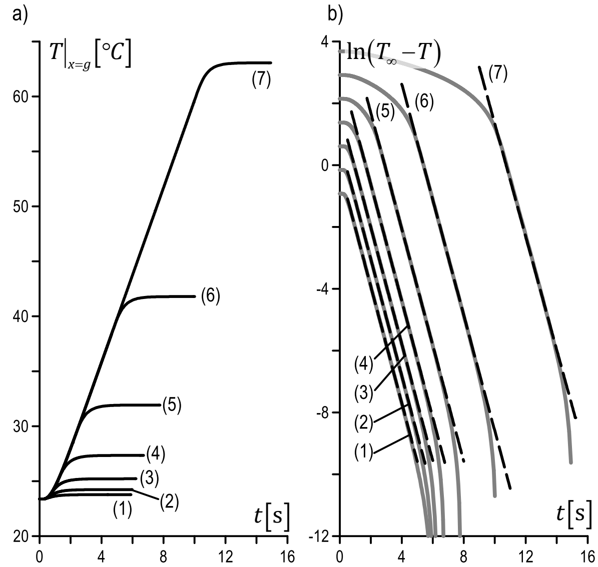

Fig. 10. Impulse heating (a) and determination of thermal diffusivity (b) for different values of the heat pulse duration t_f = 0.1 s (1), 0.21544 s (2), 0.46416 s (3), 1.0 s (4), 2.1544 s (5), 4.6416 s (6), 10 s (7)

Tab. 4. Summary of the results of calculations of the thermal diffusion coefficient *k* for different durations of the heat pulse t_f

Case	tf (s)	A (s–1)	r (-)	k (m 2/s)	ε (%)
(1)	0.1	1.88852	0.999999999	4.30532e-07	0.522
(2)	0.21544	1.888	0.999999999	4.30412e-07	0.55
(3)	0.46416	1.88686	0.999999998	4.30153e-07	0.61
(4)	1.0	1.88442	0.999999997	4.29597e-07	0.738
(5)	2.1544	1.8792	0.999999996	4.28406e-07	1.01
(6)	4.6416	1.86795	0.999999491	4.25842e-07	1.61
(7)	10.0	1.82894	0.99960493	4.16948e-07	3.66

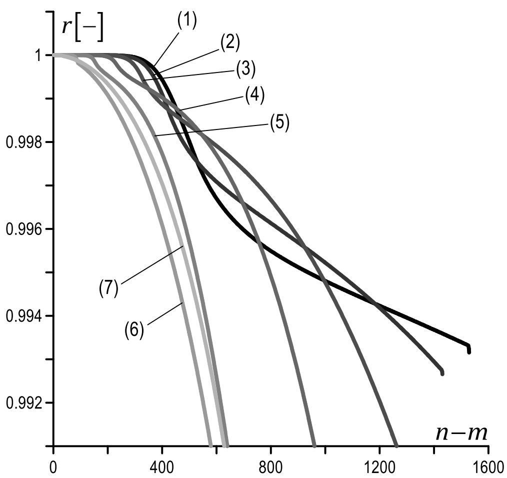

Fig. 11. Maximum regression values *r* obtained for a different number of *n* – *m* points of the linear approximation at the pulse duration t_f = 0.1 s (1), 0.21544 s (2), 0.46416 s (3), 1.0 s (4), 2.1544 s (5), 4.6416 s (6), 10 s (7)

Nevertheless, it should be recognized that the requirement to use a very short duration of the thermal excitation impulse is not a critical condition. Despite a slight reduction in accuracy, it can be assumed that the thermal excitation period is lengthened, e.g., to increase the total thermal energy supplied during the test.

In dependence (8), used to determine the value of the thermal diffusion coefficient k , there is a sample thickness parameter g. Its value can be influenced at the stage of preparing samples for conducting experimental trials. Fig. 12a shows the results of the simulation of impulse heating of samples with thicknesses varying from 0.5 mm to 5 mm while maintaining other conditions consistent with those described in Chapter 4. In the process of linear approximation of the dependence $\ln(T_\infty - T)$, lines with different directional coefficients A (Fig. 12b) were obtained, but after substitution into a dependency (8) the obtained values of the thermal diffusion coefficient (Tab. 5) do not correlate with the sample thickness.

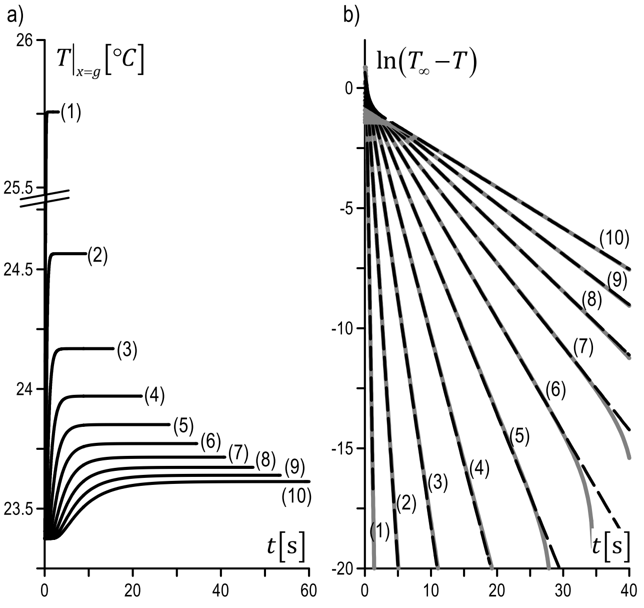

Fig. 12. Impulse heating (a) and determination of thermal diffusivity (b) for different sample thicknesses g = 0.5 (1) ÷ 5.0 (10) mm

Tab. 5. Summary of the results of calculations of the thermal diffusion coefficient k for various thicknesses of the sample g

Case	g (mm)	A (s–1)	r (-)	k (m2/s)	ε (%)
(1)	0.5	16.9289	1.000	4.28815e-07	0.92
(2)	1.0	4.2473	1.000	4.30341e-07	0.57
(3)	1.5	1.88866	1.000	4.30564e-07	0.51
(4)	2.0	1.06254	1.000	4.3063e-07	0.5
(5)	2.5	0.680069	1.000	4.30659e-07	0.49
(6)	3.0	0.472287	1.000	4.30674e-07	0.49
(7)	3.5	0.346998	1.000	4.30688e-07	0.49
(8)	4.0	0.265688	1.000	4.30718e-07	0.48
(9)	4.5	0.209969	1.000	4.30804e-07	0.46
(10)	5.0	0.170174	1.000	4.31055e-07	0.4

As in the previous calculation variant, the time of reaching the conditions set after the thermal impulse excitation is variable and in this case, depends on the thickness of the sample. Proper selection of this parameter can improve the conditions for carrying out measurements during experimental tests. For a given frequency of measurements of the sample surface temperature and for materials with a high speed of heat wave propagation, it may turn out that the number of measurements that are in the linear dependence zone $\ln(T_\infty - T)$ will be too small to accurately determine the A coefficient and to determine the value of the thermal diffusion coefficient. If it is not possible to carry out measurements at higher registration frequencies, then the test should be performed on a sample of greater thickness.

The above-mentioned conclusions regarding the thickness of the sample and the previous ones concerning the heat impulse duration and its size were formulated based on the results of simulation calculations of impulse heating in a convection-free environment. It can be expected that in real conditions, the process of heat exchange between the heated sample and the environment may affect the accuracy of the determination of the thermal diffusivity value.

The attempt to estimate this impact will be made based on the results of calculations using the finite element method of the impulse heating model of the plate at different values of the convective heat transfer coefficient h, included in the analysis in the form of boundary conditions:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = h[T(t, x = 0) - T_0], \quad t > t_0,$$
$$\left.\frac{\partial T}{\partial x}\right|_{x=g} = h[T(t, x = g) - T_0], \quad t > t_0, \tag{17}$$

assuming that the initial temperature of the plate before heating is equal to the ambient temperature: $T_0 = T_a$. The other boundary conditions comply with the conditions (4).

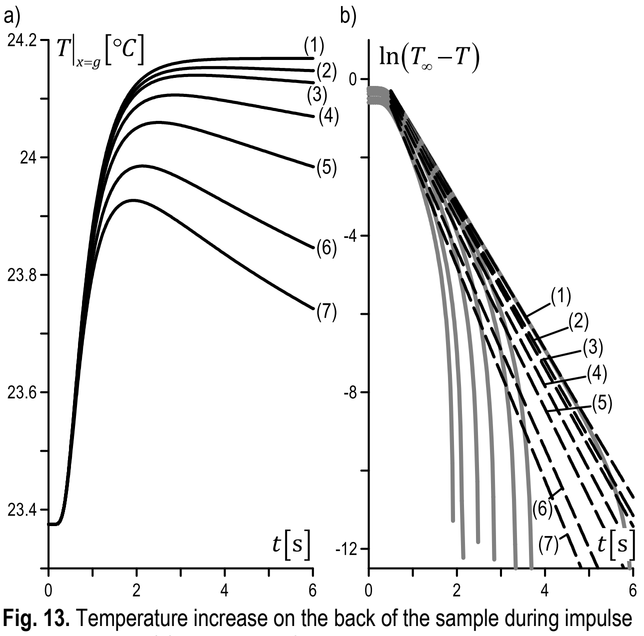

Fig. 13. Temperature increase on the back of the sample during impulse heating (a); illustration of the linear approximation procedure to determine the coefficient A and the value of thermal diffusivity k (b) for different values of the convective heat transfer coefficient h = 0÷150 W/(m^{2}K) (1–7)

The calculation results, in the form of a plot of temperature values on the surface $x = g$, are shown in Fig. 13a. For all cases where the value of the convective heat transfer coefficient other than zero was used, no temperature determination was observed in the time interval covered by the analysis. In these cases, the

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temperature $T_\infty = T_{max}$ was assumed. Dependence $\ln(T_\infty - T)$ and the results of linear approximation are shown in Fig. 13b and their cursory analysis shows that the presence of convection reduces the length of the linear segment of the dependence. This makes it difficult to carry out a linear approximation and the obtained results are subject to an error (Tab. 6), the greater, the more intense is the heat exchange process by the convection mechanism. Quality analysis of approximation (Fig. 14) confirms this thesis, the number of points lying on the numerical part of the dependence $\ln(T_\infty - T)$ decreases sharply with the onset of convection.

Tab. 6. Summary of the results of calculations of the thermal diffusion coefficient *k* for different values of the convective heat transfer coefficient *h*

Case	h (W/[m2K])	A (s–1)	r (-)	k (m2/s)	ε (%)
(1)	0	1.88807	0.999999999	4.30428e-07	0.55
(2)	5	1.97308	0.999982033	4.49809e-07	3.9
(3)	10	2.02164	0.999962527	4.60878e-07	6.5
(4)	25	2.13749	0.999904664	4.8729e-07	13
(5)	50	2.28404	0.999811522	5.20699e-07	20
(6)	100	2.53275	0.999634372	5.77397e-07	33
(7)	150	2.73303	0.999466132	6.23055e-07	44

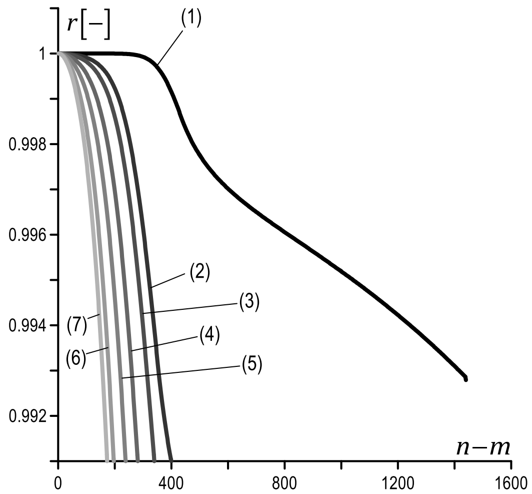

Fig. 14. The maximum regression values *r* obtained for a different number of points *n - m* of the linear approximation at different values of the convective heat transfer coefficient $h = 0 \div 150$ W/(m^{2}K) (1–7)

Based on additional calculations, not included in the paper, the following conclusions were formulated regarding the influence of the convection phenomenon on the thermal diffusivity determination procedure:

– The effect of convective cooling on the surface of the impulse-heated sample and the observed surface on the determined value of the thermal diffusion coefficient is the same.
– The error caused by the presence of convection is independent of the magnitude of the thermal excitation impulse *q*. It is not an obvious conclusion and its justification may be the fact that only a short part of the $T(t)$ heating curve is used for the thermal diffusivity value determination procedure.
– The error of the method of determining the value of the thermal diffusion coefficient caused by convection cooling is the greater, the longer the time of determining the maximum temperature on the opposite surface of the sample $T_\infty = T_{max}$. For a given material, this time is inversely proportional to the thickness of the sample. The requirement to use small sample thicknesses to minimize the effects of convection forces the use of measurement systems with a high frequency of measurement recording, as mentioned above.

An in-depth analysis of the effect of convection cooling on the method of determining the value of the thermal diffusion coefficient goes beyond the scope of this work and requires further calculations. In the literature, for natural convection in air, the value of the heat transfer coefficient *h* is estimated at the level of 5–10 W/(m^2 · K) and, as the calculations have shown, it causes a method error in the range of 3.9%÷6.5%. It should be emphasized, however, that the indicated level of convective heat transfer intensity is achieved under steady-state conditions, with the development of convective air movement. For dynamic conditions, convective heat transfer nucleation, the actual value of the heat transfer coefficient will change over time $h = h(t)$ and for short observation times, it will be lower than the indicated values [11]. It is worth pointing out that in the work of [10], the accuracy of determining the value of the thermal diffusion coefficient of 316 L steel was 0.5%.

6. SUMMARY

Based on the simulations of the impulse heating of a plane-parallel plate of material and the reproduction of the procedure for determining the value of the thermal diffusion coefficient based on the solution of the inverse heat conduction problem, the following conclusions can be drawn:

1. The method is not sensitive to the way impulse heating is carried out. The shape of the heating function, the amount of energy supplied, the size of the pulse, and its height and duration (within reasonable limits) do not affect the accuracy of the method for determining the thermal diffusion coefficient under the conditions of the numerical experiment.
2. The results of the numerical calculations did not show any correlation between the accuracy of the determination of the thermal diffusion coefficient and the characteristics of the sample, i.e. its thickness. One can confidently extend this conclusion to the thermo-physical properties of the material. The method is used both for testing good and weak heat conductors.
3. The temperature change observed during the tests is directly related to the amount of thermal energy supplied to the sample. By selecting the density of the surface heat excitation energy *q*, the impulse duration t_f, it is possible to induce an increase in temperature on the observed surface, adjusted to the sensitivity of the measuring system used.
4. With the given thermo-physical properties of the tested material, the thickness of the sample is directly related to the time of the temperature settling after the end of the impulse of the thermal excitation. This has an impact on the required speed of making and recording temperature measurements.
5. The results of the simulation of impulse heating in conditions of convective heat exchange showed a critical influence on the accuracy of the method of thermal diffusivity determina-

tion. This impact can be minimized without applying special test conditions, by appropriate selection of the test parameters, i.e. heat excitation energy and test duration.
6. The above conclusions indicate that the measuring system used in the tests should be characterized by high sensitivity and frequency of taking and recording temperature measurements. Its absolute accuracy is not critical here.

Despite the described limitations of the method, its advantages, i.e. simplicity, short implementation time, and no special requirements for sample preparation, make it a good tool for determining the value of the thermal diffusion coefficient of materials.

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Adam Adamowicz: https://orcid.org/0000-0002-4475-968X