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SUPPRESSION OF NONLINEAR EFFECTS IN ACOUSTIC RESONATORS

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This paper deals with description of finite-amplitude standing waves in both closed and semi-closed resonators. Numerical solutions of second-order one-dimensional model equation are used. The nonlinear standing waves are generated by a piston or shaker. Numerical results of multifrequency driving technique for suppressing of higher harmonics are presented.

INTRODUCTION

The problem of finite-amplitude acoustic standing waves in confined geometries is of considerable interest in different fields of physics, especially in physical acoustics. Energy stored in the form of acoustic standing waves in resonant cavity can be utilized in many branches of industry, medicine, etc. When a standing wave is driven into high amplitude, nonlinear effects couple energy from low- to high- frequency modes, ultimately resulting in shock wave formation and heightened dissipation. These nonlinear effects can be suppressed with the use of a disonant resonator in which modal frequencies are not integer multiples of fundamental mode frequency. Multifrequency drive technique can be also used for reduction of the nonlinear effects and thus more effective energy storing. For theoretical study of waveform it is necessary to solve model equations appropriately chosen for particular case.

1. MODEL EQUATIONS

An one-dimensional second order model-equation was used for description of nonlinear acoustic standing waves in constant-diameter resonator.

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2}+\rho_0\frac{\partial^2 u}{\partial x\partial t} = \frac{b}{\rho_0 c_0^4}\frac{\partial^3 p'}{\partial t^3}+\frac{\partial}{\partial t}\left(\frac{\beta-1}{\rho_0 c_0^4}\frac{\partial p'^2}{\partial t}+\frac{\rho_0}{c_0^2}\frac{\partial u^2}{\partial t}+\frac{\rho_0}{c_0^2}ua\right), \tag{1}$$

$$\frac{\partial p'}{\partial x} = -\rho_0\frac{\partial u}{\partial t}-\rho_0 a. \tag{2}$$

Derivation of this equation can be found in [3]. $p' = p - p_0$ is acoustic pressure, u is particle velocity, ρ_0 is equilibrium acoustic density of ambience, c_0 is small signal sound speed and symbol a represents resonator acceleration. Symbol β represents nonlinearity parameter defined as

$$\beta = \frac{\gamma + 1}{2} \qquad \text{and} \qquad b = \left(\zeta + \frac{4}{3}\eta\right) + \kappa\left(\frac{1}{c_V} - \frac{1}{c_p}\right) \tag{3}$$

is diffusity coefficient, where η and ζ are viscosities, κ is heat conductivity coefficient, c_p and c_V are constant pressure and constant volume specific heats, respectively, $\gamma = c_p/c_V$. This set of model equations is written in coordinates that are moving together with the resonator cavity. These equations take into account influence of viscosity and heat conduction, it assumes satisfying of resonant conditions thanks to constant temperature in resonant cavity.

For numerical purposes, it is better to rewrite equations (1), (2) in dimensionless form. Using

$$X = \frac{x}{L}, \quad T = \omega t, \quad A = \frac{a}{L\omega_0^2}, \quad U = \frac{u}{L\omega_0}, \quad P = \frac{p'}{\rho_0 L^2 \omega_0^2}, \tag{4}$$

where L is length of the resonator, ω is the angular frequency of the periodic force that shakes the resonator, $\omega_0 = \pi c_0/L$ is fundamental mode of the resonator. After rewriting into frequency domain we obtain

$$\begin{aligned} \frac{\mathrm{d}U_k}{\mathrm{d}X} &= -k^2\pi^2 G\Omega P_k - \mathrm{j}\pi^2 k\Omega P_k + \mathrm{j}k\,\pi^4\frac{(\beta-1)}{2}\Omega \sum_{i=-N+k}^{N} P_{k-i}P_i + \\ &+ \mathrm{j}k\pi^2\Omega \sum_{i=-N+k}^{N} U_{k-i}U_i + \pi^2 \sum_{i=-N+k}^{N} A_{k-i}U_i, \end{aligned} \tag{5}$$

$$\frac{\mathrm{d}P_k}{\mathrm{d}X} = -\mathrm{j}k\,\Omega U_k - A_k, \tag{6}$$

where $\Omega = \omega/\omega_0$ and $G = b\omega/\rho_0 c_0^2$.

The fifth-order predictor-corrector method was used for numerical integration. The two-point boundary value problem

$$U_k = 0, \quad \text{for} \quad X = 0, \tag{7}$$

$$U_k = 0, \quad \text{for} \quad X = 1, \tag{8}$$

where integer index k varies from 1 to N, was solved numerically using the shooting-method.

In the case of steady resonant-tube driven with vibrating piston, we obtain similar set of model equations in dimensionless form

$$\begin{aligned} \frac{\mathrm{d}U_k}{\mathrm{d}X} &= -k^2\pi^2 G\Omega P_k - \mathrm{j}\pi^2 k\Omega P_k + \mathrm{j}k\,\pi^4\frac{(\beta-1)}{2}\Omega \sum_{i=-N+k}^{N} P_{k-i}P_i + \mathrm{j}k\pi^2\Omega \sum_{i=-N+k}^{N} U_{k-i}U_i, \\ \frac{\mathrm{d}P_k}{\mathrm{d}X} &= -\mathrm{j}k\,\Omega U_k, \end{aligned} \tag{9}$$

with boundary conditions

$$U_k = -\mathrm{j}\frac{A_k}{k}, \quad \text{for} \quad X = 0, \tag{10}$$

$$U_k = 0, \quad \text{for} \quad X = 1, \tag{11}$$

where index k varies from 1 to N. Symbol A_k represents spectra of dimensionless periodic acceleration of driving piston.

2. MULTIFREQUENCY DRIVING TECHNIQUE

It was shown, that it is possible to suppress higher harmonic components in resonant cavity by means of two- or three-frequency driving force, where additional frequencies are second and third multiples of frequency fundamental, (see [4]). Providing that entire resonator is moving, it is essential to divide the resonator into two halves of different diameter to excite a waveform with second multiple of the fundamental driving frequency. However, standing wave in steady resonator is possible to be excited with arbitrary integer multiples of fundamental mode. Numerical algorithm for modelling of suppression of the higher harmonic components of standing acoustic wave is based on the shooting method. The second harmonic component of the driving force is iteratively estimated to suppress higher harmonic components of acoustic standing wave in resonator.

3. OPEN-ENDED RESONATOR

Linear boundary conditions for modelling of open-ended oscillating resonator tube were used in the form of

$$U_k = 0 \quad \text{for} \quad X = 0, \tag{12}$$

$$\frac{P_k}{U_k} = \frac{1}{\pi}\left[1 - \frac{\mathrm{J}_1(2k\pi\Omega R)}{k\pi\Omega R} + \mathrm{j}\frac{\mathrm{S}_1(2k\pi\Omega R)}{k\pi\Omega R}\right] \quad \text{for} \quad X = 0, \tag{13}$$

where $\mathrm{J}_1()$ and $\mathrm{S}_1()$ are Bessel and Struve functions of first kind and order, $R = r/L$ is radius divided by resonator length. Open-end of resonator is modelled as non-weighted piston-oscillating diaphragm placed in rigid wall. It's impedance is frequency depended, consequently conditions for resonance are not fulfilled for all frequency components and thus they are suppressed.

4. NUMERICAL RESULTS

Comparison of acoustic pressure obtained with resonator driving by vibrating piston and entire resonator shaking is shown in figure 1. Dimensionless acceleration is set to $A = 5 \times 10^{-4}$ in both cases. In all cases length of resonator is $L = 0.17$ m. First and second harmonic components are plotted here, the higher values belong to resonator driven by a shaker.

Spectra of acoustic pressure in cylindrical resonator driven by a piston is shown in figure 2 on the left. Since conditions for resonance are fulfilled for all higher harmonic components, they are excited and energy is dissipated. In figure 2 on the right side it is shown the same resonator driven by first and second harmonic component of piston driving force. Higher harmonic components are quite successfully suppressed and fundamental component of standing pressure wave is higher.

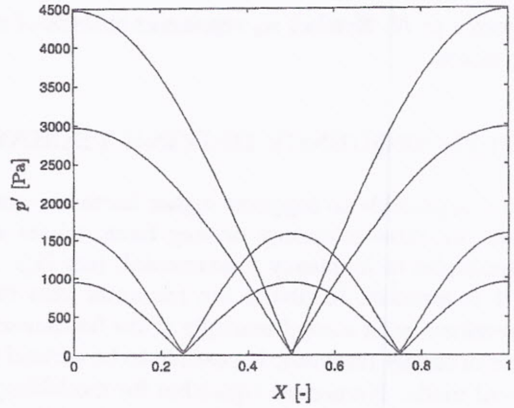

Figure 1: Comparison of first two harmonic components in resonator driven by shaking of entire resonator and by a piston.

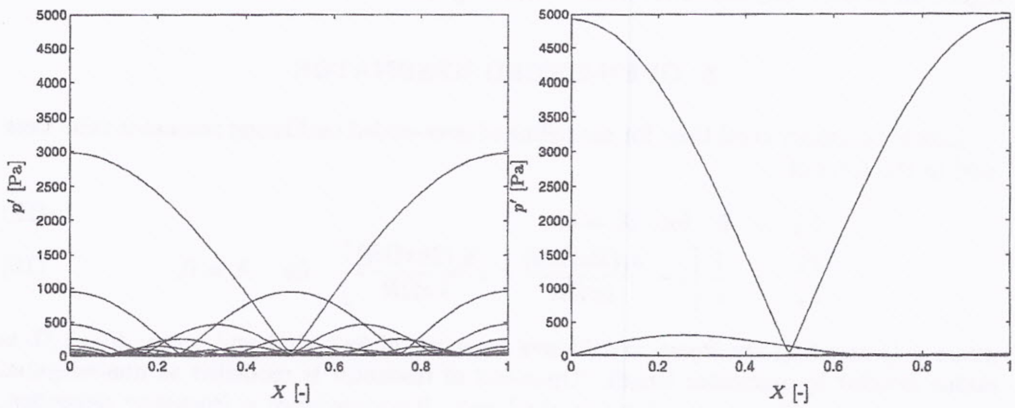

Figure 2: Spectra of acoustic pressure in cylindrical resonator (left), the same in case of multi-frequency driving (right)

Time dependence of acoustic pressure at the end of resonator in case of single-frequency and multifrequency driving is shown in figure 3. In the case of single-frequency, shock wave is generated. Thanks to multifrequency driving, higher harmonic components can be suppressed and weak harmonic distortion appears.

The dependence of higher harmonic components generation by a miscellaneous piston acceleration (acoustic saturation effect) is shown in figure 4.

Frequency response of open-ended resonator is shown in the figure 5 on the left side. Thanks to mass of co-oscillating fluid around open end, frequency of fundamental mode is decreased to approximately $\Omega = 0.41$. Providing that resonator is driven by shaking entire cavity, second multiple of fundamental mode does not excite standing sound waves. It can be seen from figure 5 on the right that owing to frequency dependence of the open-end boundary impedance, resonant conditions are not fulfilled for higher harmonic components and they are thus suppressed.

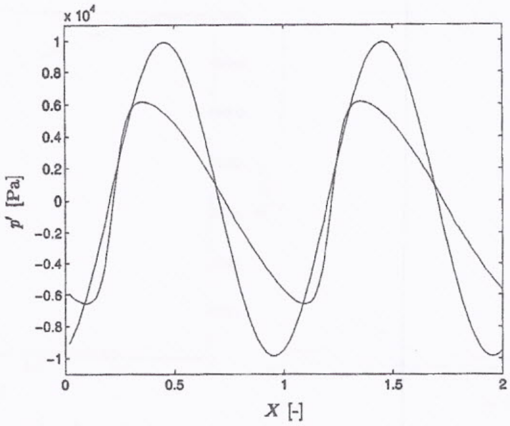

Figure 3: Time dependence of acoustic pressure at the end of the resonator in case of single-frequency and multifrequency driving.

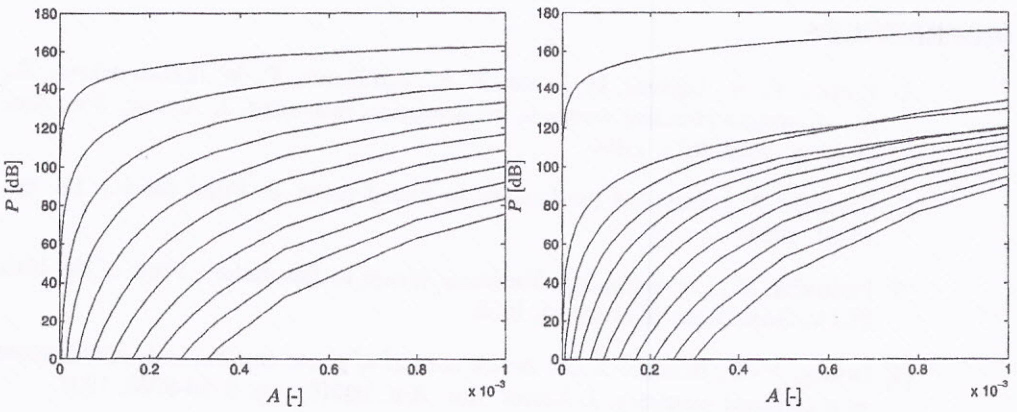

Figure 4: Acoustic saturation effect for single-frequency driving (left) and multifrequency driving (right).

CONCLUSION

Numerical solutions of one-dimensional second-order model equation in frequency domain were presented here. It's derivation can be found in [3]. Comparison of acoustic pressure spectra in resonator driven by a piston or a shaker was accomplished. It was shown that suppression of higher harmonics by multifrequency driving technique is quite a effective, comparable or more efficient than suppression performed by an axisymetric disonant resonator, see [1], [3]. It was also shown that frequency dependence of an open-ended resonator boundary value affects suppression of higher harmonic components causing higher amplitude of fundamental harmonic component excited in resonator.

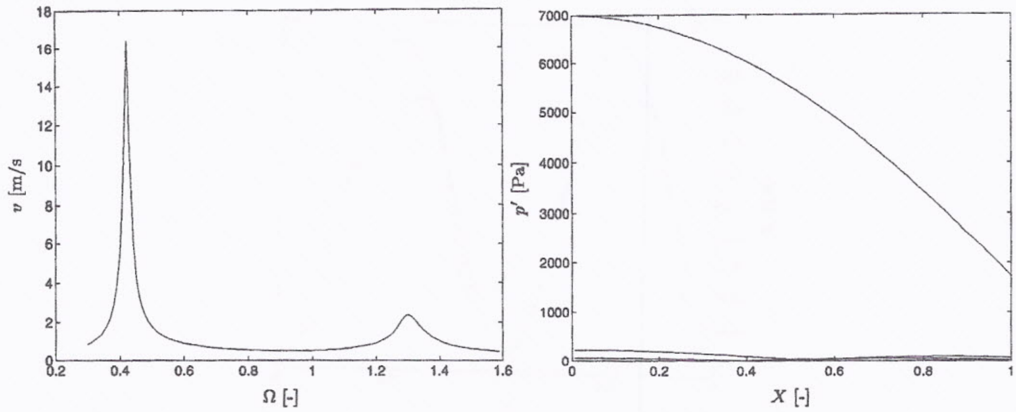

Figure 5: Frequency response of a open-ended resonator (left), distribution of acoustic pressure spectra along resonator length (right).

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