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METHODOLOGY OF THE EFFICIENCY FACTORS OF FINE GRAINED CLAYISH SUSPENSIONS SEPARATION IN MULTILEVELED HYDROCYCLONE SYSTEMS

METODYKA OCENY DOKŁADNOŚCI ROZDZIAŁU DROBNO UZIARNIONYCH ZAWIESIN ILASTYCH W WIELOSTOPNIOWYCH UKŁADACH HYDROCYKLONÓW

The necessity of obtaining many types of products of various granulation requires – more than in case of majority of other processes of mineral processing – constructing complex, multileveled technological systems of classification. They may consist of operations conducted by means of classifying devices of various types or devices of the same type and the same or various constructing parameters.

The paper presents the results of three-staged process of suspension separation of solid phase granulation < 60 μm in two series of investigation conducted in hydrocyclones of the cylindrical part diameter D = 30 mm. The effects of classification as separation efficiency, yields and losses of finest fractions in separation products, sizes of given separation particles and factors of separation efficiency were compared in case when the diameters of hydrocyclones underflow nozzles were the same for each classification stage and in case when these diameters were bigger for each individual separation level.

The main purpose of the paper was the attempt of describing real separation curves obtained experimentally by model functions for various work conditions of hydrocyclones systems.

To determine the characteristics of separation the approximation functions were used to approximate the separation curves. They were function describing Weibull distribution function for the experimental series "a" (equation 8) and function describing logistic distribution function for experimental series "b" (equation 9). Basing on the approximated separation curves the factors of separation efficiency were calculated, which were probable error (E_p), imperfection (I), characteristic particles (d_{25}, d_{75}) and cut points (d_{50}). The yields of fractions < 20 μm and < 2 μm were calculated in hydrocyclones overflows. It was stated that it is impossible to obtain very high (like > 80%) contents of ultrafine fraction < 2 μm in overflow of n^{th} separation level – even by high yields of this fraction in individual overflows and high separation efficiency – if the contents of this fraction in the feed of 1^{st} classification level is very small. Applying the Hancock equation (1) the technological efficiency of obtaining fraction < 20 μm and < 2 μm in hydrocyclones overflows was calculated.

Keywords: multileveled classification systems, small diameter hydrocyclones, classification efficiency, partition curve approximation

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Konieczność uzyskiwania całego szeregu gatunków produktów różniących się uziarnieniem wymaga – w znacznie większym stopniu niż w przypadku większości innych procesów przeróbczych – budowania złożonych, wielostopniowych układów technologicznych klasyfikacji. Mogą się one składać z operacji realizowanych przy użyciu urządzeń klasyfikujących różnych typów, bądź też urządzeń tego samego typu o takich samych lub różnych parametrach konstrukcyjnych.

W pracy przedstawiono wyniki badań trójstopniowego procesu rozdziału zawiesiny o uziarnieniu fazy stałej < 60 μm w hydrocyklonach o średnicy części cylindrycznej D = 30 mm w dwóch seriach badań. Porównano efekty klasyfikacji m.in. takie jak: skuteczność rozdziału, uzyski i straty klas najdrobniejszych w produktach rozdziału, wielkości otrzymywanych ziaren podziałowych i wskaźniki dokładności rozdziału w przypadku gdy średnice dysz wylewowych hydrocyklonów były takie same w każdym stopniu klasyfikacji oraz w przypadku gdy średnice te zwiększały się w kolejnych stopniach rozdziału.

Zasadniczym celem pracy była próba opisu rzeczywistych krzywych rozdziału uzyskanych z doświadczeń funkcjami modelowymi dla różnych warunków pracy układu hydrocyklonów. Do wyznaczenia charakterystyk rozdziału użyto funkcji aproksymujących empiryczne krzywe rozdziału. Były to: funkcja opisująca dystrybuantę rozkładu Weibulla dla doświadczeń serii „a" (wzór 8) i funkcja opisująca dystrybuantę rozkładu logistycznego dla doświadczeń serii „b" (wzór 9). W oparciu o aproksymowane krzywe rozdziału wyliczono wskaźniki ostrości rozdziału: rozproszenie prawdopodobne (E_p), imperfekcję (I) oraz ziarna charakterystyczne (d_{25}, d_{50} i d_{75}). Wyliczono uzyski klasy < 20 μm i klasy < 2 μm w przelewach hydrocyklonów. Stwierdzono, że nie jest możliwe otrzymanie bardzo wysokiej (np. > 80%) zawartości klasy bardzo drobnej < 2 mm w przelewie n – tego stopnia rozdziału, – nawet przy wysokich uzyskach tej klasy w kolejnych przelewach i wysokiej dokładności rozdziału – jeżeli zawartość tej klasy w nadawie I-go stopnia klasyfikacji jest bardzo mała. Korzystając ze wzoru Hancocka (1) wyliczono technologiczną skuteczność wydzielenia klasy < 20 μm i < 2 μm w przelewach hydrocyklonów.

Słowa kluczowe: wielostopniowe układy klasyfikacji, hydrocyklony o małych średnicach, ostrość klasyfikacji, aproksymacja krzywej rozdziału

1. Introduction

The multilevel classification systems are being used in case of necessity of obtaining separation products of various granulations, precisely determined particle fractions limits and also previously determined particle size distribution (Nowak, 1970). They can consist of operation realized by means of classification devices of various types, different characteristics or devices of the same type of the same or different construction parameters. The finer particles are expected to be separated from the initial raw material the higher must be number of separation levels. The number of classification levels necessary to obtain the final products of required granulations depends mainly on efficiency (sharpness) of separation achieved for each following operation.

Better separation efficiency (classification sharpness) is one of the most important tasks concerning flowing classification processes. It is very significant especially in case of fine- and ultrafine-grained materials like mineral excipients, sorbents, abradant powders of required particle size distribution determined by customers or even of precisely determined particle size fractions (Nowak, 2002a, 2002b; Galos & Wyszomirski, 2004). This problem concerns also classification of secondary raw materials and wastes of mineral type, usually earlier processed significantly. That is why most of them are ultrafine-grained, complex mixtures of various components, often of similar characteristics but requiring separation during utilization process.

The clay minerals are materials which feature by ultrafine particle sizes and are mixtures of various minerals which densities are quite similar. To separate these materials the hydrocyclones of small diameters of cylindrical part and underflow nozzles are being applied. It is assumed that, according to reality, about separation in hydrocyclones the radial velocity of particles decides which, in case of clayish materials, depends only on particle size and eventually on its shape. So, the beneficiation effect can be omitted. The particle size is then the separation feature deciding about how particles will separate (Brożek & Surowiak, 2010). It is worthy to notice that the shape of empirical separation curves for clayish materials has some specific character because of the range of sizes of occurring particles and their shapes – plates (Konta, 1995).

The purpose of the presented paper is to elaborate the methodology of comparing results of hydrocyclone systems work separating grained clay suspensions taking into consideration their specific character.

The final classification results in multilevel system, especially the amounts of flow, grades and granulation of separation products depend on obtained cut points (d_{50}) in following levels of classification and level of particles dissipation (determining sharpness of classification). Even small increase of separation efficiency on each classification stage can lead to elimination of at least one classification level what improves the capacity of the system and lower the costs of obtaining final products of the required granulation.

2. Factors of flowing classification process evaluation

The efficiency of classification can be evaluated, among others, on the basis of obtained values of technological factors of separation process or on the basis of characteristics of separation read from partition curve.

Dependably on requirements of hydrocyclone separation products the following factors can be accepted as evaluators of the process:

a) contents of any, required i^{th} particle size fractions (for example, fraction < 2 mm, < 20 mm, < 60 mm) in feed and separation products: overflow and underflow (respectively a_{fi}, a_{oi}, a_{ui}) [no unit or %],

b) efficiency of separation of any required i^{th} particle fraction (S) [%]

$$S = 10000 \frac{(a_{fi} - a_{ui})(a_{oi} - a_{fi})}{a_{fi}(a_{oi} - a_{ui})(100 - a_{fi})} \tag{1}$$

c) solid chase contents (α) in feed, overflow and underflow, respectively α_f, α_o, α_u [no unit or %], for example:

$$\alpha_o = \frac{q_{so}}{q_{zo}} \tag{2}$$

where: q_{so} – mass of overflow dry sample, q_{zo} – mass of overflow sample in suspension form.

The knowledge about the value of α allows to calculate other factors describing, for example, concentration of separation products: β, [kg/m^3] and density of suspension: ρ_z [kg/m^3] according to known formulas.

d) grades of suspension and solid phase in hydrocyclone overflow and underflow, respectively γ_{zo}, γ_{zu}, γ_{so}, γ_{su} [no dimension or %].

In industrial conditions when it is not possible to determine the grades of products from ratio of their masses to mass of the feed directed to the process, these factors can be determined on the basis of solid phase contents in feed and separation products or from individual particle fractions contents in them:

- Grade of suspension in underflow γ_{zu} [no unit or %]

$$\gamma_{zu} = \frac{\alpha_f - \alpha_u}{\alpha_u - \alpha_o} \tag{3}$$

- Grade of solid phase in underflow γ_{su} [no unit or %]

$$\gamma_{sw} = \frac{(\alpha_f - \alpha_u)\alpha_u}{(\alpha_u - \alpha_o)\alpha_f} \tag{4}$$

e) Yield of any i^{th} particle fraction, for example in hydrocyclone overflow ε [no unit or %]:

$$\varepsilon = \gamma_{so}\frac{a_{oi}}{a_{fi}} = \frac{a_{fi} - a_{ui}}{a_{oi} - a_{ui}} \cdot \frac{a_{oi}}{a_{fi}} \tag{5}$$

And, eventually, losses of this fraction in underflow: $\eta = 1 - \varepsilon$ [no dimension or %].

Evaluating the sharpness of classification in hydrocyclones it is very often to use the plot of separation function (Sztaba, 1956; Pudło, 1970). The separation function is set of points showing how infinitely narrow particle size fraction of mean size d_i divided into hydrocyclone overflow and underflow. It is generally accepted that the partition curve $T(d)$ for underflow and $\tau(d)$ for overflow are, respectively of growing and lowering course and are characterized by functions describing normal distribution function or one which is similar to it. Then, it is accepted that such distribution is symmetrical.

In literature, there are known examples of various models of separation functions and partition curves described by them, which shapes are different than normal distribution function. So, they are asymmetric (Zapała, 1987, 1994; Brożek & Turno, 2005). To the most often applied ones the following ones can be selected: Weibull, Gaters-Gaudin-Schuhmann, log-norm (Tarjan, 1974) and beta distribution function (Paszkowska, 1985) as well binomial distributions (Gottfried & Jacobsen, 1977; Gottfried 1978, 1981; Fallon & Gottfried, 1985; Jowett, 1986). Also, the models of three, four and five parameters are known (Tamilmani & Kapur, 1986; Mohanty et al., 2002). However, each model of process or phenomenon is better when the number of parameters is low by the same fitting of empirical data to the model. It can be then said that the two-parametric models are generally better than models containing more than two parameters.

In case when the beneficiation of material occurs in hydrocyclone (significant differences between particles densities occur) the shape of partition curves do not fulfill the conditions mentioned above (Tichonow, 1984; Sztaba, 1988).

Considering the problem of separation sharpness for ultrafine particles in hydrocyclone it occurs that the shape of partition curve is more irregular. Part of the finest particles occurs in underflow. That is why the characteristic angulation of partition curve can be observed for underflow in initial part of the plot (Sztaba, 1956).

On the basis of partition curve for classification the real cut point d_{50} size is determined, which is transferred to both products of separation with the same value of probability. This is then the value of abscissa for ordinate being equal to 0.5. The classification sharpness is described by the inclination of partition curve in ratio to abscissa axis. The bigger is inclination of this curve the better is efficiency of the process.

The most often applied factors to evaluate the separation sharpness are:

– Probable error:

$$E_p = \frac{d_{75} - d_{25}}{2} \tag{6}$$

where: d_{75}, d_{25} – particles sizes which are transferred to underflow with probability being equal to, respectively: 75% and 25%,

– Imperfection:

$$I = \frac{E_p}{d_{50}} = \frac{d_{75} - d_{25}}{2d_{50}} \tag{7}$$

where: d_{50} – cut point.

The higher is value of E_p, the lower is classification sharpness. The higher is value of I the lower is classification sharpness (by perfect separation $I = 0$).

3. Purpose and range of investigation

The purpose of the investigation was to analyze the results of multi-level classification of ultrafine grained clayish suspension and determine the influence of classification conditions on obtained separation efficiency.

The researches were conducted in hydrocyclones of small diameters (D = 30 mm). Two series of experiments of three-stage separation of suspension of solid phase granulation < 60 mm were performed. The obtaining of possibly highest contents of particles < 2 mm in hydrocyclones overflows was significant by minimum losses of this fraction in underflows. In each case the overflow of the previous separation level was the feed for the next one.

The scheme and conditions of experiments conductance were presented on Figures 1a and 1b. The feed for classification characterized by small, of about 6%, solid phase contents and about 20% contents of the fraction < 2 μm. As it can be seen on the attached schemes in case of series 1a the investigation was conducted in hydrocyclones of the same diameters of underflow nozzles (d_u = 4 mm) for each separation level and the experiments of series 1b were conducted by various values of d_u (3, 5 and 7 mm). The rest of constructive parameters of hydrocyclones were identical for each case.

During course of each of individual processes of suspension classification the samples of both products were collected with simultaneous measurement of the time of sampling. These samples were used to determine the rate of suspension flow (hydrocyclone overflows O and underflows U) and – after drying – contents and grades of solid phase in both separation products. The separate samples of classification products were collected in purpose of determining their particle size distribution. The granulation analyzes were performed by means of laser particle sizer.

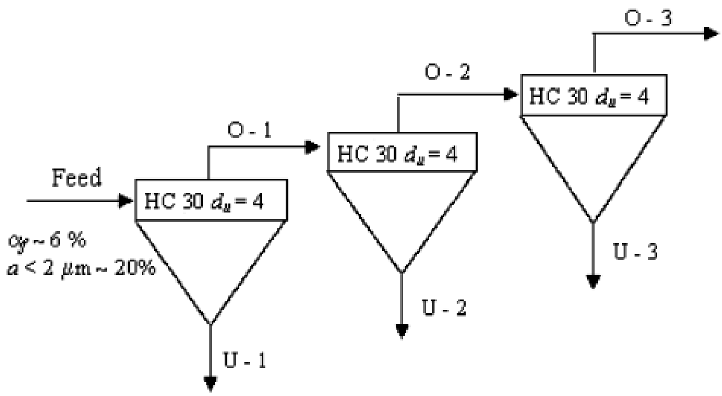

Fig. 1a. Scheme of conducting research over clayish suspension separation for three-stage classification system in hydrocyclones by constant diameter of underflow nozzle

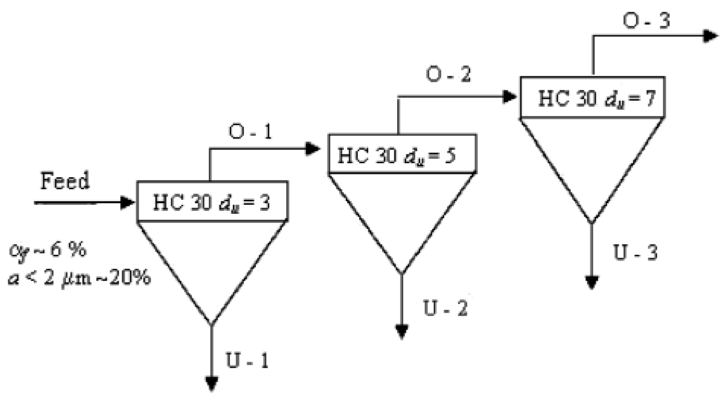

Fig. 1b. Scheme of conducting research over clayish suspension separation for three-stage classification system in hydrocyclones by various diameter of underflow nozzle

4. Results of investigation

The particle size distributions of feed and overflows obtained for individual separation levels were presented in Table 1.

Table 2 presents the technological factors of classification process, such as mass flow rate of suspension (Q_z) and solid phase (Q_s), solid phase contents in suspensions (α), grades of suspension (γ_z) and solid phase (γ_s), contents of fine particle fractions ($a_{<20\ \mu m}$, $a_{<2\ \mu m}$), yields and losses of these fractions in separation products ($\varepsilon_{<20\ \mu m}$, $\varepsilon_{<2\ \mu m}$) and also efficiency of selecting the fractions mentioned above for individual classification levels ($S_{<20\ \mu m}$, $S_{<2\ \mu m}$).

TABLE 1

Particle size distribution for feed and overflows for both series of experiments

Classification level	**Feed**	**Series 1a**			**Series 1b**		
		I	**II**	**III**	**I**	**II**	**III**
Product		**O-1**	**O-2**	**O-3**	**O-1**	**O-2**	**O-3**
***d* [µm]**	**Φ (*d*) [%]**						
< 2	19.80	26.40	29.96	36.00	25.33	29.32	34.14
< 5	51.57	66.26	72.29	75.41	65.61	70.73	80.23
< 8	67.00	82.50	88.00	91.40	84.23	88.29	94.98
< 10	72.84	87.96	92.85	94.56	90.44	93.98	98.19
< 20	87.20	96.51	98.28	99.19	99.32	99.49	100.00
< 40	96.00	99.62	99.84	99.99	99.99	100.00	–
< 60	99.46	99.94	99.97	100.00	100.00	–	–

TABLE 2

Technological factors of separation efficiency

Classification level	**Series 1a**						**Series 1b**					
	I		**II**		**III**		**I**		**II**		**III**	
Product	O-1	U-1	O-2	U-2	O-3	U-3	O-1	U-1	O-2	U-2	O-3	U-3
Parameter												
Q_z [kg·h^{-1}]	985	138	1005	128	981	124	1107	117	1059	207	654	514
Q_s [kg · h^{-1}]	49.6	21.2	45.6	9.5	44.9	7.6	56.2	29.2	48.9	13.6	27.5	26.7
α [%]	5.03	15.38	4.54	7.39	4.57	6.11	5.08	25.04	4.61	6.58	4.21	5.19
γ_z [%]	87.73	12.27	88.71	11.29	88.75	11.25	90.47	9.53	83.68	16.32	55.98	44.02
γ_s [%]	70.06	29.94	82.76	17.24	85.52	14.48	65.81	34.19	78.24	21.76	50.74	49.26
$a_{<20\ \mu m}$ [%]	96.51	64.88	98.28	87.29	99.19	93.56	99.32	61.78	99.44	95.53	100.00	99.32
$a_{<2\ \mu m}$ [%]	26.40	7.50	29.96	15.5	36.00	22.40	25.33	7.05	29.32	18.46	34.14	25.88
$\varepsilon_{<20\ \mu m}$ [%]	77.7	22.3	84.4	15.6	86.2	13.8	75.6	24.4	78.9	21.1	50.9	49.1
$\varepsilon_{<2\ \mu m}$ [%]	89.2	10.8	90.3	9.7	90.5	9.5	87.4	12.6	85.1	14.9	57.9	42.1
$S_{<20\ \mu m}$ [%]	58.82		44.09		45.13		72.27		24.09		25.13	
$S_{<2\ \mu m}$ [%]	24.12		13.81		16.00		26.60		13.34		9.63	

On the basis of data from Table 1 and grades of separation products presented in Table 2 the coordinates of partition curves $T_i(d)$ were calculated for individual classification levels in both series of experiments (Nowak & Surowiak, 2009). To compare the parameters of separation, such as characteristic particle sizes (d_{25}, d_{50}, d_{75}) and separation efficiency factors the attempt of approximation of these curves was done. Because it was impossible to get the satisfying approximation by the function representing the normal distribution function as the approximating function the Weibull distribution function was accepted of general formula:

$$T(d) = 1 - \exp\left\{ -\left[\left(\frac{d}{d_o} \right)^n \right] \right\} * 100 \tag{8}$$

where: d – particle size [µm], d_0 – characteristic particle size.

The obtained empirical partition curves are not of classical shape of letter „S" and the influence of the type of separated material and work conditions of hydrocyclones of small diameters of cylindrical part is noticeable.

The function (8) precisely fitted to the empirical points for experiments conducted in series 1a. The curvilinear correlation index in each case was equal to 0.99 and the mean standard error was equal to about 3%. what proves that the fitting of theoretical function to empirical data is highly compliant.

However, this procedure was not sufficiently good in case of experiments from series 1b. The approximation of partition curves coordinates by Weibull distribution function was not satisfactory. But, in this case the logistic distribution function gave good quality of modeling:

$$T(d) = \frac{1}{1 + \alpha e^{-\beta d}} \tag{9}$$

where: α and β are parameters of the function, d – particle size [μm].

The values of mean standard errors were equal in each case from these series of experiments to about 3%.

The function describing the logistical distribution function was not good to approximate the coordinates of partition curves for experiments of series 1a because of low fitting level. That is why it was decided to calculate the parameters characterizing separation efficiency in various way by applying proper approximation formula for each series. The difficulties in acceptance the same equation approximating real partition curve coordinates for both series of experiments occur probably from various conditions of experiments conductance (Niedoba, 2012; Nowak & Surowiak, 2011; Tumidajski, 1997, 2012). The same diameters of hydrocyclone underflow nozzles for each classification level (series 1a) do not give significant differences between probability of ultrafine particles occurring in underflow as it can be observed in case of changes the size of d_u on bigger ones in following separation levels. As an effect, the shape of partition curves in part describing ultrafine particles is different for both series of experiments. This justifies the application of different equations to approximate the experimental data.

The parameters of functions describing Weibull and logistical distribution functions were presented in Table 3.

TABLE 3

Parameters for Weibull and logistic distribution functions

	For Weibull distribution function		**For logistic distribution function**	
	d_0	n	α	β
I level	22.1	1.04	9.67	0.16
II level	42.37	0.79	6.5	0.15
III level	43.01	0.84	1.82	0.15

Approximations of partition curves coordinates for individual classification levels of both series of experiments were presented on Figures 2a and 2b, respectively for both series of investigation.

The calculated from partition curves cut point values d_{50} and also the separation efficiency factors (E_p, I) were presented in Table 4.

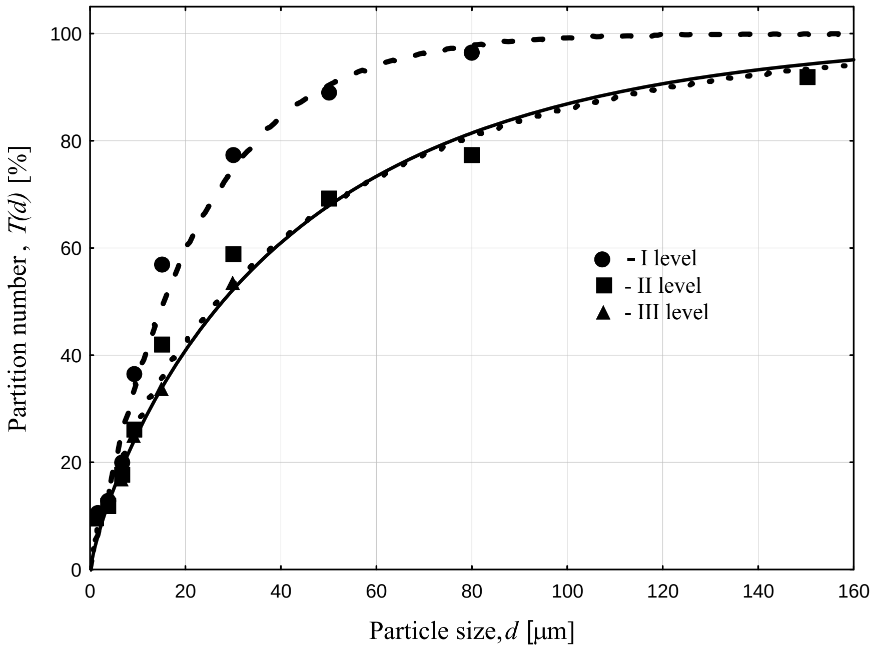

Fig. 2a. Partition curves for series 1a

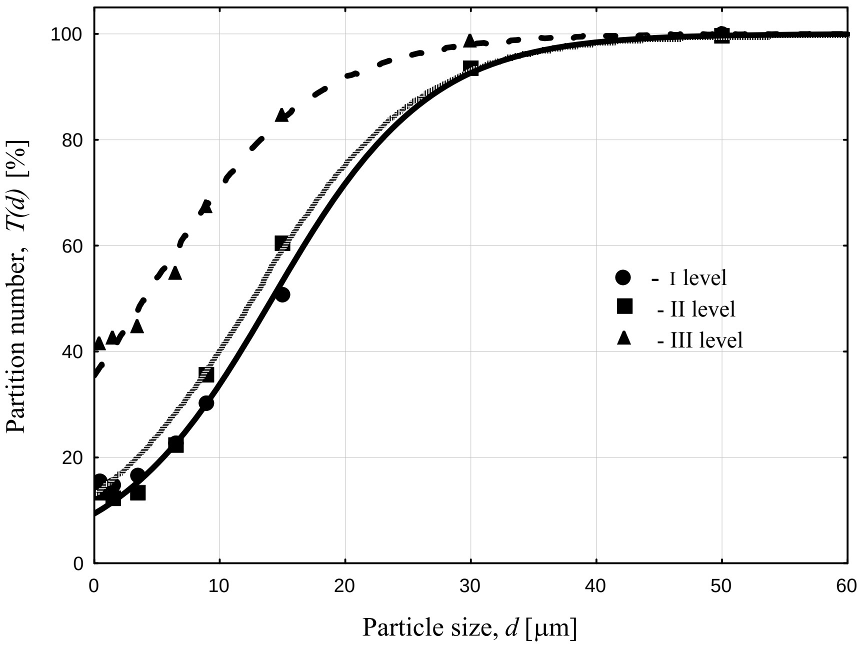

Fig. 2b. Partition curves for series 1b

TABLE 4

Factors of separation efficiency read from partition curves

Classification level	Series 1a			Series 1b		
	I	II	III	I	II	III
Parameter						
d_{50} [μm]	15.5	26.2	27.8	14.2	12.7	4.0
Ep [μm]	11.8	27.6	26.8	6.9	7.3	5.7
I [/]	0.76	1.0	0.96	0.49	0.57	0.915

Analyzing the results of investigation it can be noticed that undependably on work parameters of applied hydrocyclones and classification level the grades of solid phase for obtained overflows are significantly higher than respective grades of underflows, especially for researches of series 1a. The very high are contents of finer fraction than 20 μm in hydrocyclone overflows. In series 1a these contents exceed the value of 96% and in series 1b – 99%. Relatively low are contents of fraction finer than 2 μm in overflow products (series 1a – max 36%, series 1b – max 34%). With growth of amount of classification levels the losses of this fraction in underflows are bigger. It is worthy to notice that there are high values of yields in overflows of fractions < 20 μm and < 2 μm. The yields of fraction finer than 2 μm (ε < 2 μm) are equal to 90% in case of series 1a. In case of series 1b values of ε < 2 μm become lower with growth of amount of classification levels. This is caused by the growth of hydrocyclone underflow nozzles diameters and more ultrafine particles occur in this product. Despite high values of ultrafine fractions yields in overflows, general separation efficiency S is rather low, especially in II and III level of classification. Such observation encourage to consider the possibility of application of sedimentation centrifuge to classify clay materials. However, this requires the analysis of costs connected with certain process. The separation of suspension in case of series 1b occur by lower values of sorted grain than in case of experiments from series 1a and characterizes by higher efficiency. The separation efficiency lowers with growth of amount of classification levels.

5. Final conclusions

Analysis of the results of conducted experiments leads to the following conclusions:

The growth of ultrafine particles contents in overflows of following classification levels is low (in case of series 1a is equal to ~ 10% and in case of series 1b less than 9%). With growth of amount of classification levels the losses of this finest fraction in hydrocyclone underflows grow. These underflows, especially from II and III level, can be returned to first level of separation.

It is not possible to obtain very high (for example > 80%) contents of ultrafine fraction < 2 mm in overflow of n^{th} separation level – even by high yields of this fraction in following overflows and high separation efficiency, if the contents of this fraction in feed for I classification level is very low.

By conducting classification process in hydrocyclones of the same diameters of underflow nozzles on each classification level the size of sorted grain grows from level I till III. With growth of diameter of underflow nozzle in following classification levels the sizes of cut points lower. In each case with growth of number of separation levels the efficiency lows.

In purpose of comparing the separation effects, the coordinates of partition curve should be approximated by properly selected type of function. It seems to be justified by classification of clayish materials. To select the type of approximating function the conditions of process conductance influence which cause that various amounts of ultrafine particles occur in wrong product, what means in this case underflow instead of overflow.

Relatively low growths of finest particle fractions contents in overflow of following separation levels and low classification efficiency in hydrocyclones D = 30 mm, especially for II and III level justify to consider application of other classifying device, like sedimentation centrifuge.

The selection of classifying devices, their location in technological scheme, amount of classification levels should be conditioned by the obtained separation effects together with analysis of costs of each operation.

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