Kowalski Marcin

Magott Jan

Nowakowski Tomasz

Werbińska-Wojciechowska Sylwia
Wroclaw University of Technology, Wroclaw, Poland

Analysis of transportation system with the use of Petri nets

Keywords

logistic support system, transportation system, procurement process, Petri nets, simulation process

Abstract

The paper considers problem of city transportation system performance. Reliability analysis of such a complex system is complicated by several factors. One of them is the possibility of logistic support elements unreliability defined as unavailability of spare elements when desired, what in result may lead to decrease of performance of the system being supported. Thus, both systems must be considered in a single model. However, the simultaneous setting of all structural parameters (e.g. redundancy, repair shop capacity) and control variables (e.g. spare part inventory levels, maintenance policy parameters, time resource) is mathematically a hard problem. This paper investigates Petri net model of the system with the use of Monte Carlo simulation as a solution technique. Comparison of the simulation results with characteristics of real-life system is given.

1. Introduction

The proper organized and reliable logistic support is a composite of all the elements necessary to assure the effective and economical support of a system or its subsystems, at all levels of maintenance for its anticipated life cycle. When logistic activity is narrowed down to the supply activity, logistic support element which represents all the resources necessary to maintain and operate equipment includes: maintenance resources, support personnel, logistic information and data, spares and repair parts, and facilities [18].

To the best authors' knowledge, an effective way for achieving the reliable systems support especially bases on meeting two targets: reliability/availability and cost constraints. Reliability of the logistic support system must come before costs considerations. Every logistic system, operating under diverse system environment, may fail what in consequence may lead to:

- disruption of supporting task realization,
- inability of system to undertake a new task.

As a result, there is a need to take into account the possible unreliability of logistic support elements, which may lead to decrease of performance of the system being supported.

On the other hand, high costs motivate seeking new solutions to reliability and logistic problems for:

- enhancing reliability,
- providing on-time deliveries,
- increased equipment, spare parts and repair parts availability,
- reducing costs and problems arising from systems that fail easily.

For example, long failure free periods result in increased operational capability, fever spare parts need to be stocked, less manpower employed on maintenance activities, and hence lower costs of the whole system and its processes performance.

Most models investigated in the literature on reliability theory focus on maintenance. The prime maintenance objective is to ensure the system performs its intended function. As a result, maintenance should provide the optimal performance level as a balance between maintenance parameters (or cost of maintenance) on one side, and the performance level on the other.

The interest in development and investigation of maintenance problems has been extensively discussed in the literature since the early 1960s. The basic review in the area of maintenance modelling is prepared by Pierskalla & Voelker [20], where authors investigated discrete time vs. continuous time maintenance models, later updated by Valdez-Flores & Feldman [22]. For other surveys see e.g. [3], [14], [15], [1], [19], [21], [23], [24].

However, most of the maintenance models investigated in the literature on reliability theory assume, that all the necessary logistic support resources, which include maintenance resources, support personnel, logistic information and data, spares and repair parts, and facilities, are immediately provided when it is desired. In practice, the repair capacity is not infinite, and logistic information may be unreliable. Moreover, the influence of a spare provisioning policy on the maintenance policy also cannot be ignored, since spares are ordered and carried in the limited quantity, and the procurement lead time is not negligible.

The problem of providing an adequate and efficient supply of spare parts, in support of maintenance and repair of operational systems, has been researched for many decades. Recent overview of these models is made by Nowakowski & Werbińska-Wojciechowska in [17].

Consequently, reliability of complex systems (e.g. transportation systems, aircraft systems) can be difficult to analyze for several reasons. First, both systems, logistic and being supported are integrated and thus should be considered in a single model. However, growing body of existing literature in the investigated research area treats maintenance, replacement and inventory decisions separately [24]. Second, commonly used analytical techniques for reliability evaluation are applied probability theory, renewal reward processes, Markov decision theory, and Fault Trees. Each of these techniques has advantages and disadvantages and the choice depends on the system being modelled.

All of them require simplifying assumptions about time to failure behaviour of the system components. Moreover, Markov method analyses the system by identifying all the different states in which the system can reside and is able to produce accurate system reliability measures by assigning rates of transition between these states. However, the Markov method has its own drawbacks in its application for a relatively large system to establish the state transition model is an intractable task.

Traditional Fault Tree Analysis (FTA) [6] is probabilistic approach to safety, reliability, and risk analysis. Traditional fault trees contain Boolean gates to represent how component failures combine to produce system failures. These fault trees are now called static. In papers [4], [5], dynamic fault trees (DFTs) are presented. Gates of DFTs can express the following features:

- dynamic replacement of failed components from pools of spares,
- failures can occur only in a predefined order.

In paper [2], DFTs have been extended by repair boxes. These boxes can express a repair time of components. In this paper, transformations of different gates and repair boxes into Stochastic High Level Colored Petri Net are given.

In calculation of probabilistic characteristics of systems using DFTs, the following formalisms are used: Markov models [4], [5], Petri nets [2], Bayesian networks [13].

When analyzing the transportation system, we have to analyze not only repair and lead time, but time consuming replacement process and time resource as well. Hence, DFTs with repair boxes are not sufficient to represent the transportation system.

Moreover, Fault Tree with Time Dependencies provides an interesting solution for non-deterministic models [11], [12]. In these models, time parameters are described by minimal and maximal values, but without probabilistic characteristics. The FTTD technique has been proposed as a convenient approach to describe the values of the delay times of system of systems task performance on the ESREL conference in 2008 [10]. The investigated problem has regarded to simple logistic support model performance. Later, there is proposed method application example presented in [9]. However, this method cannot be applied when time between failures and repair times are expressed probabilistically.

In contrast to the analytic approaches Monte Carlo simulation can be broadly used. However, Monte Carlo simulation is time-consuming because of the intensive computations. This is because an extremely large number of simulated samples may be needed to estimate the reliability parameters at a high level of confidence.

Following this consideration, in the paper, Petri nets are used to support the reliability analysis of complex real-life system performance.

The primary contribution of this research is to propose a Stochastic High-level Petri Net model for presented below transportation system. This model is based on standard of High-level Petri Net [7] and on generalized stochastic Petri nets [1]. For this model, simulation experiments have been performed. Results of the experiments have been compared with real-life city transportation system.

Consequently, the rest of this paper is organized as follows: in Section 2, there is a description of tram network performance including all model assumptions. Later, there a Petri net model for the investigated system performance provided. Some comparison results with real-life city transportation system are presented. Finally, the work ends up with summary.

2. Application of tram network

2.1. Tram network performance

Analysis regards to city transportation system performing in Wroclaw city, Poland. The municipal transport services are provided by common carrier MPK Wroclaw.

During operational process of passenger transportation system performance failures of working tram may occur. These unwanted events can cause severe negative consequences for customers, like:

- shutting down of a failed tram from passenger traffic,
- delay of a failed tram,
- detour of other trams working in a system.

In order to minimize the negative consequences of tram unreliability, there are redundant trams maintained in the system.

The redundant trams have been performing in the discussed system since 1990. There is made an assumption, that during average working days there are five redundancies operating in the system, and only three in weekends. Typical allocation of redundancies in the tram network is presented in *Figure* 1. The redundancies have notations of A, B, C, D, E. More information can be found in [8].

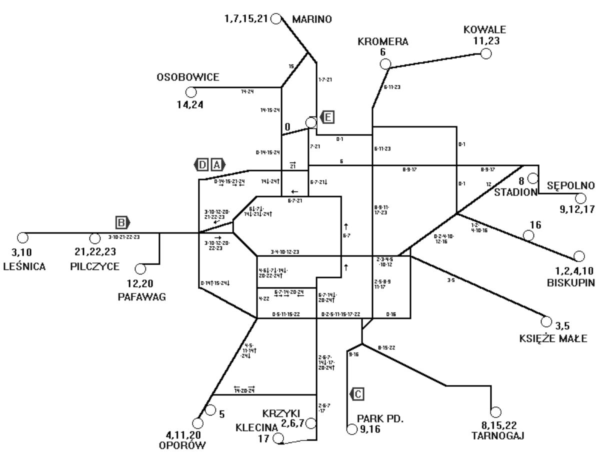

Figure 1. Allocation of redundant trams in the tram network in 2002 [8]

The exploitation process of tram in the transportation system is presented in *Figure* 2.

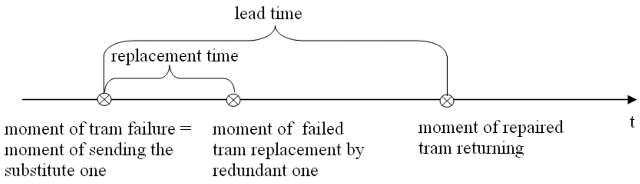

Figure 2. Exploitation process of trams performing in the system [24]

When failed tram is shot down from the system, the redundant one is sent to continue its operational tasks. The decision about this substitution is made by a dispatcher, who knows the expected replacement time, residual working time of redundancies, and other decision criteria. After repair, the substituted tram returns to operate and the redundant one return to tram depot.

Times to failure, replacement, repair and lead times are random variables. As a result, there such a situation can occur that the number of working redundancies is not enough to substitute all failed trams in the system.

There is also made an assumption, that the time resource given for putting back to service of failed tram is defined as minimal time of one tram course performance. Over crossing the defined time resource results in necessity of fine paying by the transportation company.

Following this, one of the main problems, taking into account reliability/availability of the presented system, is definition of right number of redundancies which should perform in the system. Having not enough redundant trams occur in lots of not performed tasks. On the other side, having too many of them cost lots of money.

Other problem is the right definition of the time resource. Too long tolerance time results in occurrence of many disruptions in the system. However, too short time resource increase performance costs of the system.

The application of FTTD technique to model the time relations which occur in the investigated transportation system is investigated in [9].

2.2. Tram network parameters

The operational processes performance of the chosen system of tram service can be described with the use of a simulation model of the system of systems with time dependency, where the operational system is a k out of M system ($k=M$). In the chosen model, Critical Inventory Level (s,Q) is used as a stock policy, and spare elements are equivalent to redundant trams, which are assumed to be reliable. When the tram fails, inventory level is decreased according to the occurred request. At the same moment, the "awaiting for new delivery" begins. According to this, the ordering quantity Q is equal to 1 (see *Figure* 2).

Moreover, when substitute tram is sending to replace a failed one, new "order" is activated. Thus, the time of waiting for new delivery arrival lasts from the moment when redundant tram reduces inventory level. As a result, critical inventory level is given by the following formula:

$$s = l_r - 1 \tag{1}$$

where:

l_r – number of redundant trams maintained in the system

For more information see e.g. [24], [25], [26].

Cases considered during the simulation process performance are presented in *Table 1*.

Table 1. Analyzed cases in simulation process

Case number	Number of redundant trams	Moment of system failure	Time resource limit
1	5	$t_{informing}$	$\max(T_{course})$
2			$\min(T_{course})$
3		$\max(t_{informing}, t_{turning\ off})$	$\max(T_{course})$
4			$\min(T_{course})$
5	3	$t_{informing}$	$\max(T_{course})$
6			$\min(T_{course})$
7		$\max(t_{informing}, t_{turning\ off})$	$\max(T_{course})$
8			$\min(T_{course})$

Table 2. Operational characteristics of system of tram service in Wroclaw city

Operational period of time:	21th August 2001 ÷ 28th February 2002	
	Working days	Free days and holidays
Number of redundant trams:	5	3
Minimal and maximal time resource:	41 [min] 101 [min]	
Operational time of redundant trams:	4.30 – 20.00 5.00 – 20.30 5.30 – 22.30 6.00 – 23.00 6.30 – 0.00	5.00 – 22.30 6.00 – 23.00 6.30 – 0.00
Time zones:	before 6.00 6.00 – 8.00 8.00 – 13.00 13.00 – 17.00 17.00 – 20.00 20.00 – 22.30 after 22.30	before 6.00 6.00 – 9.00 9.00 – 20.00 20.00 – 22.30 after 22.30

In the analysis, performance working days, when 5 redundant trams is in a system, and weekends, when only 3 redundant trams perform in a system, are investigated separately. Moreover, when tram fails the moment of its failure can be equal to the moment of informing the dispatcher about the occurred problem ($t_{informing}$). On the other side, the tram failure moment can be also defined as the moment when failed tram returns to tram depot ($t_{turning\ off}$).

Another problem is the definition of time resource limit. Authors defined two cases, in which the time resource limit is equal to:

- the shortest time necessary to one course performance by a train ($\min(T_{course})$)
- the longest time necessary to one course performance by a train ($\max(T_{course})$)

for the analyzed period of operational time of a system.

As a result, the transportation system characteristics can be obtained. Main random variables in the model have Weibull distribution:

$$F(t) = 1 - \exp(-B_u t^{A_u}) \qquad for\ \ t > 0 \tag{2}$$

where:

A_u – shape parameter for random variable u
B_u – scale parameter for random variable u

Parameters of transportation system' probability distributions are given in Table 3. The exemplary transportation system characteristics are presented in *Figure* 3- *Figure 4*.

Table 3. System' probability distributions' parameters

Case nr	Probability distributions' parameters					
	A_o	B_o	A_r	B_r	A_L	B_L
1 = 2	0,957	0,016	1,243	0,026	1,213	0,007
3 = 4	0,928	0,016	1,219	0,032	1,235	0,008
5 = 6	0,987	0,010	1,345	0,024	1,232	0,008
7 = 8	0,939	0,010	1,214	0,029	1,255	0,009
A_o,B_o - Weibull's parameters of time between subsequent tram failures A_r, B_r - Weibull's parameters of single operational element replacement time A_L, B_L - Weibull's parameters of lead-time time						

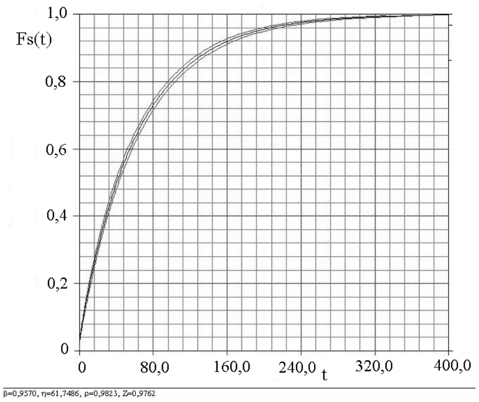

Figure 3. Cumulative distribution function of tram's time to failure (case 1)

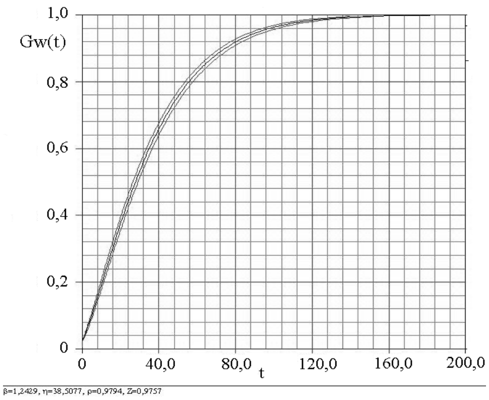

Figure 4. Cumulative distribution function of tram's replacement time (case 1)

2.3. Petri net model

We propose a Stochastic High-level Petri Net model of the investigated transportation system. This model is based on standard of High-level Petri Net [7] and on generalized stochastic Petri nets [1].

High-level Petri Net (HLPN) [7] are bi-parted graphs with two kinds of vertices: places and transitions, see *Figure* 5. Places are denoted by circles. Tokens are located in places. Tokens are denoted by dotes. Distribution of tokens in places describes a state of the net partially. Transitions are fired, what causes a change of distribution of tokens over places.

Representation of time factor is based on generalized stochastic Petri nets [1]. In these nets, there are two kinds of transitions: immediate and timed. Firing time of immediate transition is equal to zero. This transition is denoted by dash. Firing time of timed transitions is expressed by a random variable. That transition is denoted by rectangle. Special case of firing time of timed transition is time given by a real number. If immediate and timed transitions are enabled (can fire) then the immediate one is fired as first.

Meaning of the places of the HLPN from *Figure* 5 is as follows:

p_0	– tokens in this place represent future tram failures with identifier *i* of type *Integer*,
p_1	– tram is failed, token in that place expresses tram failure,
p_2	– failed tram replacement by redundant tram is being performed,
p_3	– it will be explained,
p_4	– tram in repair,
p_5	– repaired tram is ready to work,
p_6	– time resource for a failed tram has not yet passed,
p_7	– time resource for a failed tram has passed,
p_8	– redundant tram is idle,
p_9	– token in this place expresses that token from the place p_6 has been removed.

Integer is the type that is assigned to the following places: p_1, …, p_6, p_9. This type contains identifiers of tram failures. Hence, tokens that are located in these places have identifiers. Tokens in places: p_7, p_8 have no identifiers. Tokens in the place p_8 represent idle redundant trams. Tokens in the place p_7 represent tram failures for that time resource has been exceeded.

Meaning of transitions is as follows:

t_0	– tram failure,
t_1	– start of failed tram replacement by a redundant tram,
t_2	– end of failed tram replacement by a redundant tram,
t_3	– repaired tram returning (redundant tram becomes available),
t_4	– repaired tram is becoming ready to work,
t_5	– time resource has not been exceeded,
t_6	– time resource has been exceeded,
t_7	– repaired tram is ready to work before completion of failed tram replacement by a redundant one,
t_8	– repaired tram is ready to work before spare tram is available.

Now meaning of the place p_3 will be explained. Let τ(t) denote firing time instant of the transition *t*.

p_3 – token in this place expresses that: redundant tram is working if $\tau(t_2)<\tau(t_4)$, redundant tram is idle if otherwise.

Firing times of timed transitions are given as follows:

t_0 –> length of time interval between subsequent tram failures in the system; it is given by Weibull's distribution. It is not time interval between subsequent tram failures of the same tram.

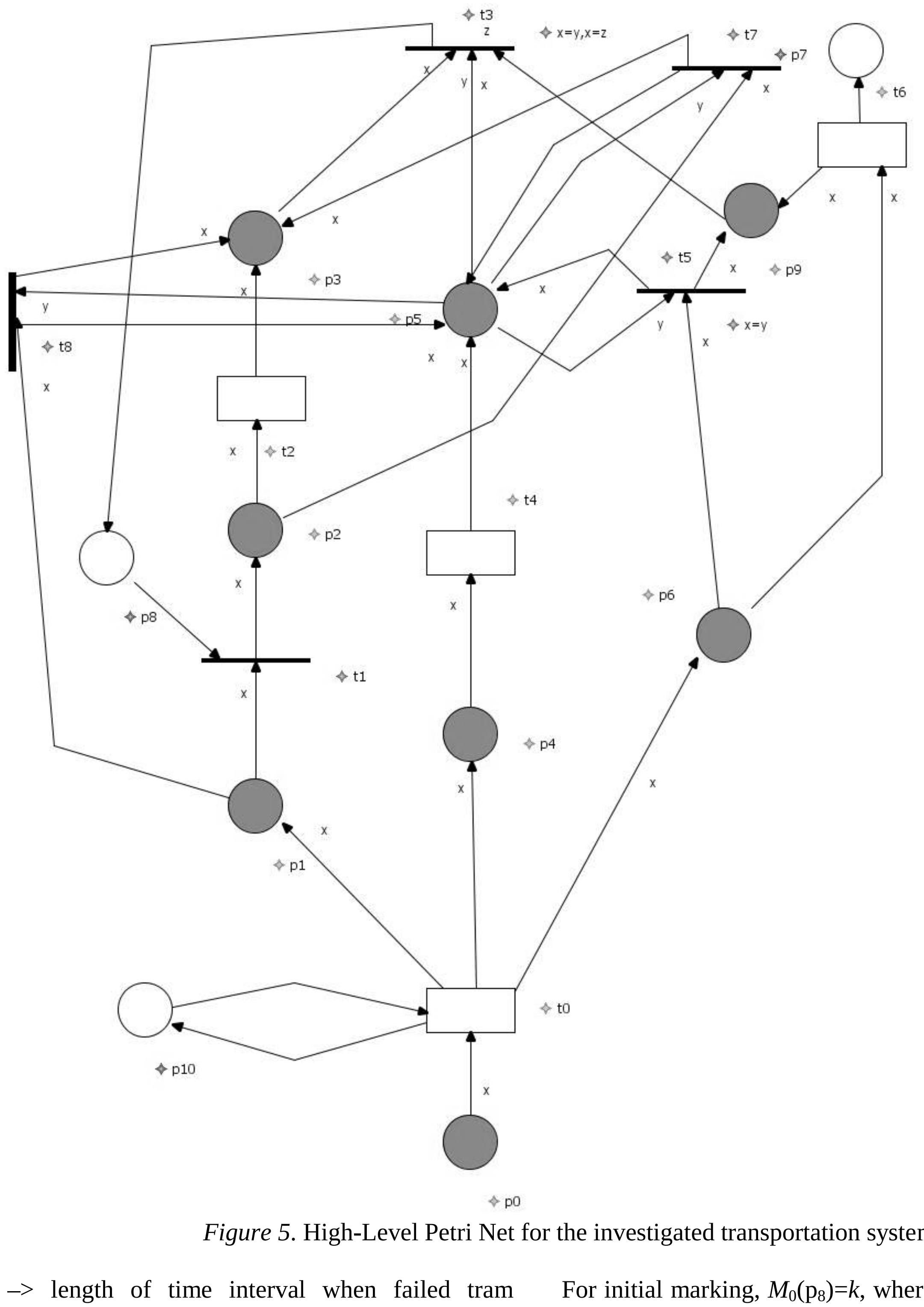

Figure 5. High-Level Petri Net for the investigated transportation system

t_2 –> length of time interval when failed tram replacement by redundant tram is being performed; it is given by Weibull's distribution.
t_4 –> sum of repair time for failed tram and lead time; it is given by Weibull's distribution.
t_6 –> time resource given by a real number.
The transitions t_2, t_4, t_6 are fired according to multiple server semantics: many firing processes can undergo in a given time instant. The transitions t_0, because of the loop around it, is fired according to single server semantics: at most one firing processes can undergo in a given time instant.

For initial marking, $M_0(p_8)=k$, where k is the number of redundant trams. Cycle of redundant tram activities is expressed by cycle of places and transitions t_1, p_2, t_2, p_3, t_3, p_8, t_1.
If the transition t_0 is fired, then tokens with tram failure identifier *i* are put in the places p_1, p_4, p_6. If there is a token in place p_8, then the transition t_1 can be fired. It represents the fact that a redundant tram can be assigned in order to replace the failed tram associated with tram failure identifier *i*.
If failed tram replacement by a redundant tram is finished before a failed one is ready to work after

repair, then redundant tram starts its work. In this case, the token with identifier *i* is added to the place p_3 earlier than the token with identifier *i* is added to the place p_5. Let there be the token with identifier *i* in the place p_3. Let us suppose that time resource for the tram failure *i* have not yet passed. Hence, there is the token with identifier *i* in the place p_6. Because there are the tokens with identifier *i* in the places p_3 and p_6, so the transition t_5 can be fired for bindings x= *i* and *y*= *i*. As a result, token with identifier *i* is added to the place p_9. Let us suppose that the transition t_4 has fired. Now, there are tokens with identifier *i* in places p_3, p_5, and p_9. Hence, the transition t_3 is fired, and the token that represents idle redundant tram is added to the place p_8. The transition t_3 is immediate. Therefore, the transition t_3 is fired in the same time instant when the transition t_4 is fired. It represents such a fact that redundant tram is becoming available immediately after the time instant when the repaired tram is ready to work.

Let us analyze similar case as before, but time resource for the tram failure *i* have passed before time instant when the redundant tram is ready to work. In this case, the transition t_6 is fired before the transition t_2 is fired. As a result of firing the transition t_6, tokens are put in the places p_7 and p_9. A token is located in the place p_3 after removing the token from the place p_6. In this case, the transition t_5 is not fired.

If failed tram is ready to work before completion of failed tram replacement by redundant tram then repaired tram should start to work, and redundant tram should become available for next failure. In such a case, transition t_4 is fired, and next transition t_7 is fired. Therefore, there is the token with identifier *i* in the place p_3. Similar analysis as before can be performed. Let us concentrate on some aspects only. Let us suppose that time resource for the tram failure *i* have not yet passed. Hence, there is the token with identifier *i* in the place p_6. The transition t_5 can be fired. Token with identifier *i* is added to the place p_9. Now, there are tokens with identifier *i* in places p_3, p_5, and p_9. Hence, the transition t_3 is fired, and the token that represents idle redundant tram is added to the place p_8. The transitions t_7, t_5, t_3 are immediate transitions. Therefore, the transition t_3 is fired in the same time instant when the transition t_4 is fired. It represents such a fact that redundant tram is becoming available immediately after the time instant when the repaired tram is ready to work.

2.4. Obtained results

First, the Monte Carlo simulation model of system of systems with time dependency performance obtained with the use of *GNU Octave*, presented in the Section 2.2, has been analyzed in addition to obtained results from real system performance data.

Examples of empirical cumulative distribution functions for the system of systems failure time are given in *Figure* 6 and *Figure 7*.

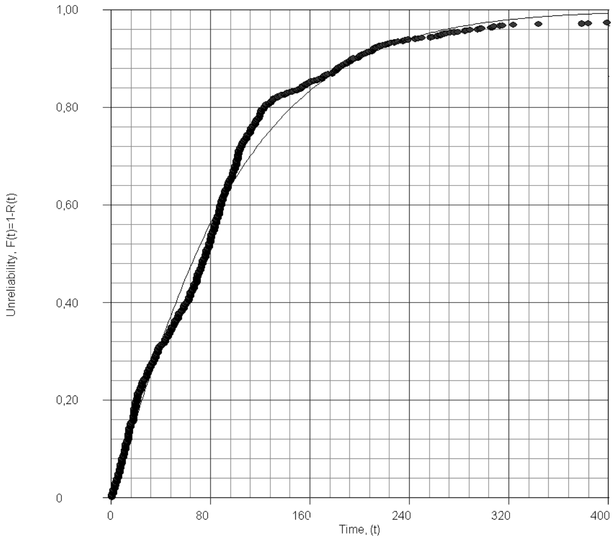

Figure 6. Empirical CDF for the system of system’s failure time - test case 1 from *Table 3*

Empirical results are convergent with simulation effects. The convergence of both the models, empirical and simulation one has been tested with Kolmogorov-Smirnov test. Calculated values of λ_{obl} for both tests do not exceed 1.57 in every trial (see *Table 4*). That testifies for well fitting both series of results at the rejection level α = 0.01 (λ_o = 1.63).

More information can be found in [24].

Table 4. Kolmogorov-Smirnov test results for the investigated cases

λ_{obl}								
Case number								λ (α_o = 0.01)
1	2	3	4	5	6	7	8	
1.48	1.38	1.35	1.57	1.61	1.09	1.46	1.12	

Moreover, there is also very important to compare the main reliability characteristics obtained from simulation performance and real life data. In *Figure 8*, there is presented a comparison of empirical and simulated system of system’s failure probability. The values of the relative errors e_{twzg} for the probability of system of systems downtime P_{nnj} do not exceed 6,5% for every analyzed cases.

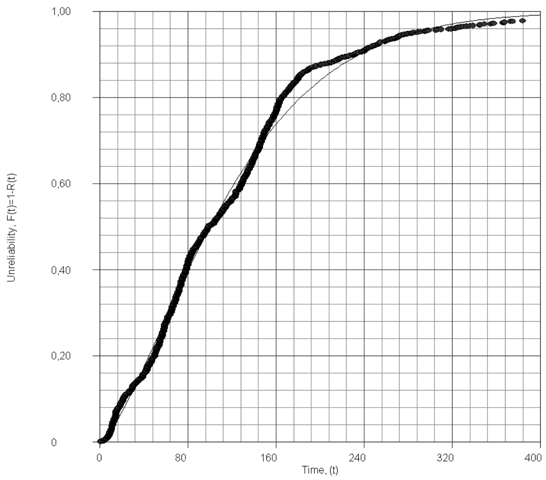

Figure 7. Empirical CDF for the system of system's failure time - test case 2 from *Table 3*

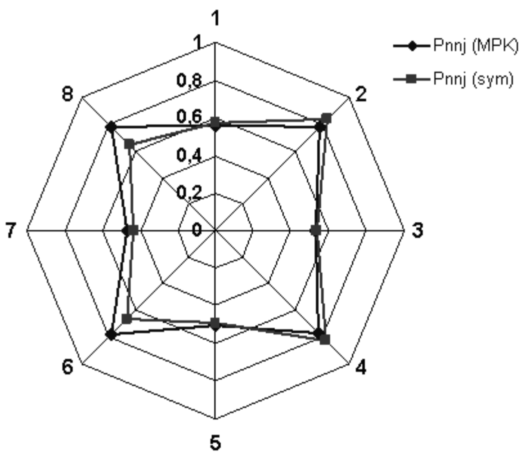

Figure 8. A comparison of empirical (Pnnj(MPK)) and simulated (Pnnj(sym)) system of system's failure probability when a tram is damaged

For summarizing the above considerations, it has to be underlined that:

- the comparison of obtained empirical and simulated results shows, that except supply process parameters, the human factor has great influence on empirical results.
- in the situation, when spare elements are ordered according to FIFO queue both series of results well fits.

For example, the developed model can be used in analysis of the following aspects:

- selection of suppliers in terms of the required delivery time,
- reliability of operational system (e.g. in terms of achieved times between failures),
- maintainability of operational system (e.g. in terms of required repair times),
- minimal CIL quantity appraisement,
- definition of minimal redundancy time.

However, presented model developed with the use of Monte Carlo simulation is time-consuming, because a large number of simulated samples have been needed to estimate the reliability parameters at a high level of confidence. Moreover, there is very difficult to simulate the real system behaviour – especially in the field of human factor influence on the obtained system of systems reliability characteristics. Thus, results from the Petri Net model have been obtained. A High Level Petri Net simulator was designed to collect data regarding execution of the net presented in the *Figure* 5. The Monte Carlo simulation's purpose is twofold. For one thing, to estimate a probability distribution function of the system of system's failure time caused by a damaged tram. Secondly, to measure a conditional probability that a failed tram will cause system of system's failure.

For the *i*th tram failure whose repair and lead time is longer than the resource time, the following calculation is made:

$x_i = \tau(t_4) - \tau(t_6)$ if $\tau(t_4) > \tau(t_6)$.

Hence, x_i denotes system of system's failure time caused by the *i*th tram failure. Consequently, by means of the statistical analysis of each tram failure, probability distributions of estimated system of system's failure time are done with the outcome presented in *Figure 9* and *Figure 10*. The results concern the test cases no. 1 and 2 from Table 3 respectively. A comparison with real system CDF is provided. There is no reason to reject the hipothesis of convergence using Kolmogorov–Smirnov test with confidence at 0.01 level.

Figure 11 and *Figure 12* refine quantitative analysis for testcase 1.

After the simulation has finished, the conditional probability is obtained through dividing a number of tokens in the place p_9 by a number of transition t_0 has fired. The *Figure* 13 displays the results.

For the sake of completeness, in the *Figure 14* expected system of systems' failure time from all test cases are compared.

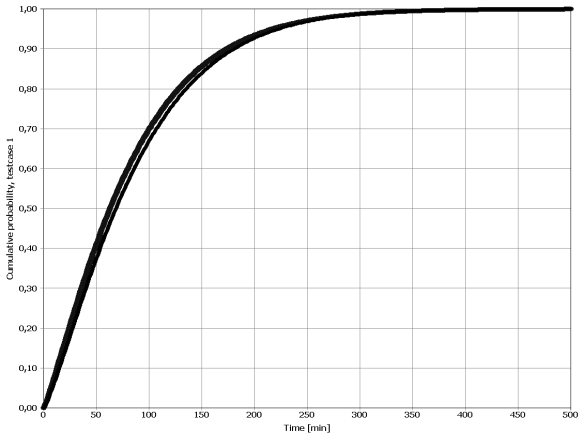

Figure 9. Petri net model (thick line) and real system CDF (thin line) for the system of system's failure time - test case 1 from *Table 3*

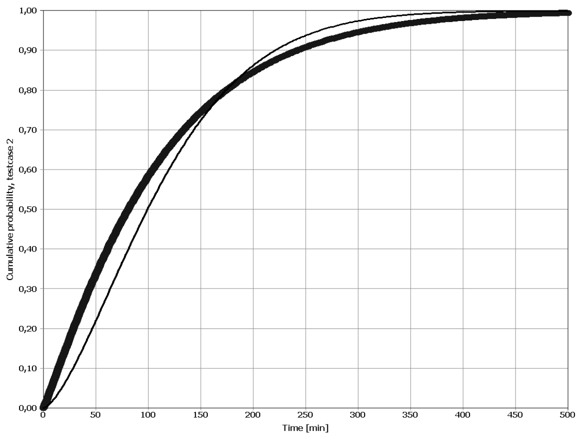

Figure 10. Petri net model (thick line) and real system CDF (thin line) for the system of system's failure time - test case 2 from *Table 3*

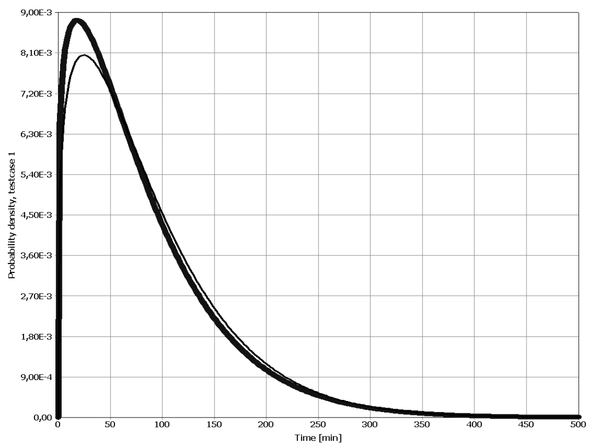

Figure 11. A PDF comparison for the system of system's failure time - test case 1 from *Table 3*

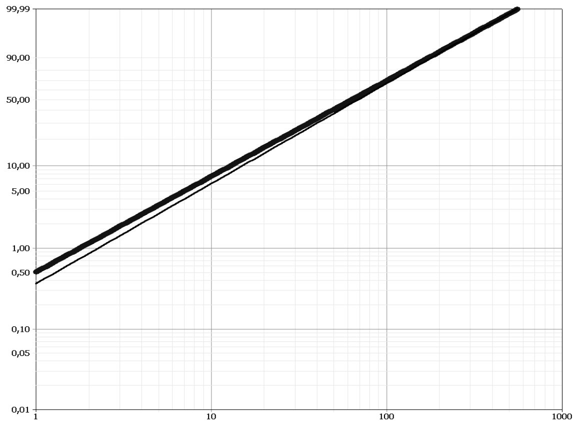

Figure 12. A Weibull probability plot comparison for the system of system's failure time - test case 1 from *Table 3*

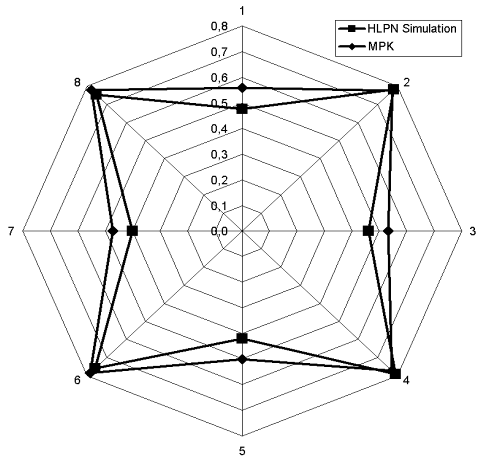

Figure 13. A comparison of system of system's failure probability when a tram is damaged

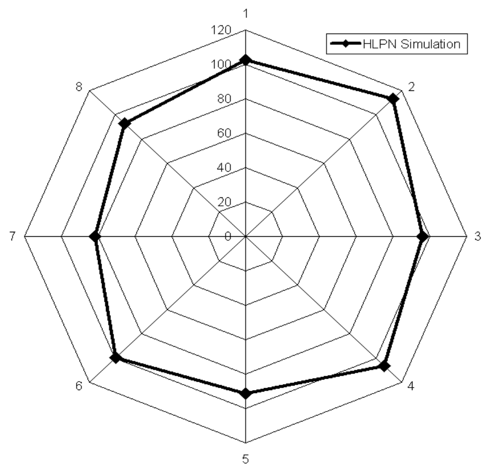

Figure 14. A comparison of average system of system's failure time

3. Conclusions

In the presented paper, there have been discussed the main limitations of known modelling methods used in real-life system reliability and supportability analysis. As a result, two modelling techniques have been applied to describe the investigated tram network performance processes.

The Petri Net model has been developed as a combination of High-level Petri Net and general stochastic Petri Net techniques. Some numerical experiments have been carried out. Obtained results confirm the convergence between both presented simulation models. The relative errors e_{twzg} for the probability of system of systems downtime P_{nnj} do not exceed 11% for every analyzed case. When comparing Petri net model to results obtained from real system performance processes, relative errors e_{twzg} for the probability of system of systems downtime P_{nnj} do not exceed 16% for every analyzed case.

Thus, the presented paper can be the starting point of consideration about searching new analytical ways of real-life system performance estimation with the use of Petri Nets.

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