Vibrations in Physical Systems Vol.26 (2014)

An Application of Longitudinal Elastic Waves for Investigation of Materials Under High Strain Rates Using the Hopkinson Bar

Tomasz SZOLC
Institute of Fundamental Technological Research of the Polish Academy of Sciences
ul.A. Pawińskiego 5B, 02-106 Warszawa
tszolc@ippt.pan.pl

Zbigniew L. KOWALEWSKI
Institute of Fundamental Technological Research of the Polish Academy of Sciences
ul.A. Pawińskiego 5B, 02-106 Warszawa
zkowalew@ippt.pan.pl

Abstract

In the paper there is presented a discrete-continuous model of the Split Hopkinson Pressure Bar (SHPB) for numerical simulations of a dynamic behaviour of material specimens under high strain-rates. For this purpose several material theories describing visco-elasto-plastic properties of the tested specimens can be applied. Using this model impact-type dynamic responses are sought by means of the longitudinal elastic wave analytical solution of the d'Alembert type. The proposed model enables us theoretical strength investigations for various elasto-plastic materials under great deformation velocities as well as structural parameter determination of the real SHPB designed to play a role of the laboratory test-rig.

Keywords: Hopkinson bar, elastic wave propagation, d'Alembert solution, numerical simulation

1. Introduction

High strain rate experimental tests are important in mechanical property analysis of materials under strongly dynamic conditions. The Split Hopkinson Pressure Bar (SHPB) has been widely used to investigate dynamic behaviour of various materials within the strain rate range of 10^2 to 10^4 s^{-1} [1-4]. In 1872 John Hopkinson investigated a stress wave propagation in a wire [1] which was the starting point for his son Bertram, who developed a measurement method for the movement recording of a cylinder during strongly dynamic conditions [2]. In 1948, Kolsky used two elastic bars instead of one with the specimen placed between them [3]. Since that date, this device has been known as the SHPB. Such experimental technique can be applied in many configurations, for example in compression, tension, torsion and shear. According to the one-dimensional elastic wave propagation theory, the "safe" maximum impact velocity is directly related to the elastic limit of the incident bar. Such condition limits the maximum strain rate in the test. As it was mentioned above, many problems appear using the SHPB technique. In order to provide better understanding of this technique, in this paper a discrete-continuous, semi-analytical model of the SHPB with an elasto-plastic material specimen has been developed as an alternative to the commonly applied, time-consuming, non-linear finite element models with huge numbers of degrees of freedom.

2. Continuous modelling and wave solution for the Split Hopkinson Pressure Bar

Since the longitudinal elastic wave propagation process is going to be investigated as a measurement tool for material specimens under high strain rates, in the proposed model the incident and transmitting bar of the SHPB are represented by continuous and homogeneous elastic cylindrical elements of mutually identical circular cross-sections and lengths l_1 and l_2, respectively. The transmitting bar is visco-elastically fixed to a rigid wall by the use of the mass-less spring of stiffness k_0 and damping coefficient c_0. The material specimen of mass $2m$, the length of which is much smaller than these of the incident and transmitting bar, can be substituted by two rigid bodies of identical masses m connected with each other by means of the mass-less, non-linear spring with response dependent visco-elastic characteristics $c(\Delta\mathring{u}(t))$ and $k(\Delta u(t))$ describing visco-elasto-plastic properties of the investigated material. The wafer has usually a cylindrical shape with length $l_0 << l_1$, l_2 and cross-sectional stiffness EA equal to these of the incident and transmitting bar. Despite of its natural continuous structure, in order to simulate the impact process, the wafer can be regarded as a rigid body of mass m_0 impacting the incident bar with initial velocity v_0 using an intervention of the mass-less spring. Stiffness k_e of this spring has been determined assuming that the incident wave excited due to the impact has a length corresponding to the double-period of the longitudinal elastic wave propagation in the wafer. Thus, $k_e = EA\pi^2/l_0$. According to the above assumptions, the proposed discrete-continuous model of the SHPB has a structure demonstrated in Fig. 1.

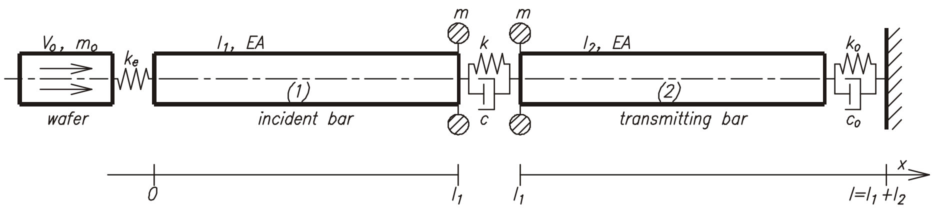

Figure 1. Discrete-continuous model of the SHPB

Motion of cross-sections of the continuous elements representing the incident and transmitting bar is governed by the following homogeneous partial differential equations

$$a^2 \frac{\partial^2 u_i(x,t)}{\partial x^2} - \frac{\partial^2 u_i(x,t)}{\partial t^2} = 0, \quad a = \sqrt{E/\rho}, \quad i = 1,2, \tag{1}$$

where E is Young's modulus, ρ denotes the material density and $u_i(x,t)$ are the longitudinal displacements of bar cross-sections, x is the spatial co-ordinate and t denotes time. Equations of motion (1) are solved with the following boundary conditions describing the support of the SHPB by the rigid wall, dynamic interaction of the elasto-plastic material specimen as well as an impact-type excitation by the wafer:

$$m_0 \frac{\mathrm{d}u_0(t)}{\mathrm{d}t^2} - k_e\left[u_0(t) - u_1(x,t)\right] = 0, \; EA\frac{\partial u_1(x,t)}{\partial x} - k_e\left[u_0(t) - u_1(x,t)\right] = 0 \text{ for } x = 0,$$

$$m\frac{\partial u_1(x,t)}{\partial t^2}+c(\Delta\dot{u})\left[\frac{\partial u_1(x,t)}{\partial t}-\frac{\partial u_2(x,t)}{\partial t}\right]+EA\frac{\partial u_1(x,t)}{\partial x}+k(\Delta u)\left[u_1(x,t)-u_2(x,t)\right]=0,$$

$$m\frac{\partial u_2(x,t)}{\partial t^2}+c(\Delta\dot{u})\left[\frac{\partial u_2(x,t)}{\partial t}-\frac{\partial u_1(x,t)}{\partial t}\right]-EA\frac{\partial u_2(x,t)}{\partial x}+k(\Delta u)\left[u_2(x,t)-u_1(x,t)\right]=0$$

for $x=l_1$,

$$EA\frac{\partial u_2(x,t)}{\partial x}+d_0\frac{\partial u_2(x,t)}{\partial t}+k_0u_2(x,t)=0 \qquad \text{for } x=l_1+l_2, \tag{2}$$

where A denotes the area of the bar cross-section and $u_0(t)$ is the generalized co-ordinate describing motion of the wafer mass center.

A dynamic response of the SHPB model excited by a wafer impact is sought using the d'Alembert wave solutions of motion equations (1) in the similar form, as in [5]:

$$u_i(x,t)=f_i\left(a(t-t_{0i})-x+x_{0i}\right)+g_i\left(a(t-t_{0i})+x-x_{0i}\right), \quad i=1,2. \tag{3}$$

The functions f_i and g_i represent longitudinal waves induced by the excitation impulse due to the wafer impact, where the function f_i represents a longitudinal wave propagating in the i-th continuous macro-element along the x-axis positive sense, Fig. 1; however, the function g_i represents a longitudinal wave propagating along the x-axis negative sense and a denotes the wave propagation velocity. According to the one-dimensional wave propagation theory, it is taken into account in (3) that the first perturbation in the i-th macro-element occurs in the cross-section of the co-ordinate x_{0i} after the finite time delay t_{0i}. Furthermore, it is assumed that the functions f_i and g_i are continuous and are null for negative arguments, i.e. before arriving the first perturbation.

Since solutions (3) identically satisfy motion equation (1), actual values of the wave functions, f_i and g_i are determined by the boundary conditions of the problem. Thus, by substituting the wave solutions (3) into the boundary conditions (2), denoting the largest argument in each equation by z, and by rearranging these equations in such a way that their right-hand sides are always known, in the considered case of the bars with a constant cross-sections we obtain the following system of ordinary differential equations of the second order with a "retarded" argument for the functions f_i and g_i, i=1,2:

$$\left(R+D_0\right)g_2''(z)+K_0g_2'(z)=\left(R-D_0\right)f_2''(z-2\lambda_2)-K_0f_2'(z-2\lambda_2),$$

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} g_1''(z+\lambda_1) \\ f_2''(z) \end{bmatrix}+\begin{bmatrix} p & -C \\ -C & p \end{bmatrix}\begin{bmatrix} g_1'(z+\lambda_1) \\ f_2'(z) \end{bmatrix}+\begin{bmatrix} K & -K \\ -K & K \end{bmatrix}\cdot\begin{bmatrix} g_1(z+\lambda_1) \\ f_2(z) \end{bmatrix}=$$
$$=\begin{bmatrix} -M\,f_1''(z-\lambda_1)+r\,f_1'(z-\lambda_1)+Cg_2'(z)+K\left[g_2(z)-f_1(z-\lambda_1)\right] \\ -Mg_2''(z)+r\,g_2'(z)+C\,f_1'(z-\lambda_1)-K\left[g_2(z)-f_1(z-\lambda_1)\right] \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} M_0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_0''(z) \\ f_1''(z) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} u_0'(z) \\ f_1'(z) \end{bmatrix} + \begin{bmatrix} K_e & -K_e \\ -K_e & K_e \end{bmatrix} \begin{bmatrix} u_0(z) \\ f_1(z) \end{bmatrix} = \begin{bmatrix} K_e g_1(z) \\ R g_1'(z) - K_e g_1(z) \end{bmatrix},$$

where:

$$R = \frac{EAl_s}{a^2 m_s}, K_0 = \frac{k_0 l_s^2}{a^2 m_s}, D_0 = \frac{d_0 l_s}{a m_s}, C(\Delta \dot{u}(z)) = \frac{c(\Delta \dot{u}(t)) l_s}{a m_s}, \lambda_i = \frac{l_i}{l_s}, \ i = 1,2,$$

$$K(\Delta u(z)) = \frac{k(\Delta u(t)) l_s^2}{a^2 m_s}, \ K_e = \frac{k_e l_s^2}{a^2 m_s}, \ M_0 = \frac{m_0}{m_s}, \ M = \frac{m}{m_s}, \ u_0(z) \equiv u_0(t),$$

p=R+C, r=R-C and l_s [m], m_s [kg] are the reference distance and mass, respectively.

The above equations have been solved numerically by means of the Newmark method using the appropriately small direct integration step in order to obtain a sufficient accuracy of results of simulation of the impact-type dynamic process. The right-hand sides of the equations with a shifted argument, which are known after each integration step, similarly as in [5], enable us their very efficient solving one after another, i.e. in the sequence defined here by (4). In the considered case, it has been assumed that when the impact process is over, i.e. when the elastic strain in the incident bar cross-section x=0 goes back to zero value, the quantities m_0 and k_e in (2) and (4) will become null during simulation.

3. Computational example

An object of consideration is the discrete-continuous model of the real laboratory test rig in the form of a classical SHPB. Here, the diameters of the incident and transmitting steel bar as well as of the wafer are equal to 0.02 m. The incident bar length l_1 is equal to 1.05 m and the length of the transmitting bar l_2=1.07 m. By means of the presented SHPB model three cylindrical specimens of diameter 0.01 m and length 0.01 m each and made of 34GS steel, M1E copper and 7075 aluminium alloy have been tested. In all cases the SHPB was impacted by the wafer of mass m_0=0.61 kg with an initial velocity 50 m/s. The characteristics $c(\Delta \mathring{u}(t))$ and $k(\Delta u(t))$ for all specimens mentioned above have been properly identified using the simplified Burgers material model, [6], which for the force equilibrium formulation in (2) can be reduced to the Voigt material model. This approach seems to be very convenient at the introductory stage of this problem investigation, where a demonstration of system dynamic responses in the form of longitudinal elastic wave propagation due to wafer impact is the main goal of the presented considerations. Nevertheless, the functions $c(\Delta \mathring{u}(t))$ and $k(\Delta u(t))$ in (2), (4) can be regarded here as properly identified constants or response dependent variables, [7].

In Fig. 2 there are shown plots of the system dynamic response in the form of time-histories of the incident bar impacted free end velocity (Fig. 2a), specimen dynamic strain (Fig. 2b) and of the specimen strain rate (Fig. 2c). In all figures the grey lines correspond to the steel specimen, the solid black lines to the copper specimen and the dashed black lines correspond to the aluminium alloy specimen. As a reference, by the dotted line in Fig. 2a there is denoted the time history of the wafer velocity which naturally tends to zero, when the impact process is over. Here, according to [4], the specimen strain and strain rate are respectively defined as:

$$\varepsilon_S(t) = -\ln(\psi(t)), \quad \dot{\varepsilon}_S(t) = \dot{\varepsilon}_p(t)/\psi(t), \quad \psi(t) = 1 - \varepsilon_p(t),$$

where $\varepsilon_p=(f_1(at\text{-}l_1)+g_1(at+l_1) - f_2(at) - g_2(at))/l_p$ and l_p denotes the initial specimen length.

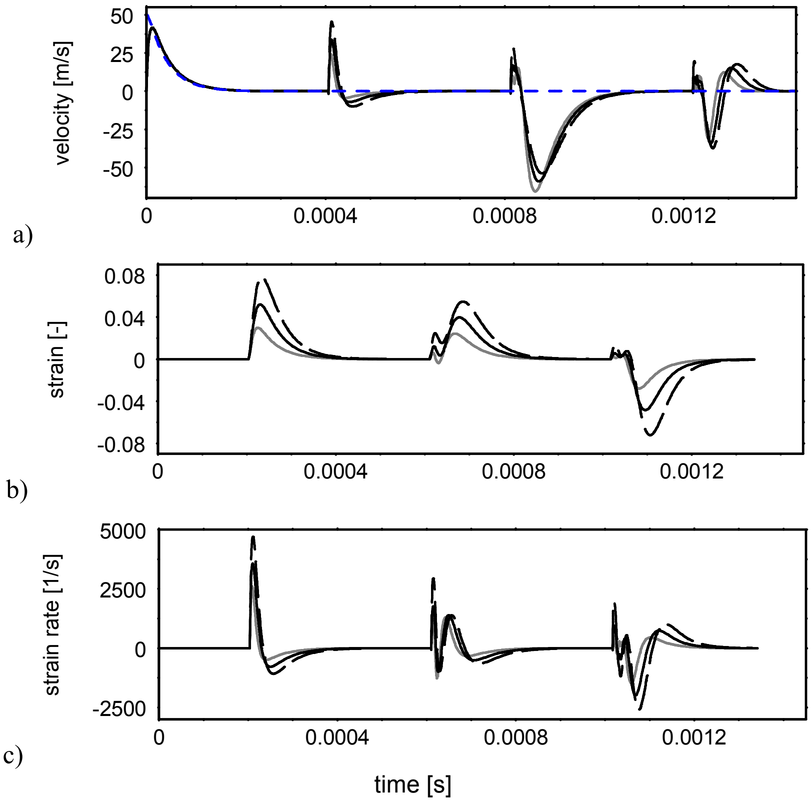

Figure 2. Dynamic response of the SHPB due to the wafer impact

In Fig. 2a there is shown a rapid increase followed by the gradual decrease of the incident bar free end velocity and then there are observed three subsequent velocity perturbations caused by successive longitudinal wave reflections upon each time interval $(l_1+ l_2)/a \cong 0.0004$ s. However, the plots in Figs. 2b and 2c are characterized by three significant perturbations, where the first ones are excited by the incident waves transmitted by the specimen after $l_1/a \cong 0.0002$ s and the two next perturbations are induced by the successive reflected waves of the strain and strain rate, respectively. It is to remark that the greatest perturbations resulting from the dynamic response of the assumed SHPB model have been obtained for the aluminium specimen which is characterized by the smallest values of functions k and c in (2) and (4). Here, the maximum strain reaches 0.08 and the greatest strain rate is close to 5000 1/s. The steel specimen, however, is the hardest one and thus, it experiences the smallest extreme values of velocity, strain and strain rate in comparison to the analogous extremes obtained for the copper specimen, see Fig. 2.

4. Conclusions

In the paper there was investigated a longitudinal elastic wave propagation process in the cylindrical homogeneous rods representing the incident and transmitting bar in the discrete-continuous model of the SHPB. For this purpose, an analytical wave solution of the d'Alembert type has been applied in order to simulate system dynamic responses obtained for various metallic specimens. Although the specimen material models assumed here require essential improvements in the next steps of research in this field, the obtained results of computations have indicated reasonable values of the commonly expected maximal strains and strain rates observed during analogous experimental measurements. According to the above, the proposed model of the SHPB, apart of theoretical investigations of material elasto-plastic properties, can be successfully used for designing of test rigs in the form of Hopkinson bars.

Acknowledgments

These investigations have been supported by the National Science Center: Research Project UMO-2012/07/D/ST-8/02668.

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