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RELIABILITY APPROACH TO RESILIENCE OF CRITICAL INFRASTRUCTURE IMPACTED BY OPERATION PROCESS

Podejście niezawodnościowe do odporności infrastruktury krytycznej uwzględniające wpływ procesu eksploatacji

Abstract: *The paper focuses on a critical infrastructure reliability and resilience to its operation process and particularly is devoted to critical infrastructure reliability and resilience indicators. First, the model of critical infrastructure reliability without considering outside impacts is proposed and applied to determine the port oil terminal reliability indicators. In the next step, the operation impact model of critical infrastructure reliability is created and applied to reliability and resilience analysis of the port oil terminal. The comparison of the port oil terminal critical infrastructure reliability indicators without considering outside impacts with indicators considering impact of its operation process is performed. A significant influence of the operation process on the port oil terminal reliability is proved.*

Keywords: critical infrastructure, reliability, operation process, risk, resilience

Streszczenie: *Artykuł koncentruje się na niezawodności i odporności infrastruktury krytycznej w procesie eksploatacji, proponując wskaźniki niezawodności i odporności tej infrastruktury. Najpierw zaproponowano model niezawodności infrastruktury krytycznej bez uwzględnienia oddziaływań zewnętrznych oraz zastosowano go do wyznaczenia wskaźników niezawodności portowej bazy paliw. Następnie utworzono model wpływu eksploatacji oraz zastosowano go do analizy niezawodności i odporności portowego terminala transportu paliwa. Porównano wskaźniki niezawodności infrastruktury krytycznej terminala bez uwzględnienia wpływów zewnętrznych ze wskaźnikami uwzględniającymi wpływ jej procesu eksploatacji. Wykazano istotny wpływ procesu eksploatacji na niezawodność portowego terminala transportu paliwa.*

Słowa kluczowe: infrastruktura krytyczna, niezawodność, proces eksploatacji, ryzyko, odporność

1. Introduction

We define critical infrastructure as a complex system in its operating environment that significant features are inside-system dependencies and outside-system dependencies that in the case of its degradation have a significant destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas [5, 15]. The safety indicators for such a system, which are crucial for its operators, can be obtained by using of an original and innovative probabilistic approach to modelling of operation process impact on its safety [8]. In the first step of the proposed approach, starting from a simplest pure safety model without considering outside impacts, we can define the critical infrastructure and its assets practically useful safety indicators SafI1-SafI7 [9, 10, 13]. This set of safety indicators can be completed by linking the safety pure model with the model of the critical infrastructure operation process [14, 11, 8]. This way created joint safety model of the critical infrastructure related to its operation process can offer additionally two resilience indicators ResI1-ResI2 which are measures of the critical infrastructure operation impact on its safety and resilience to operation [8].

The paper is devoted to modification of this safety model through the reliability approach and its practical application. In this approach, it is assumed that the critical infrastructure reliability function is exponential and that it is under the influence of its operation process. Next, the reliability and resilience indicators for this critical infrastructure under this impact are defined. The proposed reliability and resilience indicators for critical infrastructure are the simplified tools that can be practically applied to reliability, risk and resilience examination of real critical infrastructure. The way of this application is illustrated by an exemplary system reliability characteristics determination.

First, for the critical infrastructure without any outside impact the following practically useful reliability indicators are defined:

- the critical infrastructure reliability function (RelI1),
- the critical infrastructure risk function (RelI2),
- the critical infrastructure fragility curve (damage curve) (RelI3),
- the mean value of the critical infrastructure unconditional lifetime up to the failure (RelI4),
- the standard deviation of the critical infrastructure unconditional lifetime up to the failure (RelI5),
- the moment the critical infrastructure risk function exceeds a permitted level (RelI6),
- the intensity of the critical infrastructure failure (the critical infrastructure failure rate) (RelI7).

Next, taking into considerations the critical infrastructure operation process impact on the reliability of critical infrastructure, the following resilience indicators are introduced:

- the coefficient of the operation process impact on the critical infrastructure intensity of ageing (ResI1),
- the indicator of critical infrastructure resilience to operation process impact (ResI2).

The proposed modified reliability models are practically applied to reliability and resilience examination of the port oil terminal critical infrastructure.

2. Critical infrastructure reliability backgrounds

In the reliability analysis to define the critical infrastructure, we assume that:

- the critical infrastructure has the reliability state set {0,1},
- the critical infrastructure is in the reliability state 0 if it is failed,
- the critical infrastructure is in the reliability state 1 if it is not failed,
- T is a random variable representing the lifetime of the critical infrastructure in the reliability state 1, while it was not failed at the moment $t = 0$,
- $s(t)$ is the critical infrastructure reliability state at the moment t, $t \geq 0$, given that it was in the reliability state 1 at the moment $t = 0$.

As we denoted the critical infrastructure unconditional lifetime in the reliability state 1 (time to critical infrastructure failure) by T, then we define the critical infrastructure reliability function by

$$R(t) = P(s(t) = 0 \mid s(0) = 1) = P(T > t) \text{ for } t \geq 0. \qquad (2.1)$$

The exemplary graph of a critical infrastructure reliability function $R(t)$ for $t \geq 0$ is shown in fig. 1.

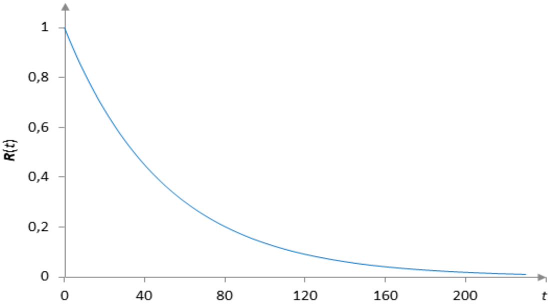

Fig. 1. The graph of a critical infrastructure reliability function $\boldsymbol{R}(t)$

From the above assumptions it follows that between a critical infrastructure reliability function $\boldsymbol{R}(t)$ and a critical infrastructure time to failure T distribution function

$$\boldsymbol{F}(t) = P(T \le t), \quad t \ge 0, \tag{2.2}$$

there exists a relationship given by

$$\boldsymbol{R}(t) = 1 - \boldsymbol{F}(t) \text{ for } t \ge 0. \tag{2.3}$$

Thus, if we define the critical infrastructure risk function

$$\boldsymbol{r}(t) = P(s(t) = 0 \mid s(0) = 1) = P(T \le t),\ t \ge 0, \tag{2.4}$$

as a probability that the critical infrastructure is in the reliability state 0, while it was in the reliability state 1 at the moment $t = 0$, then

$$\boldsymbol{r}(t) = \boldsymbol{F}(t) = 1 - \boldsymbol{R}(t),\ t \ge 0, \tag{2.5}$$

where $\boldsymbol{F}(t)$ is the distribution function given by (2.2) and $\boldsymbol{R}(t)$ is the reliability function given by (2.1).

The graph of an exemplary critical infrastructure risk function, called the fragility curve [15], is presented in fig. 2.

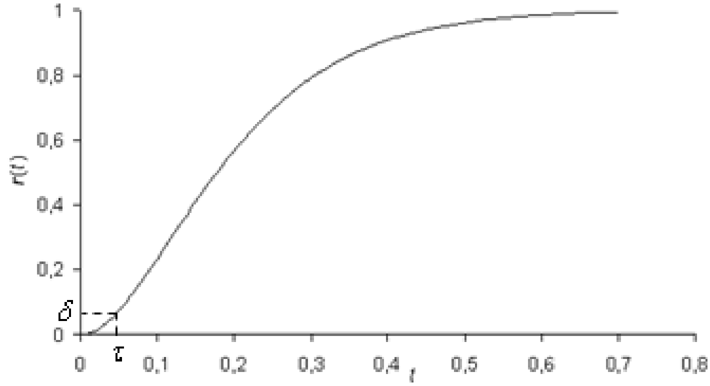

Fig. 2. The graph of an exemplary critical infrastructure risk function $\boldsymbol{r}(t)$

3. Critical infrastructure reliability indicators free of outside impacts

3.1. Reliability indicators

We suppose that the critical infrastructure is not impacted by any outside threats and further, we denote the critical infrastructure lifetime T^0 and define the first reliability indicator, the critical infrastructure reliability function (RelI1), [13]:

$$\boldsymbol{R}^0(t) = P(T^0 > t) \text{ for } t \ge 0. \tag{3.1}$$

The second reliability indicator, the critical infrastructure risk function (RelI2)

$$\boldsymbol{r}^0(t) = P(s(t) = 0 \mid s(0) = 1) = P(T^0 \le t), \; t \ge 0, \tag{3.2}$$

is defined as a probability that the critical infrastructure is in the reliability state 0 (it is failed), while it was in the reliability state 1 (it was not failed) at the moment $t = 0$ and given by

$$\boldsymbol{r}^0(t) = 1 - \boldsymbol{R}^0(t), \text{ for } t \ge 0, \tag{3.3}$$

where $\boldsymbol{R}^0(t)$ is the critical infrastructure reliability function given by (3.1). The graph of the critical infrastructure risk function (fig. 2) is the third reliability indicator called the critical infrastructure fragility curve (RelI3) [15].

The critical infrastructure reliability function (RelI1), the critical infrastructure risk function (RelI2) and the critical infrastructure fragility curve (RelI3) are proposed as main basic critical infrastructure reliability indicators.

Other practically useful critical infrastructure reliability indicators are:

- the mean value of the critical infrastructure lifetime T^0 (a time up to its failure) (RelI4) given by

$$\boldsymbol{\mu}^0 = \int_0^\infty \boldsymbol{R}^0(t)dt, \tag{3.4}$$

where $\boldsymbol{R}^0(t)$ is defined by (3.1);

- the standard deviation of the critical infrastructure lifetime T^0 (RelI5) given by

$$\boldsymbol{\sigma}^0 = \sqrt{\eta^0 - [\boldsymbol{\mu}^0]^2}, \tag{3.5}$$

where

$$\eta^0 = 2\int_0^\infty t\boldsymbol{R}^0(t)dt \tag{3.6}$$

and $\boldsymbol{R}^0(t)$ is given by (3.1) and $\boldsymbol{\mu}^0$ is given by (3.4);

- the moment τ of exceeding acceptable value of critical infrastructure risk function level δ (RelI6) given by

$$\tau^0 = (\mathbf{r}^0)^{-1}(\delta) \tag{3.7}$$

and illustrated in fig. 2, where $(\boldsymbol{r}^0)^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}^0(t)$ given by (3.3);

- the failure rate (intensity of failure) of the critical infrastructure (RelI7) given by

$$\lambda^0(t) = \frac{-\frac{d\boldsymbol{R}^0(t)}{dt}}{\boldsymbol{R}^0(t)}, \quad t \geq 0. \tag{3.8}$$

In the particular case, when the critical infrastructure has the exponential reliability function (RelI1), i.e.

$$\boldsymbol{R}^0(t) = \exp[-\lambda^0 \cdot t],\ t \geq 0,\ \lambda^0 \geq 0, \tag{3.9}$$

the failure rate (the intensity of failure) of the critical infrastructure (RelI7) is constant, i.e.

$$\lambda^0 = \frac{1}{\boldsymbol{\mu}^0}, \tag{3.10}$$

as, according to (3.4) and (3.9) the mean value $\boldsymbol{\mu}^0$ of the critical infrastructure lifetime T^0, is

$$\boldsymbol{\mu}^0 = \int_0^\infty \boldsymbol{R}^0(t)\mathrm{dt} = \int_0^\infty \exp\left[-\lambda^0 \cdot t\right]\mathrm{dt} = \frac{1}{\lambda^0}. \tag{3.11}$$

3.2. Application

We consider the port oil terminal critical infrastructure free of any outside impacts. The considered port terminal placed at the Baltic seaside is designated for receiving oil products from ships, storage and sending them by carriages or trucks to inland. The terminal can operate in reverse way as well.

The considered terminal is composed of three parts A, B and C, linked by the piping transportation system with the pier. The approximate length of the port oil piping transportation system is equal to around 25 km.

The main technical assets of the port oil terminal critical infrastructure are:
A_1 - port oil piping transportation system,
A_2 - internal pipeline technological system,
A_3 - supporting pump station,
A_4 - internal pump system,
A_5 - port oil tanker shipment terminal,
A_6 - loading railway carriage station,
A_7 - loading road carriage station,
A_8 - unloading railway carriage station,
A_9 - oil storage reservoir system.

The asset A_1, the port oil piping transportation system operating at the port oil terminal critical infrastructure consists of three subsystems:

- the subsystem S_1 composed of two pipelines, each composed of 176 pipe segments and 2 valves,
- the subsystem S_2 composed of two pipelines, each composed of 717 pipe segments and 2 valves,
- the subsystem S_3 composed of three pipelines, each composed of 360 pipe segments and 2 valves.

Its operation is the main activity of the oil terminal involving the remaining assets $A_2 - A_9$.

The port oil transportation system is a series system composed of two series-parallel subsystems S_1, S_2, each containing two pipelines (assets) and one series-“2 out of 3” subsystem S_3 containing 3 pipelines (assets).

The subsystems S_1, S_2 and S_3 are forming a general series port oil pipeline system structure presented in fig. 3. However, the pipeline system structure and its subsystems and components safety depend on changing in time operation states and the climate-weather states at its operating area.

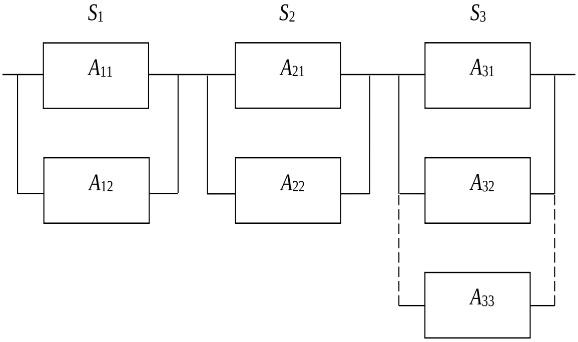

Fig. 3. General scheme of the port oil piping transportation system structure

Considering that the port oil terminal critical infrastructure is a three-state ($z = 2$) series system and assuming that the assets have exponential reliability functions, its reliability function (RelI1) determined after application of GMU Critical Infrastructure Reliability Interactive Platform [4] is given by

$$\boldsymbol{R}^0(t) = \exp[-0.115873t],\ t \geq 0. \tag{3.12}$$

The graph of the oil terminal critical infrastructure reliability function is shown in fig. 4.

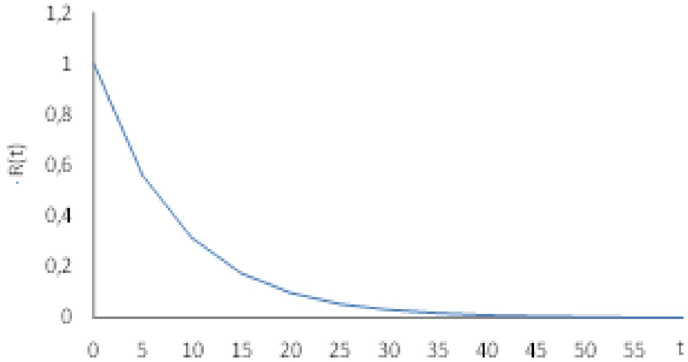

Fig. 4. The graph of the port oil terminal critical infrastructure reliability function

The graph of the intensity of failure of the oil terminal critical infrastructure is given in fig. 5.

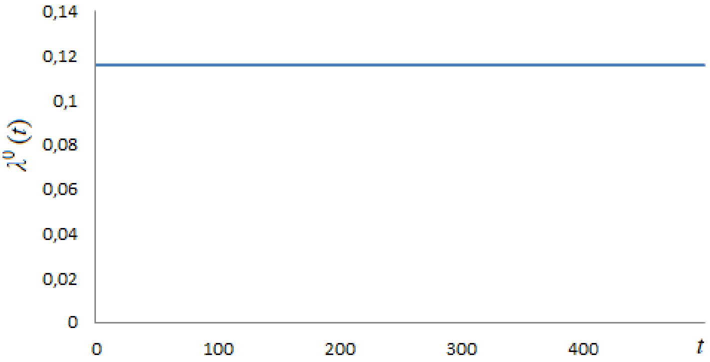

Fig. 5. The graph of the intensity of failure of the port oil terminal critical infrastructure

As the critical state is $r = 1$, then by (3.12), the port oil terminal critical infrastructure risk function (RelI2), is given by

$$\boldsymbol{r}^0(t) = 1 - \boldsymbol{R}^0(t) = 1 - \exp[-0.115873t], \text{ for } t \geq 0. \qquad (3.13)$$

The graph of the risk function $\boldsymbol{r}(t)$ of the oil terminal critical infrastructure, the fragility curve (RelI3), is shown in fig. 6.

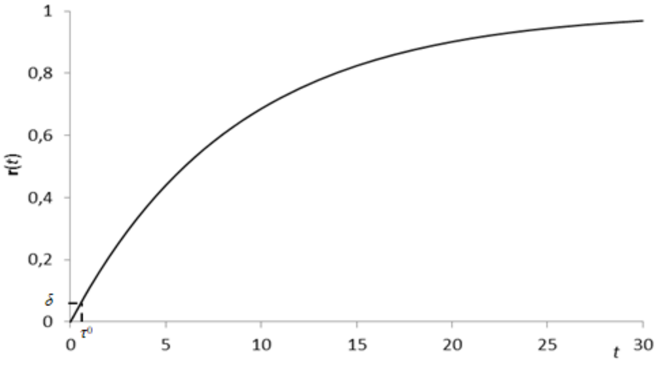

Fig. 6. The graph of the risk function $\boldsymbol{r}(t)$ (the fragility curve) of the port oil terminal critical infrastructure

By (3.11) and (3.12), the oil terminal critical infrastructure mean lifetime up to exceeding critical reliability state $r = 1$ (RelI4), is

$$\boldsymbol{\mu}^0 \cong 8.63. \qquad (3.14)$$

Hence, by (3.5)-(3.6) and (3.11)-(3.12) the standard deviation of the port oil terminal critical infrastructure lifetime up to exceeding critical reliability state $r = 1$ (RelI5), is

$$\boldsymbol{\sigma}^0 \cong 8.63. \tag{3.15}$$

From (3.13) and applying (3.7), the moment when the oil terminal critical infrastructure risk function exceeds a permitted level $\delta = 0.05$ (RelI6), is

$$\tau^0 = -\frac{1}{0.115873}\ln(1 - 0.05) \cong 0.44 \text{ years.} \tag{3.16}$$

According to (3.10) and (3.14), the oil terminal critical infrastructure intensity of failure (RelI7), is:

$$\lambda^0(t) = 0.115873. \tag{3.17}$$

4. Critical infrastructure reliability related to its operation process

4.1. Critical infrastructure operation process

We consider the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, impacted in a various way at its operation states z_b, $b = 1,2,\dots,v$. We assume that the changes of the operation states of the critical infrastructure operation process $Z(t)$ have an influence on the critical infrastructure reliability structure and on the reliability of the critical infrastructure assets A_i, $i = 1,2,\dots,n$, as well [11].

The following critical infrastructure operation process parameters (OPP) can be identified either statistically using the methods given in [11, 6] or evaluated approximately by experts:

- the number of operation states (OPP1) v;
- the vector

$$[p_b(0)]_{1\,x\,v} = [p_1(0), p_2(0),\dots, p_v(0)] \tag{4.1}$$

of the initial probabilities (OPP2)

$$p_b(0) = P(Z(0) = z_b),\ b = 1,2,\dots,v,$$

of the critical infrastructure operation process $Z(t)$ staying at particular operation states z_b at the moment $t = 0$;

- the matrix

$$[p_{bl}]_{1\,x\,v} = \begin{bmatrix} p_{11}\ p_{12}\ \dots\ p_{1v} \\ p_{21}\ p_{22}\ \dots\ p_{2v} \\ \dots \\ p_{v1}\ p_{v2}\ \dots\ p_{vv} \end{bmatrix} \qquad (4.2)$$

of probabilities of transition (OPP3) p_{bl}, $b, l = 1,2,\dots,v$, of the critical infrastructure operation process $Z(t)$ between the operation states z_b and z_l;

- the matrix

$$[M_{bl}]_{1x\,v} = \begin{bmatrix} M_{11}\ M_{12}\ \dots M_{1v} \\ M_{21}\ M_{22}\ \dots M_{2v} \\ \dots \\ M_{v1}\ M_{v2}\ \dots M_{vv} \end{bmatrix} \qquad (4.3)$$

of mean values of conditional sojourn times (OPP4)

$$M_{bl} = E[\theta_{bl}],\ b,l = 1,2,\dots,v,$$

of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation state z_b when the next state is z_l.

The following critical infrastructure operation process characteristics (OPC) can be either calculated analytically using the above parameters of the operation process or evaluated approximately by experts [11, 6]:

- the vector

$$[p_b]_{1\,x\,v} = [p_1, p_2,\dots, p_v]$$

of the limit values of transient probabilities (OPC1)

$$p_b(t) = P(Z(t) = z_b),\ t \geq 0,\ b = 1,2,\dots,v,$$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b given by

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l},\ b = 1,2,\dots,v,$$

where M_b, $b = 1,2,\ldots,v$, are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl},\ \mathrm{b} = 1,2,\ldots,\mathrm{v},$$

while the steady probabilities π_b of the vector $[\pi_b]_{1\,x\,v}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1 \end{cases} ; \tag{4.4}$$

– the vector

$$\left[\hat{M}_b\right]_{1\,x\,v} = \left[\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_v\right]$$

of the mean values (OPC2)

$$\hat{M}_b = E[\hat{\theta}_b] \cong p_b\theta,\ b = 1,2,\ldots,v, \tag{4.5}$$

of the total sojourn times $\hat{\theta}_b$ of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$, during the fixed critical infrastructure opetation time θ.

4.2. Reliability and resilience indicators

We denote the critical infrastructure conditional lifetime while its operation process $Z(t)$, $t \geq 0$, is at the operation state z_b, $b = 1,2,\ldots,v$, by $[T^1]^{(b)}$ and the conditional reliability function of the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, by

$$[\boldsymbol{R}^1(t)]^{(b)} = P([T^1]^{(b)} > t \mid Z(t) = z_b) \tag{4.6}$$

for $t \geq 0$, $b = 1,2,\ldots,v$.

The reliability function $[\boldsymbol{R}^1(t)]^{(b)}$, is the conditional probability that the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, lifetime $[T^1]^{(b)}$ is greater than t, while the critical infrastructure operation process $Z(t)$, $t \geq 0$, is at the operation state z_b.

Next, we denote the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, unconditional lifetime by T^1 and the unconditional reliability function (RelI1) of the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, by

$$\boldsymbol{R}^1(t) = P(T^1 > t) \text{ for } t \geq 0. \tag{4.7}$$

In the case when the critical infrastructure operation time θ is large enough, the unconditional reliability function of the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, defined by (4.7), is given by [11]:

$$\boldsymbol{R}^1(t) \cong \sum_{b=1}^{v} p_b \left[\boldsymbol{R}^1(\mathrm{t})\right]^{(b)} \text{ for } t \geq 0, \tag{4.8}$$

where $[\boldsymbol{R}^1(t)]^{(b)}$, $b = 1,2,\ldots,v$, are the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, conditional reliability function defined by (4.7) and p_b, $b = 1,2,\ldots,v$, are the critical infrastructure operation process $Z(t)$, $t \geq 0$, limit transient probabilities at the operation states z_b, $b = 1,2,\ldots,v$, defined by (4.4).

The second reliability indicator of the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, the risk function (RelI2)

$$\boldsymbol{r}^1(t) = P(s(t) = 0 \mid s(0) = 1) = P(T^1(0) \leq t),\ t \geq 0, \tag{4.9}$$

is defined as a probability that the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, is in the reliability state 0 while it was in the reliability state 1 at the moment $t = 0$ and given by [11]:

$$\boldsymbol{r}^1(t) = 1 - \boldsymbol{R}^1(t),\ t \geq 0, \tag{4.10}$$

where $\boldsymbol{R}^1(t)$ is the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, unconditional reliability function given by (4.7). The graph of the critical infrastructure risk function $\boldsymbol{r}^1(t)$, $t \geq 0$, defined by (4.10), is the reliability indicator called the fragility curve (RelI3) of the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$.

Other practically useful reliability indicators of the critical infrastructure related to the operation process $Z(t)$, $t \geq 0$, are:

– the mean value of the critical infrastructure unconditional lifetime T^1 (a time up to its failure) (RelI4) given by

$$\boldsymbol{\mu}^1 = \int_0^\infty \boldsymbol{R}^1(t)dt \cong \sum_{b=1}^{v} p_b \left[\boldsymbol{\mu}^1\right]^{(b)}, \tag{4.11}$$

where $[\boldsymbol{\mu}^1]^{(b)}$ are the mean values of the critical infrastructure conditional lifetimes $[T^1]^{(b)}$ at the operation states z_b, $b = 1,2,\ldots,v$, given by

$$\left[\boldsymbol{\mu}^1\right]^{(b)} \int_0^\infty \left[\boldsymbol{R}^1(t)\right]^{(b)} dt, b = 1,2,\ldots,v, \tag{4.12}$$

and $[\boldsymbol{R}^1(t)]^{(b)}$, $b = 1,2,\dots,v$, are defined by (4.7) and p_b are defined by (4.4);

- the standard deviation of the critical infrastructure lifetime T^1 (RelI5) given by

$$\boldsymbol{\sigma}^1 = \sqrt{\eta^1 - [\boldsymbol{\mu}^1]^2}, \tag{4.13}$$

where

$$\eta^1 = 2\int_0^\infty t\boldsymbol{R}^1(t)dt, \tag{4.14}$$

and $\boldsymbol{R}^1(t)$ is defined by (4.7) and $\boldsymbol{\mu}^1$ is given by (4.11);

- the moment τ^1 of exceeding acceptable value of critical infrastructure risk function level δ (RelI6) given by

$$\tau^0 = (\boldsymbol{r}^1)^{-1}(\delta), \tag{4.15}$$

where $(\boldsymbol{r}^1)^{-1}(\delta)$ is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (4.10);

- the failure rate (the intensity of failure) of the critical infrastructure given by (RelI7)

$$\lambda^1(t) = \frac{-\dfrac{d\boldsymbol{R}^1(t)}{dt}}{\boldsymbol{R}^1(t)}, \quad t \ge 0. \tag{4.16}$$

The critical infrastructure impacted by operation process resilience indicators are:

- the coefficient of operation process impact on the critical infrastructure failure rate (ResI1)

$$\boldsymbol{\rho}^1(t) = \frac{\lambda^1(t)}{\lambda^0(t)} = \frac{\boldsymbol{\mu}^0(t)}{\boldsymbol{\mu}^1(t)}, \quad t \ge 0, \tag{4.17}$$

i.e.

$$\lambda^1(t) = \boldsymbol{\rho}^1(t) \cdot \lambda^0(t), \quad t \ge 0, \tag{4.18}$$

where $\lambda^0(t)$, $t \ge 0$, is the failure rate of the critical infrastructure without of operation process impact, and $\lambda^1(t)$, $t \ge 0$, $u = 1,2,\dots,z$, is the intensity of degrade failure rate of the critical infrastructure with the operation process impact;

- the indicator of critical infrastructure resilience to operation process impact (ResI2) defined by

$$\boldsymbol{RI}^1(t) = \frac{1}{\boldsymbol{\rho}^1(t)}, \quad t \ge 0, \tag{4.19}$$

where $\boldsymbol{\rho}^1(t)$, $t \geq 0$, is the coefficient of operation process impact on the critical infrastructure failure rate given by (4.18), i.e.

$$\boldsymbol{RI}^1(t) = \frac{\lambda^0(t)}{\lambda^1(t)} = \frac{\boldsymbol{\mu}^1(t)}{\boldsymbol{\mu}^0(t)},\ t \geq 0. \tag{4.20}$$

In the case, the critical infrastructure has the exponential reliability function, i.e.

$$\boldsymbol{R}^1(t) = \exp[\,-\lambda^1 \cdot t],\ \ t \geq 0, \lambda^1 \geq 0 \tag{4.21}$$

the critical infrastructure reliability indicators defined by (4.16)-(4.20) take forms:

- the failure rate of the critical infrastructure related to the operation process impact is constant and

$$\lambda^1 = \frac{1}{\boldsymbol{\mu}^1}, \tag{4.22}$$

- the coefficient of the operation process impact on the critical infrastructure failure rate

$$\boldsymbol{\rho}^1 = \frac{\lambda^1}{\lambda^0} = \frac{\boldsymbol{\mu}^0}{\boldsymbol{\mu}^1} \tag{4.23}$$

and λ^0 is the failure rate of the critical infrastructure without of operation process impact and λ^1 is the failure rate of the critical infrastructure related to the operation impact;

- the indicator of critical infrastructure resilience to operation process impact (ResI2) defined by

$$\boldsymbol{RI}^1 = \frac{1}{\boldsymbol{\rho}^1}, \tag{4.24}$$

where $\boldsymbol{\rho}^1$ is the coefficient of operation process impact on the critical failure rate given by (4.23), i.e.

$$\boldsymbol{RI}^1 = \ = \frac{\lambda^0}{\lambda^1} = \frac{\boldsymbol{\mu}^1}{\boldsymbol{\mu}^0},\ t \geq 0. \tag{4.25}$$

4.3. Application

We consider the port oil terminal critical infrastructure defined in sub-section 3.2. impacted by its operation process.

The asset A_1, the port oil piping transportation system, operation is the main activity of the port oil terminal involving the remaining assets A_2 – A_9 and determining their operation processes.

On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the following unknown basic parameters of the oil terminal critical infrastructure operation process:

the number of operation process states (OPP1) $v = 7$ and the operation process states:

- the operation state z_1 - transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3 of the asset A_1 illustrated in fig. 7 and assets A_2, A_4, A_6, A_7, A_9;

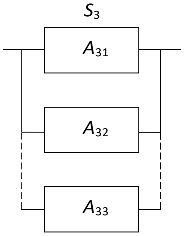

Fig. 7. The scheme of the port oil piping transportation system at the operation state z_1

- the operation state z_2 - transport of one kind of medium from the terminal part C to part B using one out of three pipelines of the subsystem S_3 of the asset A_1 illustrated in fig. 8 and assets A_2, A_4, A_8, A_9;

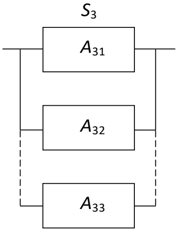

Fig. 8. The scheme of the port oil piping transportation system at the operation state z_2

- the operation state z_3 - transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 illustrated in fig. 9 and assets A_2, A_4, A_5, A_9;

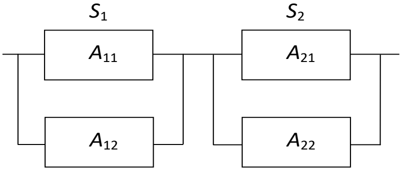

Fig. 9. The scheme of the port oil piping transportation system at the operation state z_3

- the operation state z_4 - transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines of the subsystem S_3 of the asset A_1 illustrated in fig. 10 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9;

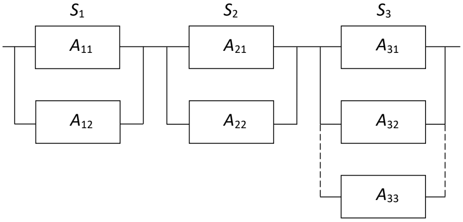

Fig. 10. The scheme of the port oil piping transportation system at the operation state z_4

- the operation state z_5 - transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 illustrated in fig. 11 and assets A_2, A_3, A_4, A_5, A_9;

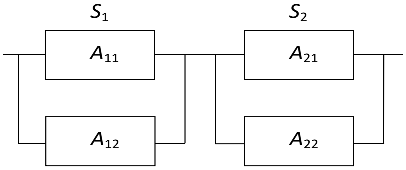

Fig. 11. The scheme of the port oil piping transportation system at the operation state z_5

- the operation state z_6 - transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 illustrated in fig. 12 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9;

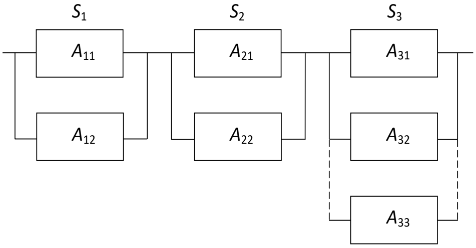

Fig. 12. The scheme of the port oil piping transportation system at the operation state z_6

- the operation state z_7 - transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3 of the asset A_1 illustrated in fig. 13 and assets A_2, A_4, A_6, A_7, A_8, A_9.

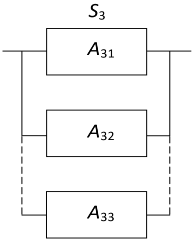

Fig. 13. The scheme of the port oil piping transportation system at the operation state z_7

The port oil terminal critical infrastructure operation process $Z(t)$ characteristics are [9]:

- the limit values of transient probabilities (OPC1) of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,7$, [12]:

$$p_1 = 0.395,\ p_2 = 0.060,\ p_3 = 0.003,\ p_4 = 0.002,\ p_5 = 0.20,\ p_6 = 0.058,\ p_7 = 0.282. \quad (4.26)$$

The coefficients of the operation process impact on the port oil terminal critical infrastructure intensities of ageing at the operation states z_b, $b = 1,2,\ldots,7$, are as follows [12]:

$$[\rho^1]^{(b)} = 1.10,\ b = 1, 2, 7,$$
$$[\rho^1]^{(b)} = 1.20,\ b = 3, 5,$$
$$[\rho^1]^{(b)} = 1.30,\ b = 4, 6. \quad (4.27)$$

Hence and from (3.17), applying (4.18) we get:

$$
\begin{aligned}
&[\lambda^1]^{(1)} = 1.10 \cdot 0.115873 = 0.1274603,\\
&[\lambda^1]^{(2)} = 1.10 \cdot 0.115873 = 0.1274603,\\
&[\lambda^1]^{(3)} = 1.20 \cdot 0.115873 = 0.1390476,\\
&[\lambda^1]^{(4)} = 1.30 \cdot 0.115873 = 0.1506349,\\
&[\lambda^1]^{(5)} = 1.20 \cdot 0.115873 = 0.1390476,\\
&[\lambda^1]^{(6)} = 1.30 \cdot 0.115873 = 0.1506349,\\
&[\lambda^1]^{(7)} = 1.10 \cdot 0.115873 = 0.1274603.
\end{aligned}
\tag{4.28}
$$

From the results (4.26) and (4.28), applying (4.6) we have:

$$
\begin{aligned}
&[\boldsymbol{R}^1(t)]^{(1)} = \exp[-0.1274603\mathrm{t}],\\
&[\boldsymbol{R}^1(t)]^{(2)} = \exp[-0.1274603\mathrm{t}],\\
&[\boldsymbol{R}^1(t)]^{(3)} = \exp[-0.1390476\mathrm{t}],\\
&[\boldsymbol{R}^1(t)]^{(4)} = \exp[-0.1506349\mathrm{t}],\\
&[\boldsymbol{R}^1(t)]^{(5)} = \exp[-0.1390476\mathrm{t}],\\
&[\boldsymbol{R}^1(t)]^{(6)} = \exp[-0.1506349\mathrm{t}],\\
&[\boldsymbol{R}^1(t)]^{(7)} = \exp[-0.1274603\mathrm{t}],\\
&\qquad t \geq 0.
\end{aligned}
\tag{4.29}
$$

Hence, applying (4.22), the expected values of the port oil terminal critical infrastructure lifetimes at the operation states z_b, $b = 1,2,\ldots,v$, respectively are:

$$
\begin{aligned}
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(1)} \cong 7.85,\\
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(2)} \cong 7.85,\\
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(3)} \cong 7.19,\\
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(4)} \cong 6.64,\\
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(5)} \cong 7.19,\\
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(6)} \cong 6.64,\\
&[\boldsymbol{\mu}^{\mathbf{1}}]^{(7)} \cong 7.85.
\end{aligned}
\tag{4.30}
$$

Further, by (4.8) and from the results (4.26) and (4.29) we get the reliability function:

$$
\begin{gathered}
\boldsymbol{R}^1(t) = 0.395\exp[-0.1274603t] + 0.060\exp[-0.1274603t]\\
+ 0.003\exp[-0.1390476t] + 0.002\exp[-0.1506349t] + 0.20\exp[-0.1390476t]\\
+ 0.058\exp[-0.1506349t] + 0.282\exp[-0.1274603t],\\
t \geq 0.
\end{gathered}
\tag{4.31}
$$

The graph of the reliability function is show in fig. 14.

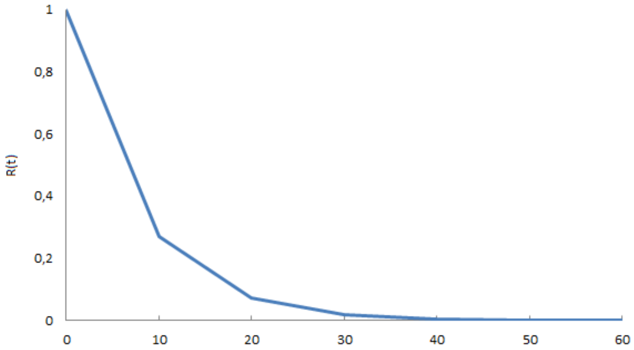

Fig. 14. The graph of the oil terminal critical infrastructure reliability function $\boldsymbol{R}^1(t)$ coordinate

Considering (4.26) and (4.30) and applying (4.11), the expected value and standard deviation of the port oil terminal critical infrastructure lifetimes respectively is:

$$\boldsymbol{\mu}^1 \cong 0.395\cdot 7.85 + 0.06\cdot 7.85 + 0.003\cdot 7.19 + 0.002\cdot 6.64 + 0.20\cdot 7.19 + 0.058\cdot 6.64 + 0.282\cdot 7.85 = 7.64 \text{ years}, \tag{4.32}$$

and applying (4.13)-(4.14)

$$\eta^1 \cong 117.017, \tag{4.33}$$

$$\boldsymbol{\sigma}^1 = \sqrt{\eta^1 - [\boldsymbol{\mu}^1]^2} \cong 7.66. \tag{4.34}$$

As the critical state is r =1, then by (4.10) and (4.31), the port oil terminal critical infrastructure risk function (RelI2), is given by

$$\boldsymbol{r}^1(t) = 1 - \{0.395\exp[-0.1274603t] + 0.060\exp[-0.1274603t] + 0.003\exp[-0.1390476t] + 0.002\exp[-0.1506349t] + 0.20\exp[-0.1390476t] + 0.058\exp[-0.1506349t] + 0.282\exp[-0.1274603t]\} \text{ for } t \geq 0, \tag{4.35}$$

The graph of the risk function $\boldsymbol{r}^1(t)$ of the oil terminal critical infrastructure, the fragility curve (RelI3), is shown in fig. 15.

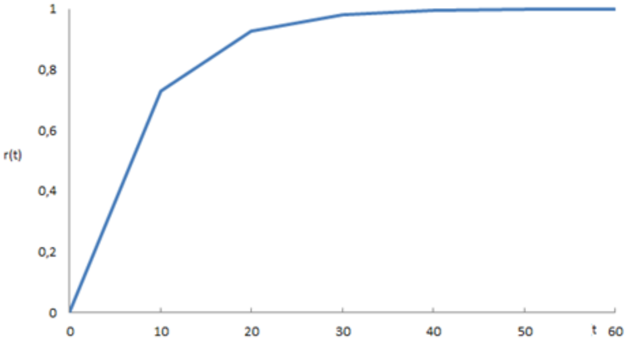

Fig. 15. The graph of the risk function $\boldsymbol{r}^1(t)$ (the fragility curve) of the oil port terminal critical infrastructure

From (4.15) and (4.35), the moment when the oil terminal critical infrastructure risk function exceeds a permitted level $\delta = 0.05$ (RelI6), is

$$\tau^1 = (\boldsymbol{r}^1)^{-1}(\delta) \cong 0.3689 \text{ years.} \tag{4.36}$$

Applying (4.22), the oil terminal critical infrastructure intensity of failure (RelI7), is

$$\lambda^1(t) = \frac{1}{\boldsymbol{\mu}^1} \cong \frac{1}{7.64} \cong 0.1309. \tag{4.37}$$

The graph of the intensity of failure of the oil terminal critical infrastructure is shown in fig. 16.

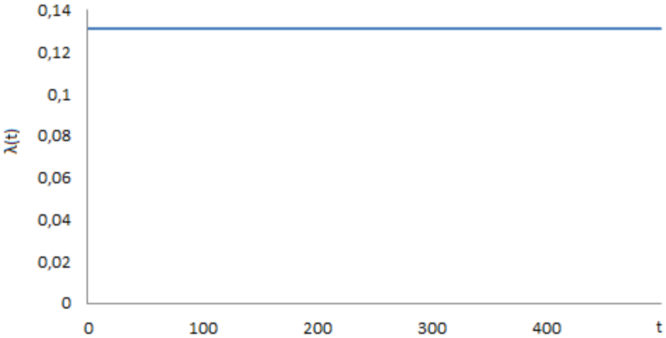

Fig. 16. The graph of the intensity of failure of the oil terminal critical infrastructure

Considering (3.17) and (4.37) and applying (4.23), the coefficient of the operation process impact on the oil terminal critical infrastructure intensity of failure, is

$$\boldsymbol{\rho}^1(t) = \frac{\lambda^1(t)}{\lambda^0(t)} = \frac{0.1309}{0.1159} \cong 1.1294. \tag{4.38}$$

Finally, by (4.24) and (4.38), the port oil terminal critical infrastructure resilience indicator (ResI2), i.e. the coefficient of the port oil terminal critical infrastructure resilience to operation process impact, is

$$\boldsymbol{RI}(t) = \frac{1}{\boldsymbol{\rho}^1(t)} = \frac{1}{1.1294} \cong 0.8854 = 88.54\%,\ t \geq 0. \tag{4.39}$$

The comparison of reliability indicators (3.14)-(3.17) and (4.32)-(4.37) prove on a significant influence of the operation process on the port oil terminal reliability what is clearly expressed in the resilience indicator (4.39).

5. Conclusions

The proposed reliability models of critical infrastructure safety without considering outside impacts and the critical infrastructure impacted by its operation process can be applied to the reliability and resilience analysis of various critical infrastructures. They, together with the newest results on reliability of systems with ageing and dependent components presented in [16-18] and [2, 1] respectively, can be the basis for analyzing reliability of critical infrastructures composed of ageing and dependent assets. Further research can be related with considering other impacts and solving the problems of critical infrastructure reliability optimization and finding the optimal values of reliability and resilience indicators [11, 7]. These results can help to mitigate critical infrastructure accident consequences and to enhance critical infrastructure resilience to operation and other impacts [3]. This research can also result in the backgrounds for business continuity and cost-effectiveness analysis of critical infrastructures under operation and other impacts.

6. References

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