Calculation of discrete states probability of the system with four units of harvesting machines

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Abstract. The model example is considered. In particular, the system composed of four units of harvesting techniques is considered. The states of the corresponding system are being analyzed. The probability of state i is the probability of the fact that the system will be in the state S_i in the instant t. The state probabilities can be found as time functions. For this the system of differential equations with unknown functions is obtained. The intensities of events flow appear in the system. They cause the rejection of unit i of harvesting machines and the intensities of events flow after "finishing of repairing" of unit i of harvesting techniques. The intensities of rejections as time function are considered. The system is solved using numerical methods. The probabilities of finding the system in the discrete states are calculated. The tabulated solution functions and their graphs are given. It is shown how the probabilities of states depending on time change.

Key words: project management, configuration, discrete states, intensities of rejections.

INTRODUCTION

It is necessary to apply methods and models that allow increasing the efficiency of technical potential use of harvesting machines for solving the problem of optimal technical support of agricultural production processes.

ANALYSIS OF THE LATEST RESEARCHES AND PUBLICATIONS

The application of mathematical methods and models in agro-industrial projects management aimed at the increase of the number harvesting machines is the topical scientific and practical problem [1]. In the latest researches [1, 2, 3, 4, 5] and others models of productivity indicators of harvesting t machines are considered. However, similar models do not take into consideration the probable character of the factors which affect the ag-ricultural goods harvesting process and it declines the modeling accuracy.

MATERIALS AND METHODS

In [6, 7, 8] it is grounded the advisability and methodology of applying accidental processes in models of harvesting machines productivity determination in the management of agricultural goods harvesting projects. Such approach takes into consideration the probabilistic nature of the factors influencing the activity of physical systems.

The productivity of harvesting machines during the operations of agricultural production harvesting projects is of probabilistic nature. That is why it is relevant to apply the stochastic tasks involving the usage of random markovian processes in the models of definition of agricultural machines productivity; the optimization of their quantity for certain projects and the calculation of their economic efficiency.

The model example for the system created of four units of harvesting techniques is considered [8; 9; 10]. The probability of machinery failure is taken into consideration. We have to deal with undeterminate factors of machinery failure. These factors are random quantities. The probabilistic characteristics of these quantities can be obtained empirically (using statistical methods). The process under study is a process with discrete states and permanent time. It means that
1) the given physical system can be in different states, that can be enumerated in advance; the states are characterized by failure resulting from machinery breakage and unplanned repairs of a certain number of harvesting equipment units; the transition from the state of functionality into the state of failure occurs suddenly; 2) the transitions from one state into another are indeterminate that is why they are possible in any moment.

The states of the corresponding system were analyzed. Later the graph was made up [8; 9].

On its basis the mathematical model of the systems with discrete states has been built. In this case system *S* has been considered, that has sixteen possible states: S_i (*i* = 1, …, 16). The probability of *i*-state is the probability of the fact that the system will be in the state S_i in the instant t. For every instant of time the sum of all state probabilities equals unit. The state probabilities can be found as time functions.

The possible discrete states of this system: S_1 – all four units are serviceable; S_2 – the first unit is being repaired, the second, the third and the fourth ones are serviceable; S_3 - the second unit is being repaired, the first, the third and the fourth ones are serviceable; S_4 - the third unit is being repaired, the first, the second and the fourth ones are serviceable; S_5 - the fourth unit is being repaired, the first, the second and the third ones are serviceable; S_6 - the first and the second units are being repaired, the third and fourth ones are serviceable; S_7 - the first and the third units are being repaired, the second and fourth ones are serviceable; S_8 - the first and the fourth units are being repaired, the second and third ones are serviceable; S_9 - the second and the third units are being repaired, the first and fourth ones are serviceable; S_{10} – the second and the fourth units are being repaired, the first and third ones are serviceable; S_{11} – the third and the fourth units are being repaired, the first and second ones are serviceable; S_{12} – the first, the second and the third units are being repaired, the fourth one is serviceable; S_{13} – the first, the second and the fourth units are being repaired, the third one is serviceable; S_{14} – the second, the third and the fourth units are being repaired, the first one is serviceable; S_{15} – the first, the third and fourth units are being repaired, the second one is serviceable; S_{16} – all four units are being repaired.

We suppose that the average time of repairing harvesting machines unit does not depend on the fact whether one unit is being repaired or several units are being repaired at once. We also believe that the transition of the system from the state S_1 in the state S_6 is possible only through the states S_2, S_3 and S_{12}. That is why we consider that all units can fail independently, we neglect that it can be simultaneously.

Let the system is in the state S_1. It is obvious that events which promote the rejection of the first unit of harvesting machines transfer it into the state S_2. Its intensity λ_1 is equal to the unit that is divided into the average time of infallible work of the first machinery unit. In the reverse direction the system is transferred from the state S_2 in the state S_1 due to the flow of "finishing of repairing" of the first unit of harvesting technique. Its intensity μ_1 is equal to the unit that is divided into the average time of repairing of the first technique unit. The intensities of events flows that transfer the system from

the state into the state are similarly calculated. The system transfers in different states are represented by the appropriate graph of states [8, 9]. Having marked graph of states one can find probabilities of states p_i (*i* = 1, …, 16) as the function of time. For this reason it is compiled the system of Kolmogorov equations in order to search probabilities p_i (*i* = 1, …, 16) of system stay in every state S_n (*n* = 1, …, 16) [10] (1).

Here λ_i (*i* = 1, …, 4) - intensities of events flow that promote the rejection of *i* (*i* = 1, …, 4) harvesting technique unit; μ_i (*i* = 1, …, 4) - intensities of events flow of "finishing of repairing" of *i* (*i* = 1, …, 4) harvesting technique unit.

MAIN PRESENTATION

It is necessary to set the initial conditions in order to solve Kolmogorov equation and find states probabilities. If the initial state of the system S_1 is known then we can suppose, for example, that in time moment *t* = 0,1 p_1 (0,1) = 1 all other initial probabilities are equal to zero. In our case we can assume that in time moment *t* = 0,1 all four machinery units are serviceable that is we solve the system (1) under such initial conditions:

$$p_1(0,1) = 1 \, p_2(0,1) = p_3(0,1) = p_4(0,1) =$$

$$= p_5(0,1) = p_6(0,1) = p_7(0,1) = p_7(0,1) =$$

$$p_8(0,1) = p_9(0,1) = p_{10}(0,1) = p_{11}(0,1) = \quad (2)$$

$$= p_{12}(0,1) = p_{13}(0,1) = p_{14}(0,1) = p_{15}(0,1) =$$

$$= p_{16}(0,1) = 0.$$

We will consider the rejection intensities $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ as functions from time. It is confirmed by observations data [11; 12; 13; 14]. The functions of intensities are modeled as:

$$\lambda(t) = \lambda_0 \, \alpha \, t^{\alpha-1},$$

where: λ_0 and α are some numerical parameters [15].

We use mathematically processed statistical data [14] and method of least squares for determining parameters λ_0 and α of the function $\lambda(t)$. After finding parameters the functions $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ will be as following:

$$\lambda_1(t) = 877,964 \cdot t^{-1,88193},$$

$$\lambda_2(t) = 816,84 \cdot t^{-1,85615}, \quad (3)$$

$$\lambda_3(t) = 838,6609 \cdot t^{-1,91494},$$

$$\lambda_4(t) = 838,6609 \cdot t^{-1,91494}.$$

$$\frac{dp_1}{dt} = \mu_1 p_2 + \mu_2 p_3 + \mu_3 p_4 + \mu_4 p_5 - (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) p_1,$$

$$\frac{dp_2}{dt} = \lambda_1 p_1 + \mu_2 p_6 + \mu_3 p_7 + \mu_4 p_8 - (\lambda_2 + \lambda_3 + \lambda_4 + \mu_1) p_2,$$

$$\frac{dp_3}{dt} = \lambda_2 p_1 + \mu_1 p_6 + \mu_3 p_9 + \mu_4 p_{10} - (\lambda_1 + \lambda_3 + \lambda_4 + \mu_2) p_3,$$

$$\frac{dp_4}{dt} = \lambda_3 p_1 + \mu_1 p_7 + \mu_2 p_9 + \mu_4 p_{11} - (\lambda_1 + \lambda_2 + \lambda_4 + \mu_3) p_4,$$

$$\frac{dp_5}{dt} = \lambda_4 p_1 + \mu_1 p_8 + \mu_2 p_{10} + \mu_3 p_{11} - (\lambda_1 + \lambda_2 + \lambda_3 + \mu_4) p_5,$$

$$\frac{dp_6}{dt} = \lambda_2 p_2 + \lambda_1 p_3 + \mu_3 p_{12} + \mu_4 p_{13} - (\lambda_3 + \lambda_4 + \mu_1 + \mu_2) p_6,$$

$$\frac{dp_7}{dt} = \lambda_3 p_2 + \lambda_1 p_4 + \mu_2 p_{12} + \mu_4 p_{14} - (\lambda_2 + \lambda_4 + \mu_1 + \mu_3) p_7,$$

$$\frac{dp_8}{dt} = \lambda_4 p_2 + \lambda_1 p_5 + \mu_2 p_{13} + \mu_3 p_{14} - (\lambda_2 + \lambda_3 + \mu_1 + \mu_4) p_8$$

$$\frac{dp_9}{dt} = \lambda_3 p_3 + \lambda_2 p_4 + \mu_1 p_{12} + \mu_4 p_{15} - (\lambda_1 + \lambda_4 + \mu_2 + \mu_3) p_9,$$

$$\frac{dp_{10}}{dt} = \lambda_4 p_3 + \lambda_2 p_5 + \mu_1 p_{13} + \mu_3 p_{15} - (\lambda_1 + \lambda_3 + \mu_2 + \mu_4) p_{10},$$

$$\frac{dp_{11}}{dt} = \lambda_4 p_4 + \lambda_3 p_5 + \mu_1 p_{14} + \mu_2 p_{15} - (\lambda_1 + \lambda_2 + \mu_3 + \mu_4) p_{11},$$

$$\frac{dp_{12}}{dt} = \lambda_3 p_6 + \lambda_2 p_7 + \lambda_1 p_9 + \mu_4 p_{16} - (\lambda_4 + \mu_1 + \mu_2 + \mu_3) p_{12},$$

$$\frac{dp_{13}}{dt} = \lambda_4 p_6 + \lambda_2 p_8 + \lambda_1 p_{10} + \mu_3 p_{16} - (\lambda_3 + \mu_1 + \mu_2 + \mu_4) p_{13},$$

$$\frac{dp_{14}}{dt} = \lambda_4 p_7 + \lambda_3 p_8 + \lambda_1 p_{11} + \mu_2 p_{16} - (\lambda_2 + \mu_1 + \mu_3 + \mu_4) p_{14},$$

$$\frac{dp_{15}}{dt} = \lambda_4 p_9 + \lambda_3 p_{10} + \lambda_2 p_{11} + \mu_1 p_{16} - (\lambda_1 + \mu_2 + \mu_3 + \mu_4) p_{15},$$

$$\frac{dp_{16}}{dt} = \lambda_4 p_{12} + \lambda_3 p_{13} + \lambda_2 p_{14} + \lambda_1 p_{15} - (\mu_1 + \mu_2 + \mu_3 + \mu_4) p_{16}$$

$$(1)$$

Let we make model assumption that events flow intensity promoting exit from the state of breaking does not depend on time that is we find the significance μ_1, $\mu_2, \mu_3,$ μ_4. For this reason we consider the specific brand of combine harvesters. At today's market of Ukraine there is a great number of combine harvesters which differ by technical and value indicators [11, 12, 13, 14].

Statistical data on the particular indicators of harvesters (removal of technological rejections) are collected and mathematically processed on the basis of chronometric observations after the work of combine harvesters under conditions of agricultural enterprises of Lviv region [11, 12, 13, 14]. In particular

$$\mu_1 = 1,75 \ , \ \mu_2 = 2, \ \mu_3 = 2,25, \ \mu_3 = 2,5. \qquad (4)$$

We get the system (1) – the system of differential equations with non-linear coefficients $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$.

So, under the initial conditions (2) and appropriate meanings of technological indicators $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$, μ_1, μ_2, μ_3, μ_4 (3) and (4) we solve the system (1).

The system (1) is solved by numerical methods with the help of the program package Maple. We present the tabulated functions of solution (Table) and their graphs (Fig. 1).

Table. Tabulated functions of solution

| t	0,1	10,1	20,1	30,1	40,1	50,1	60,1	70,1	80,1	90,1	100,1
p_1	1	0,001	0,028	0,129	0,270	0,405	0,5172	0,6055	0,6741	0,7275	0,7694
p_2	0	0,017	0,054	0,052	0,036	0,023	0,0145	0,0094	0,0062	0,0043	0,0030
p_3	0	0,013	0,042	0,039	0,027	0,017	0,0104	0,0067	0,0044	0,0030	0,0021
p_4	0	0,105	0,098	0,046	0,019	0,008	0,0036	0,0018	0,0009	0,0005	0,0003
p_5	0	0,111	0,098	0,044	0,018	0,007	0,0033	0,0016	0,0008	0,0005	0,0003
p_6	0	0,088	0,075	0,033	0,013	0,005	0,0024	0,0011	0,0006	0,0003	0,0002
p_7	0	0,076	0,066	0,029	0,012	0,005	0,0022	0,0010	0,0005	0,0003	0,0002
p_8	0	0,004	0,051	0,109	0,133	0,131	0,1182	0,1035	0,0895	0,0773	0,0670
p_9	0	0,004	0,045	0,096	0,118	0,117	0,1068	0,0938	0,0815	0,0706	0,0614
p_{10}	0	0,003	0,034	0,072	0,087	0,085	0,0766	0,0667	0,0574	0,0494	0,0427
p_{11}	0	0,003	0,034	0,070	0,083	0,079	0,0705	0,0606	0,0517	0,0441	0,0378
p_{12}	0	0,023	0,080	0,081	0,058	0,038	0,0244	0,0160	0,0108	0,0075	0,0053
p_{13}	0	0,018	0,062	0,061	0,043	0,027	0,0175	0,0114	0,0076	0,0053	0,0037
p_{14}	0	0,020	0,062	0,059	0,041	0,026	0,0161	0,0104	0,0069	0,0047	0,0033
p_{15}	0	0,016	0,054	0,054	0,038	0,025	0,0158	0,0103	0,0069	0,0048	0,0034
p_{16}	0	0,499	0,119	0,025	0,006	0,002	0,0005	0,0002	0,0001	0,0000	0,0000
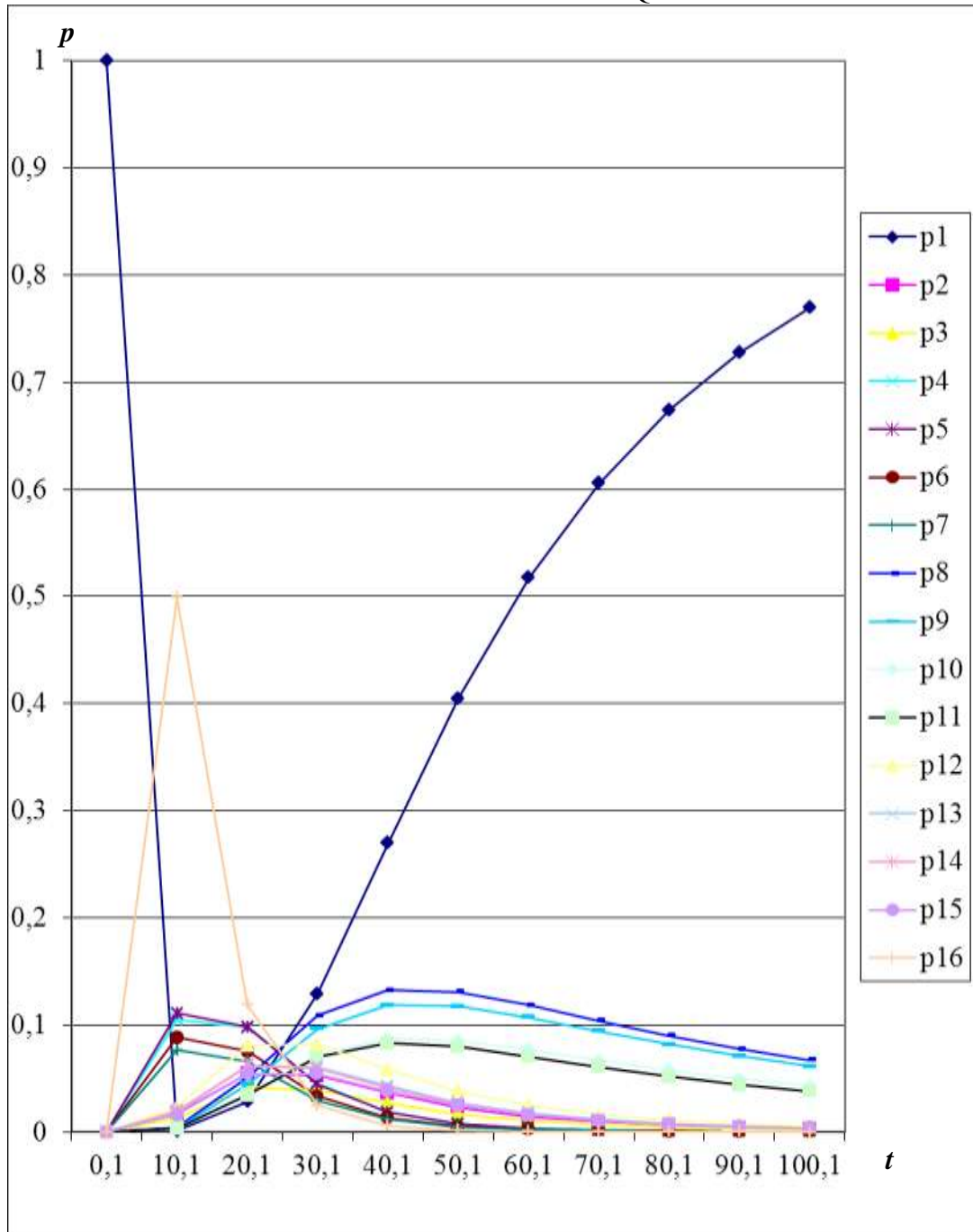
Fig. 1. The graph of system states probabilities

On the basis of interrelation of probabilities the picture 2 it is determined how system states probabilities p_i ($i = 1, …, 16$) differ over time of the system work during 100 hours. The received solutions allow collecting agricultural goods, to evaluate the average efficiency of harvesting machines system work, to optimize the number of units, to determine productivity indicators, to calculate economic efficiency.

CONCLUSIONS

It is shown that during different periods of time the probabilities of stay of three units system in appropriate discrete states differ significantly. In particular it is determined that p_1 – the probability of the fact that all three units of harvesting machines will be serviceable is the biggest one however it decreases rapidly, during the first 10 hours of the system work. During next 20 hours of the work the significance of this probability reaches its minimum. After 30 hours of the system work, p_1 increases fast, exceeding probabilities of other states. The probabilities of other states have increasing character during the first 20-40 hours of the system work, then they decrease gradually.

Model solutions may be applied in certain circumstances of agricultural production harvesting, during the optimization of the quantity of machinery units for the certain conditions of agricultural production harvesting, the definition of the productivity indicators, the calculation of economic efficiency etc.

The designed mathematical model may be used for the development and realization of management projects. Nowadays it opens up the possibility of prediction of the development of the systems with discrete states and provides efficient functioning of such systems.

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