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Geometrically Nonlinear Free Transversal Vibrations of Thin-Walled Elongated Panels with Arbitrary Generatrix

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Abstract

The algorithm for finding a finite number of the first values of natural frequencies and forms of geometrically nonlinear free transverse vibrations of thin-walled elongated panels with arbitrary generatrix is proposed and verified. Under normal coordinate quadratic the approximation of displacements is used. Along the tangential coordinates used one-dimensional finite elements. The discrete variation problem is built. For its solving the perturbation method is applied. The numerical results are compared with previously obtained by other authors.

Keywords: elongated panels, vibrations, nonlinearity, perturbations method

1. Introduction

Thin elongated panels with various curves as generatrix medial surfaces are widely used in the construction and hardware for various purpose. In the operating conditions they are subjected to intense dynamic loading, in particular, cyclic. These loads are causing in panels the normal displacement commensurate with their thickness. The last are causing to their geometrically nonlinear dynamic stress-strain state.

To avoid resonance phenomena for the actions of cyclic loading is necessary at the design stage to determine the spectrum of frequencies of said structural elements. Issues of geometrically nonlinear vibrations of plate and shell elements of the constructions on the basis classical and shear theories thoroughly examined in [11] for the definition of the fundamental frequency. Significant progress in this field together with experimental approaches is done in [1, 2] and some analytical results are given in [8]. However, for nonlinear oscillations in many cases it is necessary to define a number of first frequencies and forms to detect the phenomena of internal, subharmonic and combination resonances [4]. A numerical method for determining the first several frequencies and forms at geometrically nonlinear vibrations of shells is proposed in the work [5].

In this paper is developed and verified algorithm for determining a finite number of natural frequencies and forms elongated thin panels for geometrically nonlinear vibrations. For the primary relations is taken spatial equation geometrically nonlinear dynamic theory of elasticity. Used quadratic approximation of displacements by the normal coordinate and finite-element by tangential. The discrete variation problem is built. For its solving the method of perturbations is applied.

2. Problem statement

Curved anisotropic elastic layer with thickness h we take to natural mixed system of coordinate α_1, α_2, α_3 on the median surface. This surface is formed by the motion of the line $\alpha_1 = 0$; $\alpha_3 = 0$ on the segment of arbitrary generatrix. We consider that layer is significantly larger along the axis α_2 to the length of the section arc $\alpha_2 = 0$ of the middle surface $\alpha_3 = 0$. So we have an elongated panel. If the conditions of fixing the ends of the panel $\alpha_1 = \pm\alpha_1^0$ and the initial conditions are independent of the coordinate α_2, then through little influence of conditions fixing the edges $\alpha_2 = \pm\alpha_2^0$, the functions, that determine the characteristics of geometrically nonlinear vibration processes in the plane of the middle section, are dependent from α_1, α_3. To find these functions are [9]:

- motion equations

$$div\,\hat{S} = \rho \frac{\partial^2 U}{\partial t^2}\,; \tag{1}$$

- elasticity relations

$$\hat{\Sigma} = \tilde{A} \otimes \hat{\varepsilon}\,; \tag{2}$$

- deformation relation between the strain tensor components $\hat{\varepsilon}$ and the components of the elastic displacement vector $\vec{U} = u_i \vec{e}_i \vec{e}_j$

$$\varepsilon_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i + \nabla_i u^k \nabla_j u_k)\,; \tag{3}$$

- relation between the components S^{ij} of the nonsymmetrical Kirchhoff stress tensor $\hat{S}$ and the components σ^{ik} of the symmetric Piola stress tensor $\hat{\Sigma}$

$$S^{ij} = \sum_k \sigma^{ik}(\delta_k^j + \nabla_k u^j)\,. \tag{4}$$

In equations (1) and (2) $\tilde{A}$ – tensor of elastic properties of anisotropic layer, and ρ – its density.

Boundary conditions on the front surface of the panel $\alpha_3 = \pm h/2$ for the free vibrations has the form

$$S^{31}(\alpha_1, \pm h/2, t) = S^{33}(\alpha_1, \pm h/2, t) = 0\,, \quad |\alpha_1| \le \alpha_1^0\,. \tag{5}$$

At the elongated ends of the panel $\alpha_1 = \pm\alpha_1{}^0$under the conditions of the fixing the hinge on the lower surface of the front $\alpha_2 = -h/2$ boundary conditions has the form

$$S^{1i}(a, \alpha_3, t) = 0\,, \tag{6}$$

$$u_i(a, \pm h/2, t) = 0\,, \quad |\alpha_3| \le h/2\,, \quad i = 1, 3\,, \quad a = 0, l\,. \tag{7}$$

The motion equations (1) together with relations (2)–(4) and boundary conditions (5)–(7) are describe geometrically nonlinear transverse vibrations of the middle section of the panel, if the initial conditions specify as follows:

$$u_i(\alpha_1, \alpha_3, t)\Big|_{t=t_0} = v_i^0(\alpha_1, \alpha_3)\,, \quad \frac{\partial u_i(\alpha_1, \alpha_3, t)}{\partial t}\Bigg|_{t=t_0} = v_i^1(\alpha_1, \alpha_3)\,, \quad i = 1,\ 3\,, \tag{8}$$

$$\left|v_3^0(\alpha_1, \alpha_3)\right| >> \left|v_1^0(\alpha_1, \alpha_3)\right|, \quad (\alpha_1, \alpha_3) \in \Omega = [-\alpha_1^0, \alpha_1^0] \times [-h/2, h/2]\,. \tag{9}$$

3. Discretezed problem

Considered above differential formulation of the problem of geometrically nonlinear free vibrations is equivalent to the problem of minimizing the functional L [10]:

$$\begin{aligned} L &= -\int_\Omega \sum_i \sum_j u_i \frac{\partial S^{ij}}{\partial x_j} d\Omega - \int_\Omega \rho \frac{\partial^2 U^T}{\partial t^2} \cdot U d\Omega = \\ &= -\int_\Omega \sum_i \sum_j S^{ij} \frac{\partial u_i}{\partial x_j} d\Omega - \int_\Omega \rho \frac{\partial^2 U^T}{\partial t^2} \cdot U d\Omega \to \min. \end{aligned} \tag{10}$$

Boundary conditions (5) and (6) for the variation formulation of the problem is a natural [10], and condition (7) must take into account during its solution.

Assuming that the considering panel is thin-walled, approximate the unknown displacement at transverse coordinate [7]:

$$u_i(\alpha_1, \alpha_2) = \sum_{k=1}^{2} u_{ik}(\alpha_1) p_k(\alpha_3)\,, \quad i = 1, 3\,, \tag{11}$$

where

$$p_0(\alpha_3) = \frac{1}{2} - \frac{\alpha_3}{h}\,, \quad p_1(\alpha_3) = \frac{1}{2} + \frac{\alpha_3}{h}\,, \quad p_2(\alpha_3) = 1 - \left(\frac{2\alpha_3}{h}\right)^2.$$

For finding unknown coefficients $u_{ik}(\alpha_1)$ in (11) we use approximation by the tangential coordinate α_1 on one-dimensional izoparametrical linear finite elements [10]:

$$u_{ik}^{(e)} = \sum_{k,m}^{2} u_{ikm}^{(e)}(\alpha_1)\varphi_m^{(e)}(\xi)\,, \quad \xi = \frac{2\alpha_1}{\alpha_{12}^{(e)} - \alpha_{11}^{(e)}} - 1\,, \tag{12}$$

where e – element number; $u_{ikm}^{(e)} = u_{ik}(\alpha_{1m}^{(e)})$; $\alpha_{1m}^{(e)}(\alpha_1)$, $m = 1, 2$ – coordinates of the element nodes; $\varphi_1^{(e)}(\xi) = \frac{1}{2}(1-\xi)$; $\varphi_2^{(e)}(\xi) = \frac{1}{2}(1+\xi)$.

After substituting (11) into (4) and the result together with (11) into (10) we obtain:

$$L^{\Delta} = \{u\}^T K_L \{u\} + \{u\}^T K_{NL}(u)\{u\} + \{u\}^T M\{\ddot{u}\} \to \min\,, \tag{13}$$

where $\{u\}$ = $\{u\}(t)$ – vector of values of the coefficients $u_{ikm}^{(e)}$ at points in the finite-element partition of the section $[-\alpha_1^0, \alpha_1^0]$; K_L – linear, and K_{NL} – nonlinear components of stiffness matrix; M – matrix of mass [10].

Non-linear component of stiffness matrix K_{NL} presented in the form

$$K_{NL}(\{u\}(t)) = B(\{u\}(t)) \cdot \{u\}^T(t) \cdot B^T(\{u\}(t))\,. \tag{14}$$

Matrix $B(\{u\}(t))$ we obtain by integrating in (10) members, who are the product of partial derivatives, the displacement u_i [10].

Minimum of discrete functional (13) is achieved at the point $\{u\}(t)$, where the equation is satisfied

$$K_L(\{u\}(t)) + K_{NL}(\{u\}(t))\{u\}(t) + M(\{\ddot{u}\}(t)) = 0\,. \tag{15}$$

4. The method of perturbations

The system of nonlinear equations (15) is written as

$$K_L(\{u\}(t)) + \mu K_{NL}(\{u\}(t))\{u\}(t) + M(\{\ddot{u}\}(t)) = 0\,, \tag{16}$$

where μ $(0 \le \mu \le 1)$ – the parameter perturbation. At $\mu = 0$ have a system of linear algebraic equations for the vector $\{u\}$, while $\mu = 1$ the nonlinearity is taken into account fully. The method of perturbations the desired vector of functions $\{u\}(t)$ and matrix K_L presented in the form

$$\{u\}(t) = \{u\}_0(t) + \mu\{u\}_1(t) + \ldots\,,$$

$$K_L = K - \mu K_{L1} - \ldots\,. \tag{17}$$

The result of substituting (17) into (16) and grouping expressions under the same powers of μ are the equations

$$M\{\ddot{u}\}_0(t) + K\{u\}_0(t) = 0\,, \tag{18}$$

$$M\{\ddot{u}\}_1(t) + K\{u\}_1(t) - K_{L1}(\{u\}_0(t)) + K_{NL}(\{u\}_0(t))\{u\}_0(t) = 0\,. \tag{19}$$

Solution of equation (18) is written as

$$\{u\}_0(t) = \tilde{u}\cos\omega t\ , \tag{20}$$

and the solution of (19) can be written as follows. Consider equation (17), which seeks a solution in the form

$$\{u\}_1(t) = \{u\}_1^{*}(t) + \{u\}_1^{\times}(t)\,, \tag{21}$$

where $\{u\}_1^{*}(t)$ i $\{u\}_1^{\times}(t)$ – solutions of homogeneous and inhomogeneous equations (19).

According to [6] matrix K_{L1} can be represented as

$$K_{L1} = \frac{3}{4}B\{\tilde{u}\}\{\tilde{u}\}^T B^T\ . \tag{22}$$

After substituting (20) and (22) into (19) and taking into account formula [3]

$$4\cos^3\omega t = 3\cos\omega t + \cos 3\omega t\,, \tag{23}$$

for finding $\{u\}_1^{\times}(t)$ we obtain the equation

$$M\{\ddot{u}\}_1^{\times}(t) + K\{u\}_1^{\times}(t) = -\frac{1}{4}B\{\tilde{u}\}\{\tilde{u}\}^T B^T\{\tilde{u}\}\cos 3\omega t\ , \tag{24}$$

solution of which we take as

$$\{\tilde{u}\}_1^{\times}(t) = \tilde{c}\cos 3\omega t\ . \tag{25}$$

After substituting (25) into equation (24) we arrive at a relations for determination of parameter $\tilde{c}$:

$$(K - 9\omega^2 M)\tilde{c} = -\frac{1}{4}B\{\tilde{u}\}\{\tilde{u}\}^T B^T\{\tilde{u}\}\,. \tag{26}$$

If the initial moment the panel is deformed on a certain law, which describes the first formula in (8) and is stationary, then the initial conditions for the functions $\{u\}_0(t)$ and $\{u\}_1(t)$ can be represented as

$$\{u\}_0(0) = \overline{A}\ ,\quad \{\dot{u}\}_0(0) = 0\,, \tag{27}$$

$$\{u\}_1(0) = 0\ ,\quad \{\dot{u}\}_1(0) = 0\ . \tag{28}$$

In view of (21), we write:

$$\{u\}(t) = (\overline{A} - \tilde{c})\cos\omega t + \tilde{c}\cos 3\omega t\ . \tag{29}$$

This allows you to build an algorithm for partial finding a finite number of the first natural frequencies and amplitudes geometrically nonlinear vibrations of the panel:

1. Set $r = 1$ and $a_{(0)} = 0$.

2. Compute $K_{(r-1)} = K + \frac{3}{4} B a_{(r-1)} a_{(r-1)}^T B^T$.
3. Find the eigenvalues $\omega_{(r)}$ and eigenvectors $a_{(r)}$ from the system
   $(K_{(r-1)} - \omega_{(r)} M) a_{(r)} = 0$.
4. If the conditions are satisfied $\left\| a_{(r)} - a_{(r-1)} \right\| / \left\| a_{(r)} \right\| \le \varepsilon_1$, $\left\| \omega_{(r)} - \omega_{(r-1)} \right\| / \left\| \omega_{(r)} \right\| \le \varepsilon_2$,
   where ε_1 and ε_2 – specified accuracy, then go to step 5, otherwise $r := r+1$, and go to step 2.
5. As the solution we accept $a := a_{(r)}$, $\omega := \omega_{(r)}$ and find a vector $\tilde{c}$ having solved the a system of algebraic equations
   $(K_{(r)} - 9\omega^2 M)\tilde{c} = -\frac{1}{4} B a a^T B^T a$.

5. Analysis of results and conclusions

To verify the proposed algorithm practicing it for problem, where are known analytical and numerical solutions [6]. We consider an isotropic plate-strip elongated edges which are fixed by with stationary hinges on the lower front of the plane (see Fig. 1), with characteristics: geometric l = 1 m; h = 0.1 m and mechanical E = 40000 N/m^2; ν = 0.3.

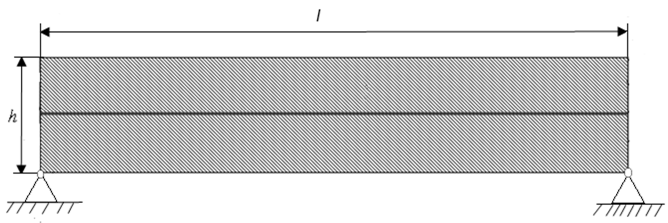

Figure 1. The plate-strip with stationary hinges on the elongated edges

In Figure 2 shows graphs of free vibrations of the point that has coordinates $(l/2; 0)$ for linear (●), analytical (▲) and obtained using the proposed algorithm (■).Sufficiently good correlation with the analytical solution is marked.

In Figure 3 shows the first four own forms (modes) for geometrically nonlinear vibrations the considered plate-strip.

In Figure 4 shows the skeletal curves [11], constructed using the proposed method (dashed line) and the results presented in the work [5] (solid line). The maximum relative error does not exceed 9%, indicating a sufficiently good approximation property of the proposed method. Subsequently, it is advisable to perform a similar study for a wider class of thin-walled elements of constructions and anisotropy of mechanical properties.

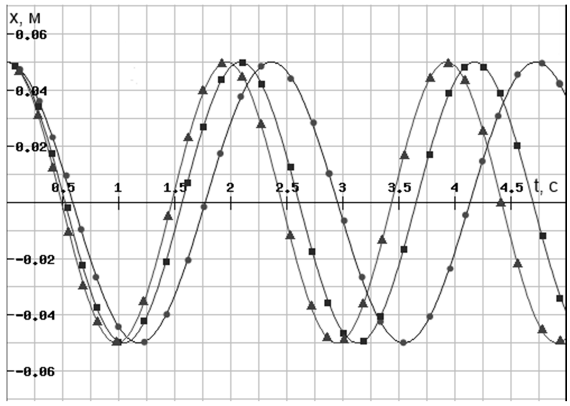

Figure 2. Free vibrations of point $(l/2;0)$

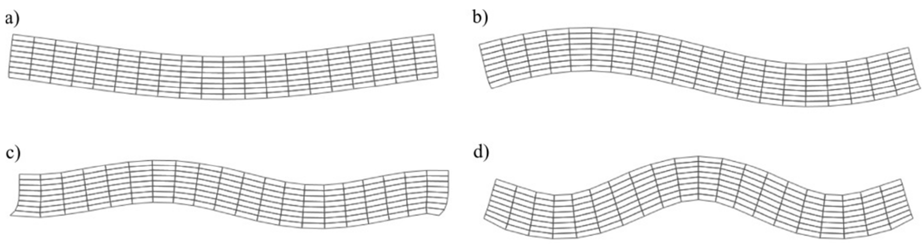

Figure 3. View panels in different modes: a) – the first mode; b) – second; c) – third; d) – the fourth

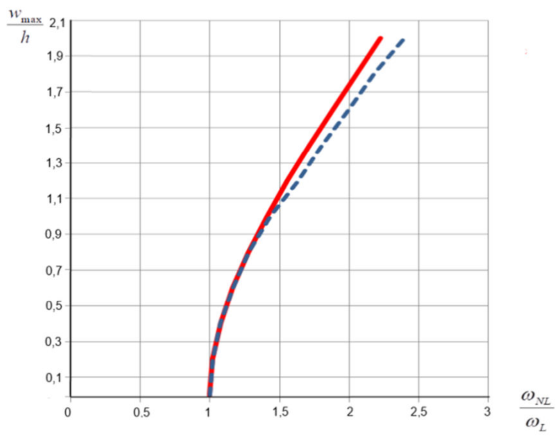

Figure 4. Comparison of amplitude-frequency characteristics obtained from the use of perturbation method and the results of the work [5]

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