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MATHEMATICAL MODELING OF THE MOTION PARAMETERS OF A DISABLED PERSON IN A BUS

Summary. With a view to determining the dynamics and possible trauma sustained by a disabled person in a wheelchair in the passenger compartment of a bus, a physical model has been developed. The system, as the time function, generalized in the coordinates $X(t)$, $\varphi(t)$, was described by second-order Lagrangian differential equations. The proposed mathematical model allows for determining the motion characteristics of disabled people in a wheelchair in a bus that can cause injuries. This paper presents the curves of the change in coordinates of the angular velocity of the head and its center of gravity, as well as the curve of the change in the angle between the femur and the wheelchaior seat, through mathematical modeling and an experiment.

1. INTRODUCTION

Passenger transportation by bus is of great importance for the country's economy, as well as for satisfying better the growing demands for sound and rapid transport of passengers. Despite this, there are many serious problems, one of which is safety. Passenger transport safety is one of the most important pre-conditions for protecting the life and health of the driver and passengers.

The safety level of passenger transportation in Georgia significantly lags behind the global standards. The main indicators to increase the safety level of passenger transportation might include travel time, comfort in travel and delay, travel safety and so on.

According to the 2015 World Health Organization report, every year, 250 thousand people die as a result of road traffic crashes, on average, worldwide. From 20 to 50 million people have nonfatal injuries [1]. All this costs the developed countries on average 3% of the gross domestic product, while in the low- and middle-income countries, it costs 5% of the GDP.

According to statistical data in foreign countries, the proportion of traffic accidents involving buses in the total number of road traffic accidents is approximately 5.5-6% [2].

From the statistics of road traffic accidents in Georgia, it is not clear how many traffic accidents were caused by passenger transport. Despite this, they play a significant role in the safety of interurban bus transportation. The probability of road accidents involving suburban, interurban and especially urban passenger transport is high. According to the results of the study, it has been established that drivers of both interurban and urban passenger vehicles drive carelessly or perform risky maneuvers.

All the standards related to passenger transport services include paying particular attention to safety requirements for passenger transportation.

100% of the respondents indicated the lack of technical equipment for persons with disabilities in buses.

Both urban and interurban buses are considered to be dangerous vehicles. Most passengers (66%) do not feel safe in terms of travel conditions. According to the persons interviewed, increased risk is associated with road conditions (38%) prevalent in the route network and multiple violations of traffic rules, especially by mini-buses, whose drivers often violate the rules of maneuvering to board the passengers, and for the same reason, cases of fast start and braking are also too frequent, as well as a lack of experience on the part of drivers (49%), and 54% of the respondents is fully or partially satisfied by the safety of passenger compartments (seats, temperature, lighting, noise). The respondents were especially questioned on how convenient the seats are arranged in mini buses, including on the distance between these seats, as well as on condition of these seats and interior in medium-capacity buses [3]. All of the aforementioned are particularly relevant for transporting disabled persons in wheelchairs by urban passenger buses.

Special attention is paid to selection of safe seats for disabled persons in wheelchairs in the passenger compartments of urban buses. The best option for them is to select seats in the middle of the passenger compartment, in the right row, away from the window. Seats located near the bus exit are also considered to be safe, since they allow for getting off the bus rapidly in case of an accident.

Providing safe transportation for passengers is a systemic problem, in which vehicles' bench seats play a key role. The vehicle bench seat must be properly fixed because, at collision, its mass should not be added to the passenger's safety belt. During a collision, the seat system should not allow passenger movement and it should prevent injury to his/her body.

During transportation of disabled people by bus, the wheelchair seat serves as a regular car seat as described above. The wheelchair seat intended for disabled persons is required to ensure safe and comfortable pick-up and set-down of passengers while traveling and during collision of the vehicle [4, 5].

Statistical data indicate that the most common type of road traffic accidents is collision: they always take place for various reasons, in different forms, and often with grave consequences.

In recent years, human motion models have been used in simulations for crash experiments. For example, Bertocci [6] has used a dynamic lumped mass crash simulator to develop a model of a restrained occupant subjected to a 0g/30mph frontal motor vehicle crash. Also, Moorcroft [7] demonstrated that motor vehicle crashes could be simulated with the use of MADYMO, a program designed specifically for occupant safety analysis.

Thus, the crashworthiness of the wheelchair is discussed in more detail [8]. The most dangerous type of accident is the frontal impact scenario. Frontal impact tests are carried out as sledge or direct impact processes [9 - 11]. In the sledge impact test, the chair is mounted on a platform using a seat belt system.

2. SUBJECT AND METHODS OF RESEARCH

At present, an increase in the vehicle fleet and, consequently, an increase in traffic density are leading to increases in the number of road traffic accidents, such as collisions with a fixed obstacle. In some countries, collision of cars (including with other parked vehicles) amounts to roughly 10-12% of the total number of road accidents.

Analysis of international and local documents regulating the safety of transportation of persons with disabilities showed that the most important argument for their possible restriction is the difficulty of safe use of a wheelchair in buses. At the same time, the analysis revealed that modern buses cannot fully ensure safe travel for persons with disabilities.

In research investigations carried out abroad [12 - 14], a computer model has been developed of the movement of a disabled person in a wheelchair during a sudden stop of vehicles and in road traffic accidents. The effectiveness of a safety belt for disabled people in wheelchairs has been described.

It is noteworthy that in cities of most of the countries abroad, as well as in urban areas of Georgia, public transport seats are not equipped with safety belts at all, and the probability of their use is very minimal by persons with disabilities in wheelchairs.

This research is focused on determining the characteristics of the possible movement of disabled people in wheelchairs in the passenger compartment, using the physical model of their dynamics, that can lead to injuries.

3. MATHEMATICAL MODELING OF THE SYSTEM OF URBAN BUS SERVICES

The goal of this research is to determine the motion parameters of a disabled person in a wheelchair in a bus, for which a generalized (modernized) model was developed based on a physical model used in the work [14]. This generalized model (Fig. 2) provides for the angle φ_4 formed between the seat and the femur of the dummy in a traffic accident. On the basis of the assumption, this model does not provide for the frictional force friction arising between the femur and the seatpad.

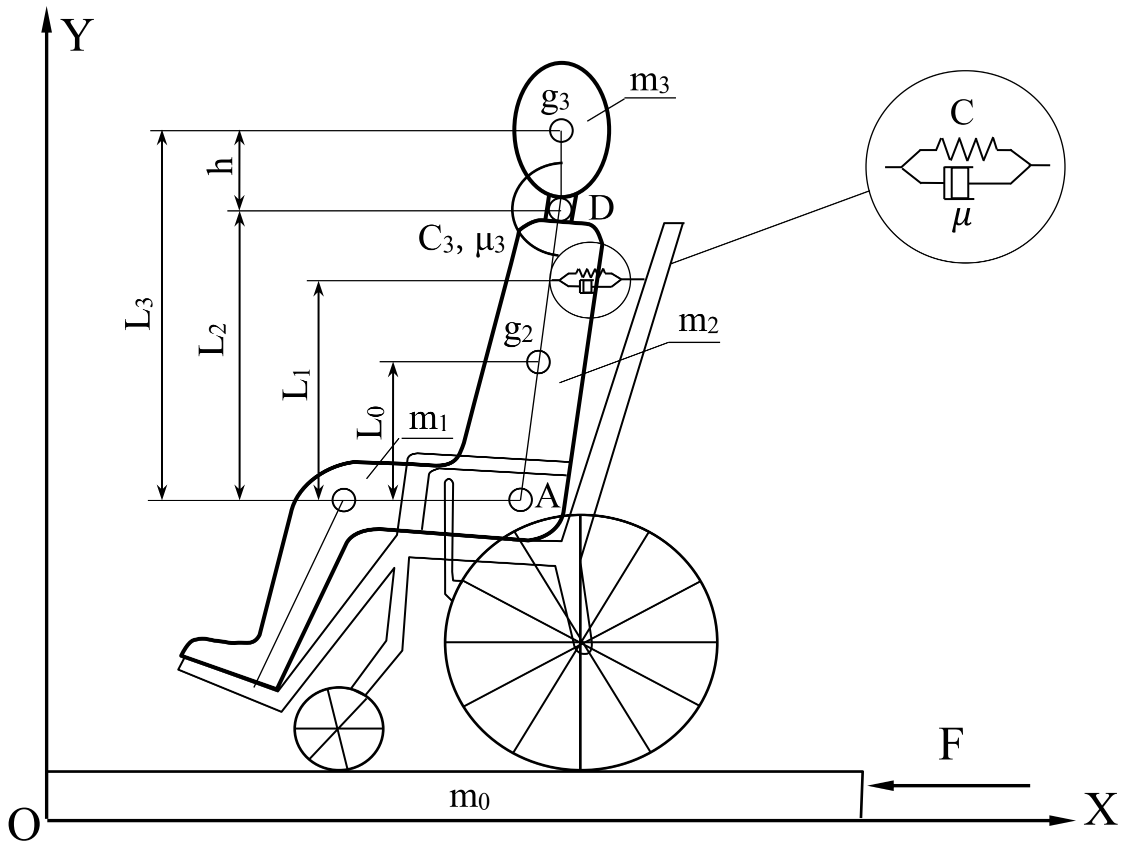


Fig. 1. A physical model of the system “bus-disabled person-wheelchair”

The following parameters are present in the model [Fig.1]:

- m_0- the mass of the bus together with the mass of a disabled person in a wheelchair;
- m_1- the mass of the passenger’s hip and shin;
- m_2- the mass of the passenger’s body and hand; and
- m_3- the mass of the passenger’s head and neck.

Member m_1 is able to move longitudinally together with joint A. Member m_2 is able to turn around joint A, while member m_3 is able to turn around conditional joint D. The bus is moving in the direction of the OX axis.

The following parameters are present in a physical model:

the center of gravity of the members g_2, g_3- m_2 and m_3;

L_0 L_1 L_2 L_3 – the distance between the conditional joint and the respective point;

h- the distance between the conditional joint D and the center of gravity of the head;

C, μ – the reduced stiffness and damping coefficient of the wheelchair back; and

C_3, μ_3 – the roll stiffness and damping coefficient of the neck.

The analysis of conducted studies revealed that the direct and backward movements of passenger in the bus compartment are adequate. Thus, we shall consider the bus movement with passengers at V_0 speed, which is braked by F force, while the slowdown changes according to trapezoid characteristics.

As is known, the second-order Lagrangian differential equations are used to study the mechanism of any mechanical system independent of the number of masses and freedom degrees [15, 16].

The model reviewed (Fig. 1) has five degrees of freedom. It consists of the head, body and femur. The center of gravity of each of them is located in their centers. As generalized coordinates, it is convenient to use the following parameters: x – the absolute motion of a wheelchair; x_1 - the relative motion of m_1 link (toward the bus); φ_2 the turning angle of m_2 link (the body turning angle); φ_3- the turning angle of m_3 link (the head turning angle); φ_4-the turning angle of the m_1 link (the femur turning angle).

The turning angles are calculated in a clockwise (positive) direction.

In this case, to solve the problem of the system, it is necessary to find the time function of this system in generalized X (t), φ (t) coordinates by the second-order Lagrangian differential equations:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x} = Q_0 \quad (1)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_1} - \frac{\partial T}{\partial x_1} = Q_1 \quad (2)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_2} - \frac{\partial T}{\partial \varphi_2} = Q_2 \quad (3)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_3} - \frac{\partial T}{\partial \varphi_3} = Q_3 \quad (4)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_4} - \frac{\partial T}{\partial \varphi_4} = Q_4 \quad (5)$$

where $T$is the critical energy of the system expressed in the generalized coordinates and velocities; Q_0, Q_1, Q_2, Q_3 and Q_4 – are the respective generalized forces in the selected generalized coordinates.

According to the scheme shown in Fig. 2, we calculate kinetic energy, on which the coordinates are counted from the initial locations of members

$$T = \frac{1}{2}m_0\dot{x}^2 + \frac{1}{2}m_1(\dot{x} - \dot{x}_1)^2 + \frac{1}{2}m_2V_2^2 + \frac{1}{2}J_2\dot{\varphi}_2^2 + \frac{1}{2}m_3V_3^2 + \frac{1}{2}J_3\dot{\varphi}_3^2 + \frac{1}{2}J_4\dot{\varphi}_4^2 \quad (6)$$

where:

V_2 is the velocity of the center of gravity of the body and hand;

V_3 is the velocity of the center of gravity of the head and neck;

$J_2 = \frac{1}{3}m_2|AG_2|^2$, $J_3 = \frac{2}{5}m_3h^2$ are the moments of inertia of the body and head toward the horizontal axis passing through their center of gravity; and

$J_4 = \frac{1}{3}m_1\ell^2$ is the moment of inertia of the femurs toward the knee joint.

By the expression of velocities V_2 and V_3 in the generalized coordinates of the system, we obtain

$${V_2}^2 = {\dot{x}_2}^2 + {\dot{y}_2}^2, \quad {V_3}^2 = {\dot{x}_3}^2 + {\dot{y}_3}^2 \quad (7)$$

where $\dot{x}_2$, $\dot{y}_2$, $\dot{x}_3$, $\dot{y}_3$, - g_2, g_3 *are* the absolute coordinates of the centers of gravity.

By inserting the values determined according to the scheme shown in Figure 2

$$x_2 = x - x_1 - L_0 sin\varphi_2 - \ell cos\varphi_4 \quad (8)$$

$$y_2 = L_0 cos\varphi_2 + \ell sin\varphi_4 + H \quad (9)$$

$$x_3 = x - x_1 - L_2 sin\varphi_2 - h sin\varphi_3 - \ell cos\varphi_4 \quad (10)$$

$$y_3 = L_2 cos\varphi_2 + h cos\varphi_3 + \ell sin\varphi_4 + H \quad (11)$$

where $(x_2; y_2)$ and $(x_3; y_3)$ are the absolute coordinates of points g_2 and g_3.

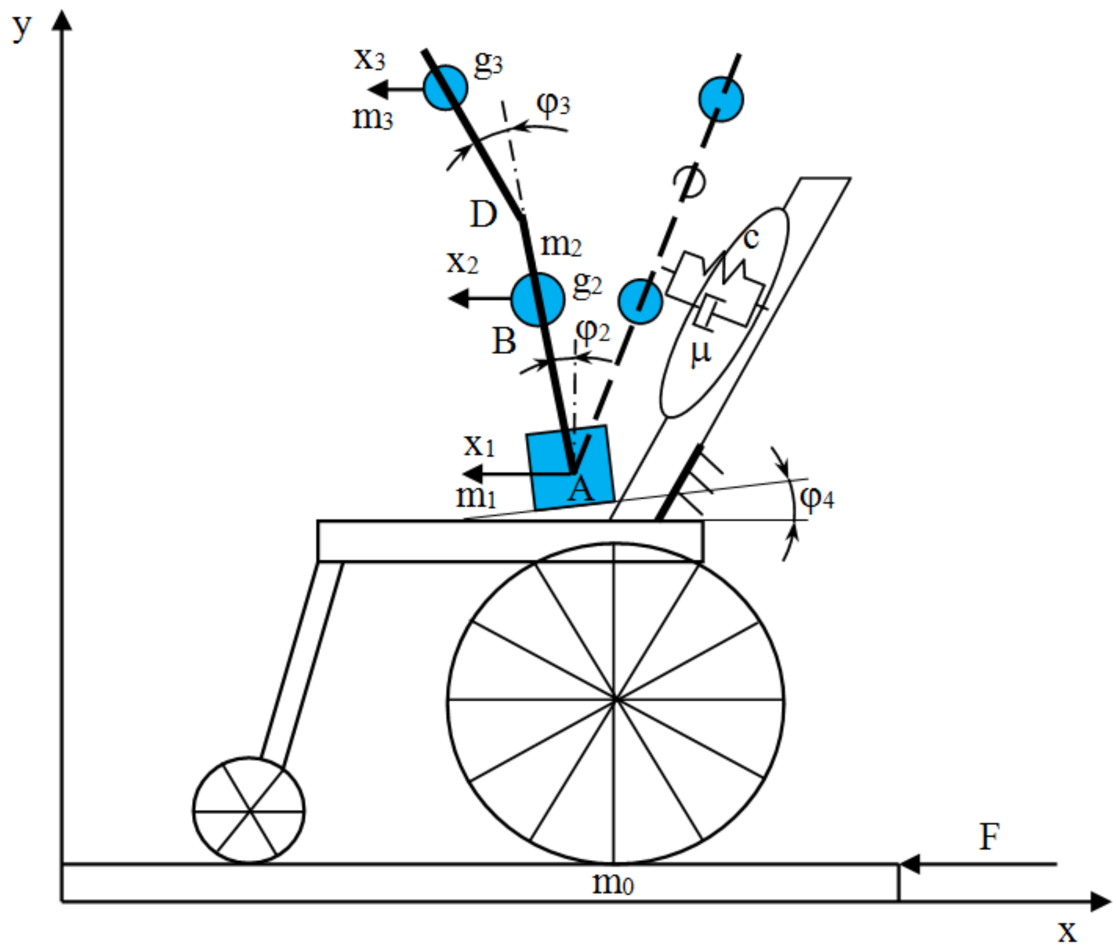


Fig. 2. Variation characteristics of the locations of members of the system “bus-disabled person in wheelchair-wheelchair” as a result of the applied force

By time differentiation of these formulas, and then by inserting into formula (7), we will obtain the expressions of the appropriate point velocities as follows:

$$\dot{x}_2 = \dot{x} - \dot{x}_1 - L_0\dot{\varphi}_2 cos\varphi_2 + \ell\dot{\varphi}_4 sin\varphi_4 \quad (12)$$

$$\dot{y}_2 = -L_0\dot{\varphi}_2 sin\varphi_2 + \ell\dot{\varphi}_4 cos\varphi_4 \quad (13)$$

$$\dot{x}_3 = \dot{x} - \dot{x}_1 - L_2\dot{\varphi}_2 cos\varphi_2 - h\dot{\varphi}_3 cos\varphi_3 + \ell\dot{\varphi}_4 sin\varphi_4 \quad (14)$$

$$\dot{y}_3 = -L_2\dot{\varphi}_2 sin\varphi_2 - h\dot{\varphi}_3 sin\varphi_3 + \ell\dot{\varphi}_4 cos\varphi_4 \quad (15)$$

$$V_2^2 = (\dot{x} - \dot{x}_1 - L_0\dot{\varphi}_2 cos\varphi_2 + \ell\dot{\varphi}_4 sin\varphi_4)^2 + (-L_0\dot{\varphi}_2 sin\varphi_2 + \ell\dot{\varphi}_4 cos\varphi_4)^2 \quad (16)$$

$$V_3^2 = (\dot{x} - \dot{x}_1 - L_2\dot{\varphi}_2 cos\varphi_2 - h\dot{\varphi}_3 cos\varphi_3 + \ell\dot{\varphi}_4 sin\varphi_4)^2 + (-L_2\dot{\varphi}_2 sin\varphi_2 - h\dot{\varphi}_3 sin\varphi_3 + \ell\dot{\varphi}_4 cos\varphi_4)^2 \quad (17)$$

By inserting the expressions of V_2 and V_3 velocities into formula (6), we will obtain the expression of the model’s kinetic energy as follows:

$$T = \frac{1}{2}m_0\dot{x}^2 + \frac{1}{2}m_1(\dot{x} - \dot{x}_1)^2 + \frac{1}{2}m_2(\dot{x} - \dot{x}_1)^2 + \frac{1}{2}m_2L_0^2\dot{\varphi}_2^2 + \frac{1}{2}m_2\ell^2\dot{\varphi}_4^2 - m_2L_0\dot{\varphi}_2(\dot{x} - \dot{x}_1)cos\varphi_2 + m_2\ell\dot{\varphi}_4(\dot{x} - \dot{x}_1)sin\varphi_4 - m_2\ell L_0\dot{\varphi}_2\dot{\varphi}_4 sin(\varphi_2 + \varphi_4) + \frac{1}{6}m_2\dot{\varphi}_2^2 + \frac{1}{2}m_3(\dot{x} - \dot{x}_1)^2 +$$

$\frac{1}{2}m_3L_0^2\dot{\varphi}_2^2+\frac{1}{2}m_3h^2\dot{\varphi}_3^2++\frac{1}{2}m_3\ell^2\dot{\varphi}_4^2-m_3L_2\dot{\varphi}_2(\dot{x}-\dot{x}_1)cos\varphi_2-m_3h\dot{\varphi}_3(\dot{x}-\dot{x}_1)cos\varphi_3+$
$m_3\ell\dot{\varphi}_4(\dot{x}-\dot{x}_1)sin\varphi_4++m_3hL_2\dot{\varphi}_2\dot{\varphi}_3cos(\varphi_3-\varphi_2)-m_3\ell L_2\dot{\varphi}_2\dot{\varphi}_4sin(\varphi_2+\varphi_4)-$
$m_3\ell h\dot{\varphi}_3\dot{\varphi}_4sin(\varphi_3+\varphi_4)+\frac{1}{5}m_3h^2\dot{\varphi}_3^2+\frac{1}{6}m_1\ell^2\dot{\varphi}_4^2$ (18)

For the calculation of generalized forces, we shall use the following formula:

$$Q_j=\frac{\sum_{i=1}^{n}\delta A_i}{\delta q_j}\ (j=1,\ldots,5) \tag{19}$$

where Q_j is a generalized force corresponding to the generalized coordinate q_j. We will obtain the following:

$$Q_0=-F \tag{20}$$

$$Q_1=0 \tag{21}$$

$$Q_2=-c(x_1+L_1\varphi_2)L_1\text{-}\mu(\dot{x}_1+L_1\dot{\varphi}_2)L_1\text{-}c_3(\varphi_3-\varphi_2)-\ \mu_3(\dot{\varphi}_3-\dot{\varphi}_2) \tag{22}$$

$$Q_3=c_3(\varphi_3-\varphi_2)+\ \mu_3(\dot{\varphi}_3-\dot{\varphi}_2) \tag{23}$$

$$Q_4=(0.5m_1+m_2+m_3)g\ell sin\varphi_4 \tag{24}$$

Based on the structure of the second-order Lagrangian equations, the expression of kinetic energy must first be differentiated by the generalized velocities, while the expressions obtained must be differentiated by time and the kinetic energy must be differentiated by the generalized coordinates. All of the above gives us five second-order differential equations, that is, a nonlinear system of the tenth-order differential equations. The obtained system describes both forward and backward motions of a disabled person in a wheelchair:

$(m_0+m_1+m_2+m_3)\ddot{x}\ -(m_1+m_2+m_3)\ddot{x}_1-(m_2L_0+m_3L_2)\ddot{\varphi}_2cos\varphi_2+(m_2L_0+$
$m_3L_2)\dot{\varphi}_2^2sin\varphi_2-m_3h\ddot{\varphi}_3cos\varphi_3+m_3h\dot{\varphi}_3^2sin\varphi_3+(m_2+m_3)\ell\ddot{\varphi}_4sin\varphi_4+(m_2+m_3)\ell\dot{\varphi}_4^2cos\varphi_4=$
$-F$ (25)

$-(m_1+m_2+m_3)(\ddot{x}-\ddot{x}_1)+(m_2L_0+m_3L_2)\ddot{\varphi}_2cos\varphi_2\text{-}(m_2L_0+m_3L_2)\dot{\varphi}_2^2sin\varphi_2-$
$m_3h\ddot{\varphi}_3cos\varphi_3+m_3h\dot{\varphi}_3^2sin\varphi_3-(m_2+m_3)\ell\ddot{\varphi}_4sin\varphi_4-(m_2+m_3)\ell\dot{\varphi}_4^2cos\varphi_4=0$ (26)

$\left(m_2L_0^2+\frac{1}{3}m_2+m_3L_2^2\right)\ddot{\varphi}_2\text{-}(m_2+m_3)\ell L_0\dot{\varphi}_4(\dot{\varphi}_2+\dot{\varphi}_4)cos(\varphi_2+\varphi_4)-(m_2L_0+m_3L_2)(\ddot{x}-$
$\ddot{x}_1)cos\varphi_2+(m_2L_0+m_3L_2)(\dot{x}-\dot{x}_1)\dot{\varphi}_2sin\varphi_2+m_3hL_0\ddot{\varphi}_3\cos(\varphi_2-\varphi_3)\text{-}\ m_3hL_0\dot{\varphi}_3(\dot{\varphi}_2-$
$\dot{\varphi}_3)\sin(\varphi_2-\varphi_3)+m_2\ell L_0\dot{\varphi}_2\dot{\varphi}_4cos(\varphi_2+\varphi_4)-m_2L_0\dot{\varphi}_2(\dot{x}-\dot{x}_1)sin\varphi_2-m_3L_2\dot{\varphi}_2(\dot{x}-\dot{x}_1)sin\varphi_2+$
$m_3hL_2\dot{\varphi}_2\dot{\varphi}_3\sin(\varphi_2-\varphi_3)+m_3\ell L_2\dot{\varphi}_2\dot{\varphi}_4\cos(\varphi_2+\varphi_4)=\text{-c}(x_1+L_1\varphi_2)/L_1\text{-}\ \mu(\dot{x}_1+L_1\dot{\varphi}_2)/L_1\text{-}$
$c_3(\varphi_3-\varphi_2)-\mu_3(\dot{\varphi}_3-\dot{\varphi}_2)$ (27)

$\frac{7}{5}m_3\ddot{\varphi}_3-m_3\text{h}(\ddot{x}-\ddot{x}_1)\cos\varphi_3+m_3\text{h}(\dot{x}-\dot{x}_1)\dot{\varphi}_3sin\varphi_3+m_3hL_2\ddot{\varphi}_2\cos(\varphi_2-\varphi_3)-m_3hL_2\dot{\varphi}_2(\dot{\varphi}_2-$
$\dot{\varphi}_3)\sin(\varphi_2-\varphi_3)\text{-}\ m_3\ell h\ddot{\varphi}_4\sin(\varphi_3+\varphi_4)\text{-}m_3\ell h\dot{\varphi}_4(\dot{\varphi}_3+\dot{\varphi}_4)\cos(\varphi_3+\varphi_4)-m_3\text{h}(\dot{x}-$
$\dot{x}_1)\dot{\varphi}_3sin\varphi_3-m_3hL_2\dot{\varphi}_2\dot{\varphi}_3\sin(\varphi_2-\varphi_3)+m_3\ell h\dot{\varphi}_3\dot{\varphi}_4\cos(\varphi_3+\varphi_4)=\ c_3(\varphi_3-\varphi_2)-$
$\mu_3(\dot{\varphi}_3-\dot{\varphi}_2)$ (28)

$(m_2+m_3)\ell(\ddot{x}-\ddot{x}_1)+(m_2+m_3)\ell(\dot{x}-\dot{x}_1)\dot{\varphi}_4cos\varphi_4+\left(m_2+\frac{1}{3}m_1+m_3\right)\ell^2\ddot{\varphi}_4\text{-}(m_2L_0+$
$m_3L_2)\ddot{\varphi}_2\ell\sin(\varphi_2+\varphi_3)\text{-}\ (m_2L_0+m_3L_2)\ell\dot{\varphi}_2(\dot{\varphi}_2+\dot{\varphi}_3)\cos(\varphi_2+\varphi_3)\text{-}m_3\ell h\ddot{\varphi}_3\sin(\varphi_3+\varphi_4)\text{-}$

$m_3\ell h\dot{\varphi}_3(\dot{\varphi}_3+\dot{\varphi}_4)\cos(\varphi_4+\varphi_3)+m_2\ell L_0\dot{\varphi}_2\dot{\varphi}_4 cos(\varphi_2+\varphi_4)-m_2\ell L_0\dot{\varphi}_4(\dot{x}-$

$$\dot{x}_1)cos\varphi_4+m_3\ell L_2\dot{\varphi}_2\dot{\varphi}_4 cos(\varphi_2+\varphi_4)+m_3\ell h\dot{\varphi}_3\dot{\varphi}_4 cos(\varphi_3+\varphi_4)=\left(\frac{1}{2}m_1+m_2+m_3\right)g\ell sin\varphi_4 \quad (29)$$

Let us linearize the system obtained. To this end, assume that $sin\varphi_2 \cong \varphi_2$, $cos\varphi_2 \cong 1$, $sin\varphi_3 \cong \varphi_3$ and $cos\varphi_3 \cong 1$, and neglect infinitely small values of the second and higher order. In addition, let us replace the angular generalized coordinates φ_2 and φ_3 by the linear generalized coordinates x_2 and x_3 with the following formulas:

$$\varphi_2 = x_2/L_1 \quad \varphi_3=x_3/h \quad (30)$$

where x_2 is the displacement of the passenger's body point B toward the femur and x_3 is the displacement of the center of gravity of the passenger's head toward the body.

Then, we will obtain the following system of differential equations:

$$(m_1+m_2+m_3)\ddot{x}_1+(m_2L_0+m_3L_2)\ddot{x}_2+m_3\ddot{x}_3=F-(m_0+m_1+m_2+m_3)a(t) \quad (31)$$

$$(m_1+m_2+m_3)\ddot{x}-(m_1+m_2+m_3)\ddot{x}_1-\left(m_2+\frac{m_3L_2}{L_1}\right)\ddot{x}_2-m_3\ddot{x}_3=0 \quad (32)$$

$$-\left(m_2+\frac{m_3L_2}{L_0}\right)\ddot{x}_1+(m_2L_0+J_2/L_0+m_3L_2^2/L_0)/L_1\ddot{x}_2-m_3(L_2/L_0)\ddot{x}_3=$$

$$\left(m_2+\frac{m_3L_2}{L_1}\right)a(t)+c_3/(L_1L_0)x_2-c_3/(hL_0)x_3-\mu_3(L_1/L_0)\dot{x}_1-(\mu_2L_1-\mu_3/L_1)/L_0\dot{x}_2-\mu_3/(L_0h)\dot{x}_3 \quad (33)$$

$$m_3\ddot{x}_1+m_3L_2/L_1\ddot{x}_2+(m_3+J_3/h^2)\ddot{x}_3=m_3a(t)-c_3/h^2x_3+c_3/(L_1h)x_2+\mu_3/(L_1h)\dot{x}_2-\mu_3/h^2\dot{x}_3 \quad (34)$$

$$a(t)=\begin{cases} -\frac{a}{t_1}t, & 0<t<t_1, \\ -a, & t_1 \le t \le t_2, \\ -a\left(1-\frac{t-t_2}{t_3-t_2}\right), & t_2<t<t_3 . \end{cases} \quad (35)$$

with the initial data as follows:
t=0, $x_1(0)=0, x_2(0)=0,\ x_3(0)=0, \dot{x}_1(0)=0, \dot{x}_2(0)=0,\ \dot{x}_3(0)=0.$

Numerical integration of the obtained system of differential equations will be carried out using a mathematical package Mathcad. To this end, let us adjust it to to the normal system.

We introduce the designations as follows:

$$A=\begin{pmatrix} m_1+m_2+m_3 & m_2L_0+m_3L_2 & m_3 \\ -\left(m_2+\frac{m_3L_2}{L_0}\right) & (m_2L_0+J_2/L_0+m_3L_2^2/L_0)/L_1 & -m_3(\frac{L_2}{L_0}) \\ m_3 & m_3L_2/L_1 & (m_3+J_3/h^2) \end{pmatrix} \quad (36)$$

$$B=\begin{pmatrix} 0 & 0 & 0 \\ -\mu_3L_1/L_0 & (\mu_2L_1-\mu_3/L_1)/L_0 & \mu_3/L_0h \\ 0 & \mu_3/(L_1h) & -\mu_3/h^2 \end{pmatrix} \quad (37)$$

$$C=\begin{pmatrix} 0 & 0 & 0 \\ 0 & c_3/(L_1L_0) & -c_3/(hL_0) \\ 0 & c_3/(L_1h) & -c_3/h^2 \end{pmatrix} \quad (38)$$

$$Q(t)=\begin{pmatrix} F-(m_0+m_1+m_2+m_3)a(t) \\ \left(m_2+\frac{m_3L_2}{L_1}\right)a(t) \\ m_3a(t) \end{pmatrix} \quad (39)$$

The system of differential equations obtained in these designations can be written as follows:

$$A\ddot{q}=B\dot{q}+Cq+Q(t) \quad (40)$$

where $q \equiv (x_1 \quad x_2 \quad x_3)^T$; consequently, $\dot{q} \equiv \left(\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3\right)^T$, $\ddot{q} \equiv (\ddot{x}_1 \quad \ddot{x}_2 \quad \ddot{x}_3)^T$.

We can rewrite the matrix equation (40) as follows:

$$\ddot{q} = A^{-1}B\dot{q} + A^{-1}Cq + A^{-1}Q(t) \quad (41)$$

We will introduce the third-order unity E and zero O matrices (in a package Mathcad): $E3 = identity(3)$, $O3$=$E3$-$E3$, and by using these, we obtain the block matrices as follows:

$L_1 = augment(O3, E3),\ L_2 = augment(A^{-1}C, A^{-1}B),\ L = stack(L_1, L_2),$

$$W{=}stack(O3,\ A^{-1}Q(t)),\ Z = (q \quad \dot{q}),\ L = \begin{pmatrix} O3 & E3 \\ A^{-1}C & A^{-1}B \end{pmatrix} \quad (42)$$

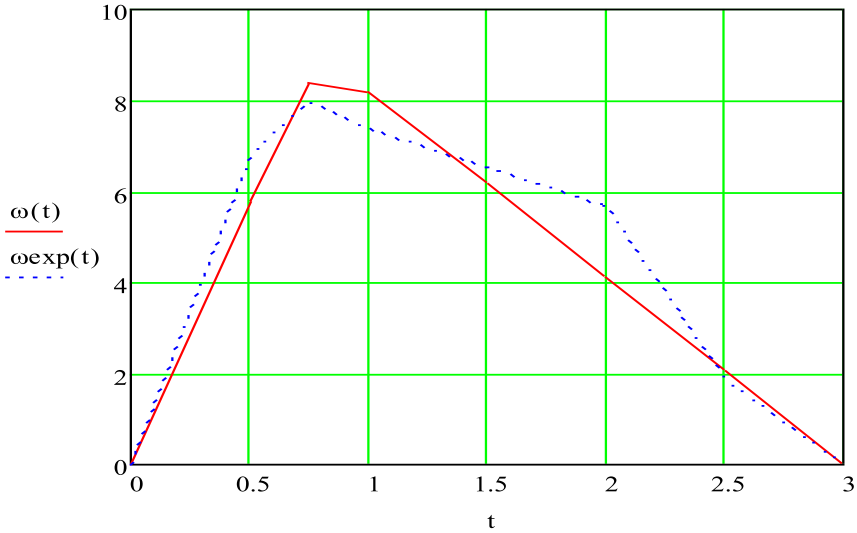
Formula (41) in a canonical form can be written as follows:

$$\dot{Z} = LZ + W \quad (43)$$

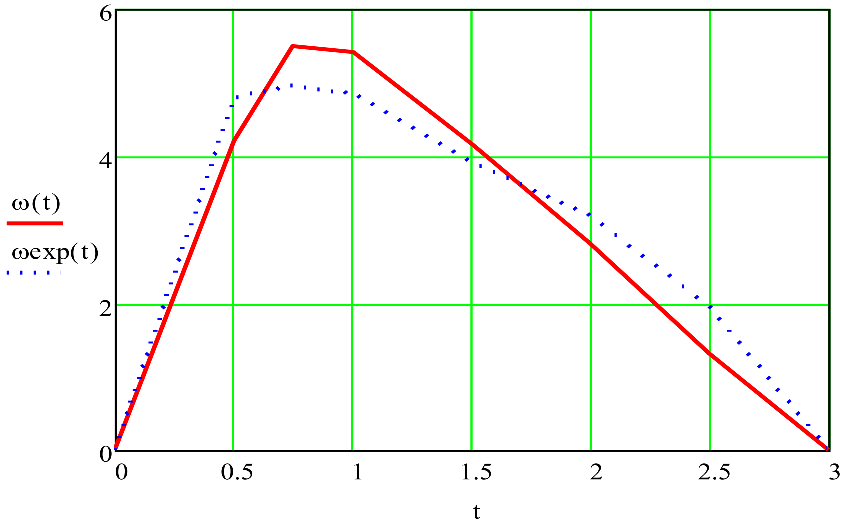
Let us integrate it using the function *rkfixed* built in a package Mathcad.

For more precise results of the theoretical calculation, an experimental study was carried out. The displacement of the center of gravity and angular velocity of the head was determined by various slowdowns of a bus using special-purpose transducers.

The curves reflecting the theoretical and experimental studies of the angular velocity of the head are shown in Fig. 3.



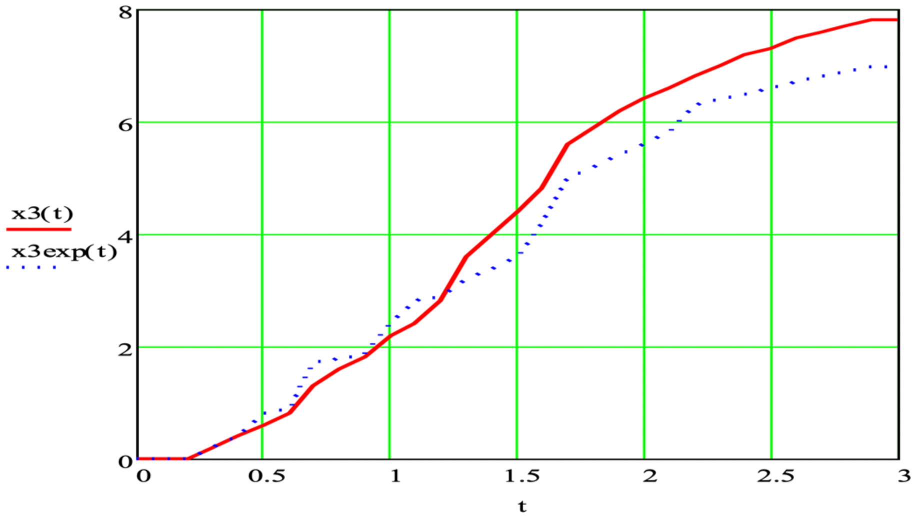
a.



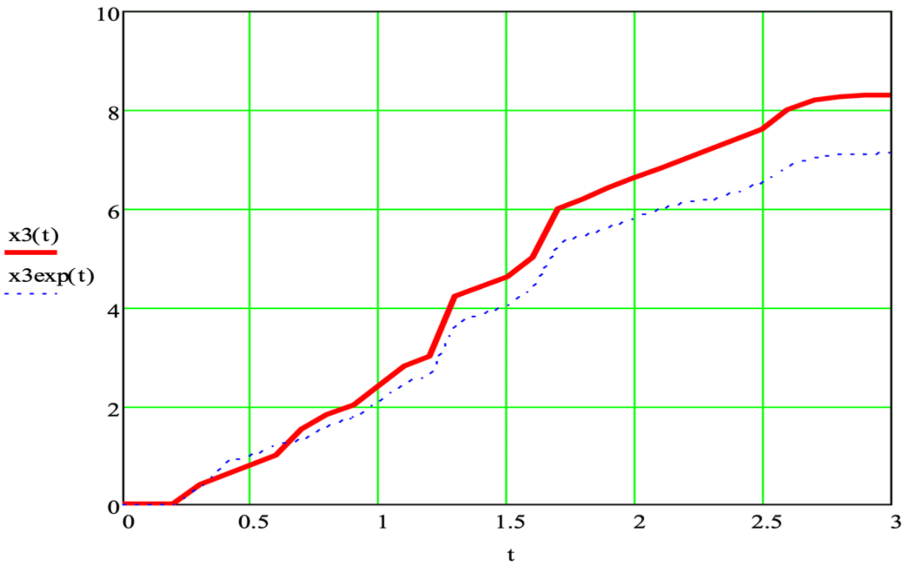
b.

Fig. 3. Comparative curves of the results obtained through mathematical modeling and the experiment of angular velocity of the head: a – bus speeding down in a collision – 150 m/sec^2; b – bus speeding down in a collision – 100 m/sec^2

The head–neck segment (Fig. 3) increases rapidly from the moment of collision for t = 0.075 seconds, then decreases steadily to 0.1 seconds, rapidly decreases from 0.1 seconds to 0.3 seconds and reaches 0. It should be noted that the maximum values of angular velocity depend on the deceleration value. The graph (Fig. 3) shows that from 0 to 0.075 sec, the head goes forward at an angular speed of 8.4 rad/sec, which may cause damage to the neck vertebrae or collision with any element of the bus compartment. As can be seen from the graphs, the difference between the results of theoretical and experimental studies is insignificant and the curves on changing the angular velocity of the head are almost identical.



a.



b.

Fig. 4. Comparative curves of the displacement of the center of gravity of the head through mathematical modeling and experiment: a – bus speeding down in a collision – 150 m/sec^2; b – bus speeding down in a collision – 100 m/sec^2

The curves reflecting the results of theoretical and experimental studies of the displacement of the center of gravity of the head are shown in Figure 4. Similar to the angular velocity of the head, the difference between the results of theoretical and experimental studies is insignificant, and the curves on changing the displacement of the center of gravity of the head are almost identical.

Through theoretical study, the change in the angle between the femur and the wheelchair seat (Fig. 5) has been determined during the frontal collision of a bus. As the figure illustrates, the angle between the femur and the wheelchair is formed from 0.05 seconds and rapidly rises to 0.48 seconds, and then after the first phase of collision, it rapidly drops to 0.

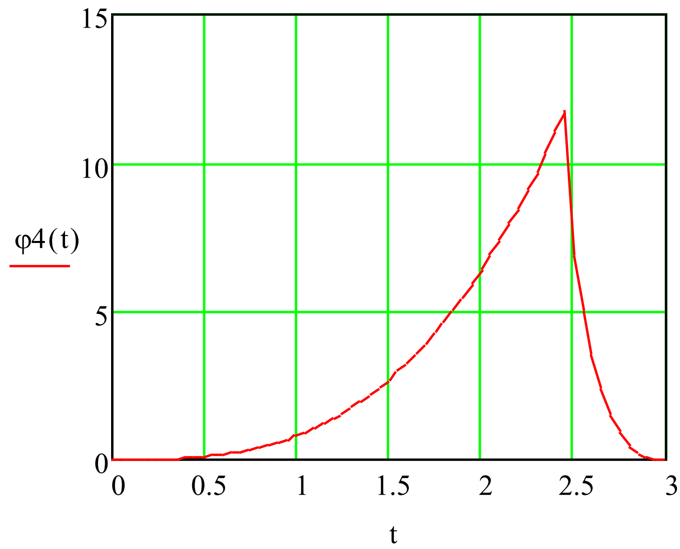


Fig. 5. Curve on changing the angle between the femur and the wheelchair seat

4. CONCLUSIONS

The severity of the injury of a disabled passenger in a wheelchair sitting in a bus in a traffic accident is mostly determined by damage to the neck vertebrae. Damage to the cervical spine depends on the angular velocity of the head and the duration of the period of loading. According to these studies, the long-lasting loading of a relatively small value of angular velocity of the head may cause serious damage to the cervical spine. Therefore, it is necessary to minimize the angular velocity of the head by structural modification of the support plate.

There is no provision in the transport legislation of Georgia on vehicles designed for transportation of wheel chair users with restricted mobility, including the equipment of their seats in the bus compartment. Studies of public buses in Georgia with seats for wheelchair users with restricted mobility revealed that they are not equipped with support plates. However, it should be borne in mind that during traffic accidents and abrupt maneuvering, persons with limited mobility may sustain heavy injuries in the head and neck vertebrae.

The parameters obtained as a result of theoretical study are of practical benefit; in particular, taking into account the parameters obtained, it is possible to produce a support plate with geometrical dimensions, rigidity and energy-absorbing properties that allows for minimizing the angular velocity of the head, and consequently, the severity of injury. The upper part of the support plate should be made of energy-intensive material. In addition, the rigidity of the support plate and the energy-capturing element must ensure that when the human head comes into contact with the support plate, the slowdown for 3 seconds should not exceed 80 g.

The support plate of an appropriate design will significantly reduce the head injury risk and the probability of injury during the head's contact with the energy-capturing surface.

In the future, by generalizing the modified model, it may be possible to carry out fundamental studies, and identify and select the optimal parameters to ensure safe and comfortable transport of persons with disabilities.

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