dr n. tech. Andrzej Antoni CZAJKOWSKI[a,b], dr Grzegorz Paweł SKORNY[b], mgr Jakub ŚLEDZIOWSKI[b]

[a] University of Szczecin, Faculty of Mathematics and Physics, Department of Informatics and Technical Education
Uniwersytet Szczeciński, Wydział Matematyczno-Fizyczny, Katedra Edukacji Informatycznej i Technicznej

[b] Higher School of Technology and Economics in Szczecin, Informatics and Technical Education
Wyższa Szkoła Techniczno-Ekonomiczna w Szczecinie, Edukacja Informatyczno-Techniczna

ANALYTICAL AND NUMERICAL SOLVING OF RIGHT-ANGLED TRIANGLES WITH APPLICATION OF NUMERICAL PROGRAMS MS-EXCEL, MATHCAD AND MATHEMATICA

Abstract

Introduction and aims: The paper shows the analytical models of solving right-angled triangles with appropriate discussion. For right-angled triangles have been discussed four cases. The main aim of this paper is not only to create some analytical algorithms for solving right-angled triangle, but also their implementation in programs *MS-Excel, MathCAD* and *Mathematica.*
Material and methods: Elaboration of four analytical cases of solving right-angled triangles has been made on the basis of the relevant trigonometric properties occurring in a right-angled triangle. In the paper have been used some analytical and numerical methods by using *MS-Excel, MathCAD* and *Mathematica* programs.
Results: As some results have been obtained numerical algorithms in the programs *MS-Excel, MathCAD* and *Mathematica* for four analytical cases of solving right-angled triangles.
Conclusion: Created numerical algorithms of solving the right-angled triangles in the programs *MS-Excel, MathCAD* and *Mathematica* allow for faster significant performance calculations than the traditional way of using logarithms and logarithmic tables.

Keywords: Trigonometry, solving of right-angled triangles, numerical algorithms, *MS-Excel, MathCAD, Mathematica.*

(Received: 01.06.2014; Revised: 15.06.2014; Accepted: 01.07.2014)

ANALITYCZNO-NUMERYCZNE ROZWIĄZYWANIE TRÓJKĄTÓW PROSTOKĄTNYCH Z ZASTOSOWANIEM PROGRAMÓW MS-EXCEL, MATHCAD I MATHEMATICA

Streszczenie

Wstęp i cele: *W pracy pokazano analityczne modele rozwiązywania trójkątów prostokątnych wraz z odpowiednią dyskusją. Dla trójkątów prostokątnych omówiono cztery przypadki. Głównym celem jest pracy jest nie tylko utworzenie algorytmów analitycznych rozwiązywania takich trójkątów lecz również ich implementacja w programach MS-Excel, MathCAD i Mathematica.*
Materiał i metody: *Opracowanie czterech analitycznych przypadków rozwiązywania trójkątów prostokątnych wykonano opierając się odpowiednich własnościach trygonometrycznych występujących w trójkącie prostokątnym. Zastosowano metodę analityczną i numeryczną wykorzystując programy MS-Excel, MathCAD i Mathematica.*
Wyniki: *Otrzymano algorytmy numeryczne w programach MS-Excel, MathCAD i Mathematica dla czterech analitycznych przypadków rozwiązywania trójkątów prostokątnych.*
Wniosek: *Utworzone algorytmy numeryczne rozwiązywania trójkątów prostokątnych w programach MS-Excel, MathCAD oraz Mathematica, pozwalają na znaczne szybsze wykonanie obliczeń niż drogą tradycyjną z użyciem logarytmów i tablic logarytmicznych.*

Słowa kluczowe: *Trygonometria, rozwiązywanie trójkątów prostokątnych, algorytmy numeryczne, MS-Excel, MathCAD, Mathematica.*

(Otrzymano: 01.06.2014; Zrecenzowano: 15.06.2014; Zaakceptowano: 01.07.2014)

1. Introduction to trigonometry

1.1. Division of triangles and basic relations

Before we begin the analysis of solving right triangles, let us look at the division triangles (Fig. 1) and the basic conditions of existence of the triangle (Theorem 1).

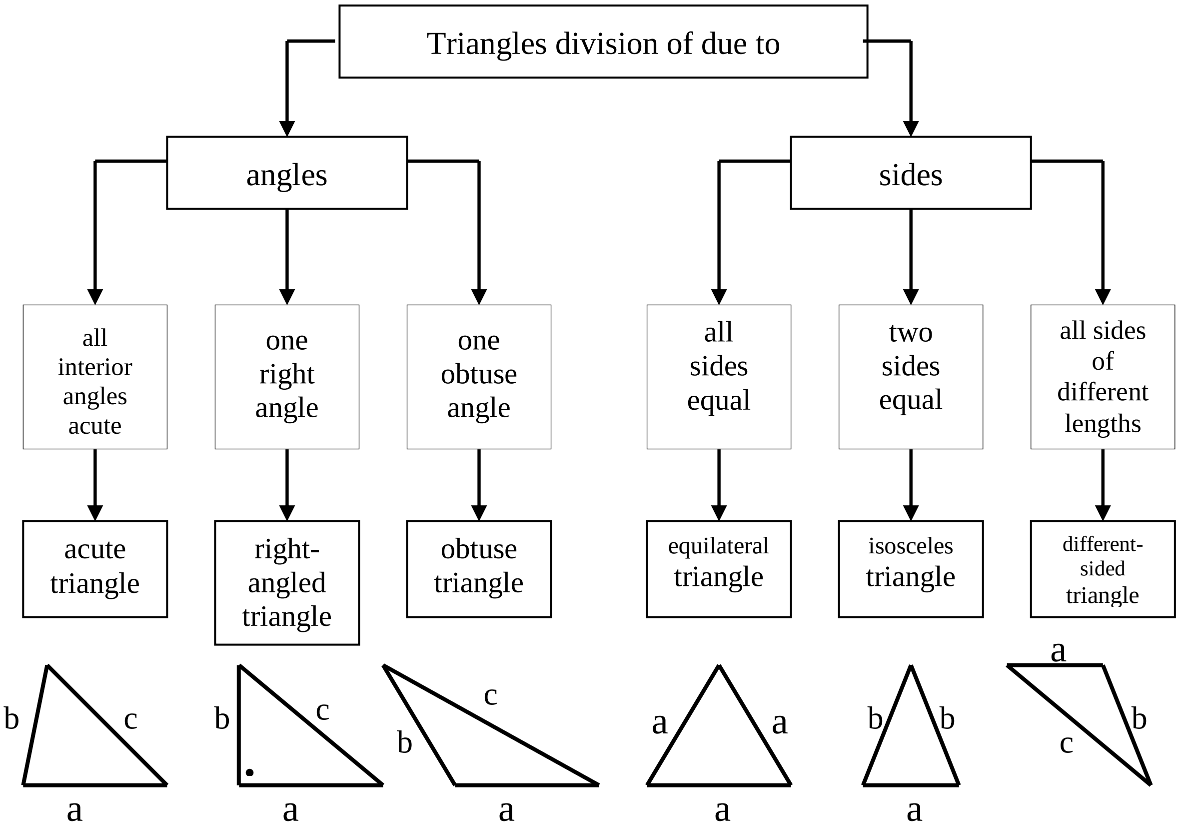

Fig. 1. Types of triangles
Source: Elaboration of the Authors basing on [2]&[7]

Theorem 1. *(Basic relationships in the triangle)* [5], [12]

In any length of each side of the triangle is less than the sum of the length of the other side and higher than the absolute value of the difference in length of the sides

This fact is written by the following conditions:

$$|b-c| < a < b+c\,, \tag{1}$$

$$|a-c| < b < a+c\,, \tag{2}$$

$$|a-b| < c < a+b\,. \tag{3}$$

1.2. The compounds in a right-angled triangle

The basic elements of the right-angled triangle are two catheti a, b (major and minor), and the hypotenuse c of the triangle and sharp angles α and β triangle (the third angle is a right angle). The solution involves the determination of the triangle 3 basic elements from 5, if the two remaining are given. To solve right-angled triangles are enough patterns arising directly from the definition of trigonometric functions.

1.2.1. Definitions of trigonometric functions

Let be given the right-angled triangle ABC (angle ACB is the right angle) (Fig. 2).

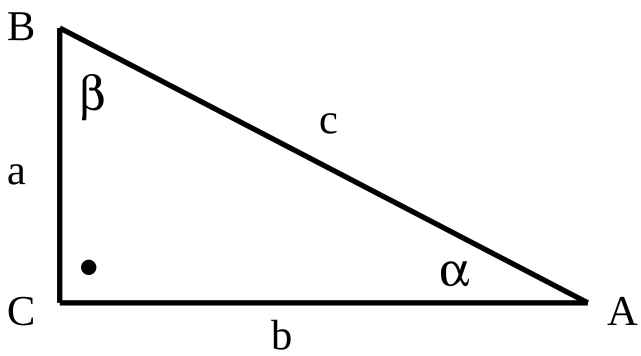

Fig. 2. The right-angled triangle with catheti a and b and hypotenuse c
Source: Elaboration of the Authors

where a, b, c mean respectively the length of shorter cathetus, longer cathetus and hypotenuse. The sides a, b and c satisfy the condition:

$$a^2 + b^2 = c^2 \tag{4}$$

and acute angles α and β satisfying the following condition:

$$\alpha + \beta = 90^\circ. \tag{5}$$

Definition 1.

The sine of acute angle is the ratio of the length of cathetus opposite this angle to the length of hypotenuse, which express the formula [4], [8], [10], [15]-[18]:

$$\sin(\alpha) = \frac{a}{c} \tag{6}$$

or

$$\sin(\beta) = \frac{b}{c}. \tag{7}$$

Definition 2.

The cosine of acute angle is the ratio of the length of cathetus adjacent this angle to the length of hypotenuse, which express the formula [4], [8], [10], [15]-[18]:

$$\cos(\alpha) = \frac{b}{c} \tag{8}$$

or

$$\cos(\beta) = \frac{a}{c}. \tag{9}$$

Definition 3.

The tangent of acute angle is the ratio of the length of cathetus opposite this angle to the length of cathetus adjacent to that angle, which express the formula [4], [8], [10], [15]-[18]:

$$tg(\alpha) = \frac{a}{b} \tag{10}$$

or

$$tg(\beta) = \frac{b}{a}. \tag{11}$$

Definition 4.

The cotangent of acute angle is the ratio of the length of cathetus adjacent this angle to the length of cathetus opposite to that angle, which express the formula [4], [8], [10], [15]-[18]:

$$ctg(\alpha) = \frac{b}{a} \tag{12}$$

or

$$ctg(\beta) = \frac{a}{b}. \tag{13}$$

1.2.2. Basic compounds in a right-angled triangle

Theorem 2.

Between the elements of a right-angled triangle, there are following dependences [15]-[18]:

$$a = c \cdot \sin(\alpha)\ , \tag{14}$$

$$b = c \cdot \sin(\beta)\ , \tag{15}$$

$$b = c \cdot \cos(\alpha)\ , \tag{16}$$

$$a = c \cdot \cos(\beta)\ , \tag{17}$$

$$a = b \cdot tg(\alpha)\ , \tag{18}$$

$$b = a \cdot tg(\beta)\ , \tag{19}$$

$$b = a \cdot ctg(\alpha)\ , \tag{20}$$

$$a = b \cdot ctg(\beta)\ . \tag{21}$$

Theorem 3.

If the elements of a right-angled triangle satisfy one of the relations (14) - (21), then this is a rectangular triangle [13], [16].

Corollary 1.

The existence in a right-angled triangle one of the relations (4) - (5) and (14) - (21) implies all other [13], [16].

1.2.3. Cases of solving right-angled triangles

For the a right-angled triangle ABC shown in figure 2, there are 4 cases of solving which are given below (Tab. 1).

Tab. 1. The instances of solving right triangles [2], [7], [15]-[17]

Case:	*Data:*	*Unknown:*
I	a, α side, angle *Cathetus shorter and the angle opposed her*	b, c, β side, side, angle *Cathetus longer and the hypotenuse and the second acute angle*
II	a, β side, angle *Cathetus shorter and the angle adjacent to it*	b, c, α side, side, angle *Cathetus longer and the hypotenuse and the second acute angle*
III	c, α side, angle *Hypotenuse and the angle*	a, b, β side, side, angle *Catheti and a second acute angle*
IV	a, b side, angle *Two catheti*	c, α, β side, angle, angle *Hypotenuse and the two acute angles*

1.2.4. Discussion of triangles solutions

In the right triangle among of 6 characterized triangle quantities (3 angles α, β, γ and three sides a, b, c) a measure of one of the angles is 90°. From the definition implies that the right triangle can be characterized by only two parameters. The other three can be determined according to the following table 2 and formula (5).

Tab. 2. The discussion of solving right triangles [2], [7], [15]-[17]

Case:	*Data:*	*Solution:*		
I	a, α	$\beta = 90° - \alpha$	$b = a \cdot ctg(\alpha)$	$c = \frac{a}{\sin(\alpha)}$
II	a, β	$\alpha = 90° - \beta$	$b = a \cdot tg(\beta)$	$c = \frac{b}{\sin(\beta)}$
III	c, α	$\beta = 90° - \alpha$	$a = c \cdot \sin(\alpha)$	$b = c \cdot \cos(\alpha)$
IV	a, b	$tg(\alpha) = \frac{a}{b}$ $\alpha = arctg\left(\frac{a}{b}\right)$	♦ $c = \frac{a}{\sin(\alpha)}$ or ♦ $c = \sqrt{a^2 + b^2}$	$\beta = 90° - \alpha$

2. Solving of right triangles

2.1. Shorter cathetus and an angle lying opposite that cathetus *(Case 1)*

➢ *Problem:*
Solve a right triangle with given a shorter cathetus and an angle lying opposite her.

2.1.1. Theoretical analysis

➢ Let be a triangle as shown in figure 3.

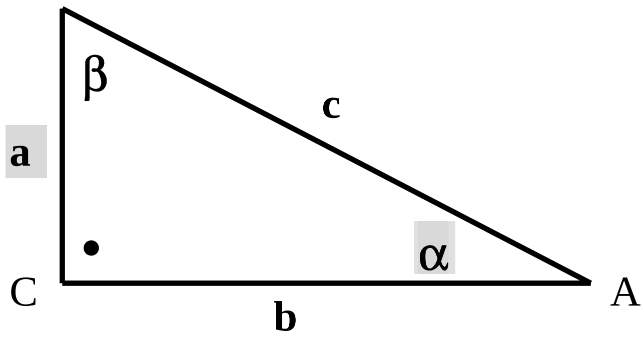

Fig. 3. The right triangle with given the cathetus a and acute angle α *(i.e. angle opposite her)*
Source: Elaboration of the Authors

➢ *Data in ΔABC:* The length of the shorter cathetus **a** and the measure of angle **α**.

➢ *Unknown in ΔABC:* A length of sides **b** and **c**, and a measure of angle **β**.

➢ *Solution:*

♦ From the properties of acute angles in a right triangle we get:

$$\beta = 90° - \alpha . \tag{22}$$

♦ From the definition of the function cotangens in a right triangle, we have:

$$b = a \cdot \mathrm{ctg}(\alpha) . \tag{23}$$

♦ From the definition of the sine function in a right triangle we get:

$$c = \frac{a}{\sin(\alpha)} . \tag{24}$$

➢ *Answer:*

The formulae for some sizes which must be calculated are defined as follows:

$$\beta = 90° - \alpha , \quad b = a \cdot \mathrm{ctg}(\alpha) , \quad c = \frac{a}{\sin(\alpha)} .$$

2.1.2. Numerical algorithms in *MS-Excel, MathCAD* and *Mathematica* programs

For numerical analysis, we assume that in the right triangle some data are: the length of shorter cathetus a = 13 and measure of the angle α = 40°. Should be determined some length of the sides b and c and measure of the angle β.

♦ Program *MS-Excel 7.0* [3], [9]

Algorithm:	*Commentary:*
B3=**13**	*Given a length of the side a*
B4=**40**	*Given a measure of the angle α [°]*
B5=B4*PI()/180	*Command for the angle α [rad]* $\alpha = 0{,}698$
B6=90-B4	*Command for the angle β [°]* $\beta = 50^{o}$
B7=B6*PI()/180	*Command for the angle β [rad]* $\beta = 0{,}872$
B8=B3*COS(B5)/SIN(B5)	*Calculation of the side b*
B9=B3*SIN(B5)	*Calculation of the side c*
50	*Result: a measure of the angle β [°]*
15,492	*Result: a length of the side b*
20,224	*Result: a length of the side c*

♦ Program *MathCAD 15* [11], [14]

Algorithm:	*Commentary:*
a:=**13**	*Given a length of the side a*
α0:=**40**	*Given a measure of the angle α [°]*
$\alpha := \frac{\alpha 0 \cdot \pi}{180} = 0.698$	*Command for the angle α [rad]* $\alpha = 0{,}698$
$\beta 0 := 90 - \alpha 0 = 50$	*Command for the angle β0 [°],* $\beta = 50^{o}$
$\beta := \frac{\beta 0 \cdot \pi}{180} = 0.872$	*Command for the angle β [rad]* $\beta = 0{,}872$
$b := \frac{a}{\tan(\alpha)} = 15.492$	*Calculation of the side b*
$c := \frac{a}{\sin(\alpha)} = 20.224$	*Calculation of the side c*
50	*Result: the measure of the angle β [°]*
15,492	*Result: the length of the side b*
20,224	*Result: the length of the side c*

♦ Program *Mathematica 7.0* [1], [6]

Algorithm:	*Commentary:*
a:=**13**	*Given a length of the side a*
A0:=**40**	*Given a measure of the angle α [°]*
A=N[A0*Pi/180];	*Command for the angle α [rad]* $\alpha = 0{,}698$
B0:=90-A0;	*Command for the angle β [°]* $\beta = 50^{o}$
B=N[B0*Pi/180];	*Command for the angle β [rad]* $\beta = 0{,}872$
b=a*Cot[A]	*Calculation of the side b*
c=a/Sin[A]	*Calculation of the side c*
50	*Result: the measure of the angle β [°]*
15,492	*Result: the length of the side b*
20,224	*Result: the length of the side c*

Numerical results: The length of the side b = 15,492 and c = 20,224.
The measure of the angle β = 50°.

2.2. Cathetus shorter and the angle attached home to it *(Case 2)*

➢ *Problem:*
Solve a right triangle with given a shorter cathetus and an angle attached home to it.

2.2.1. Theoretical analysis

➢ Let be a triangle as shown in figure 4.

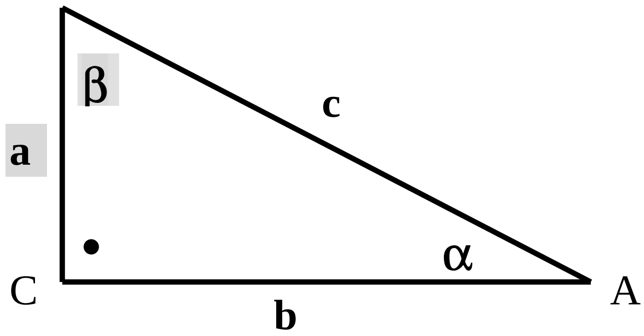

Fig. 4. The right triangle with given shorter cathetus a and acute angle β *(i.e. attached home to it)*
Source: Elaboration of the Authors

➢ *Data in ΔABC:* The length of the shorter cathetus **a** and the measure of angle **β**.

➢ *Unknown in ΔABC:* A length of sides **b** and **c**, and a measure of angle **α**.

➢ *Solution:*

♦ From the properties of acute angles in a right triangle we get:

$$\alpha = 90^\circ - \beta . \tag{25}$$

♦ From the definition of the function tangent in a right triangle, we have:

$$b = a \cdot tg(\beta) . \tag{26}$$

♦ From the definition of the cosine function in a right triangle we get:

$$c = \frac{b}{\sin(\beta)} . \tag{27}$$

➢ *Answer:*

The formulae for some sizes which must be calculated are defined as follows:

$$\alpha = 90^\circ - \beta , \quad b = a \cdot tg(\beta) , \quad c = \frac{a}{\cos(\beta)} .$$

2.2.2. Numerical algorithms in *MS-Excel, MathCAD* and *Mathematica* programs

For numerical analysis, we assume that in the right triangle some data are: the length of shorter cathetus a = 13 and measure of the angle β = 50°. Should be determined some length of the sides b and c and measure of the angle α.

♦ Program *MS-Excel 7.0* [3], [9]

Algorithm:	*Commentary:*
`B3=`**`13`**	*Given a length of the side* a
`B4=`**`50`**	*Given a measure of the angle* β [°]
`B5=B4*PI()/180`	*Command for the angle* β [rad], β = 0,872
`B6=90-B4`	*Command for the angle* α [°] α = 40°
`B7=B6*PI()/180`	*Command for the angle* α [rad] α = 0,698
`B8=B3*SIN(B5)/COS(B5)`	*Calculation of the side* b
`B9=B3*SIN(B5)`	*Calculation of the side* c
40	*Result: the measure of the angle* α [°]
15,492	*Result: the length of the side* b
20,224	*Result: the length of the side* c

♦ Program *MathCAD 15* [11], [14]

Algorithm:	*Commentary:*
a:=**13**	*Given a length of the side* a
β0:=**50**	*Given a measure of the angle* β [°]
$\beta := \frac{\beta 0 \cdot \pi}{180} = 0.872$	*Command for the angle* β [rad] β = 0,872[rad]
$\alpha 0 := 90 - \beta 0 = 40$	*Command for the angle* α0 [°], α = 40°
$\alpha := \frac{\alpha 0 \cdot \pi}{180} = 0.698$	*Command for the angle* α [rad] α = 0,698[rad]
$b := a \cdot \tan(\beta) = 15.492$	*Calculation of the side* b
$c := \frac{b}{\sin(\beta)} = 20.224$	*Calculation of the side* c
50	*Result: the measure of the angle* β [°]
15,492	*Result: the length of the side* b
20,224	*Result: the length of the side* c

♦ Program *Mathematica 7.0* [1], [6]

Algorithm:	*Commentary:*
`a:=`**`13`**	*Given a length of the side* a
`B0:=`**`50`**	*Given a measure of the angle* β [°]
`B=N[B0*Pi/180];`	*Command for the angle* β [rad], β = 0,872
`A0:=90-A0;`	*Command for the angle* α [°] α = 40°
`A=N[A0*Pi/180];`	*Command for the angle* α [rad] α = 0,698
`b=a*Tan[B]`	*Calculation of the side* b
`c=b/Sin[B]`	*Calculation of the side* c
40	*Result: the measure of the angle* α [°]
15,492	*Result: the length of the side* b
20,224	*Result: the length of the side* c

Numerical results: The length of the side b =15,492 and boku c = 20,224.
The measure of the angle α = 40°.

2.3. Hypotenuse and one of the acute angles *(Case 3)*

➢ *Problem:*
Solve a right triangle with given hypotenuse and one of the acute angles.

2.3.1. Theoretical analysis

➢ Let be a triangle as shown in figure 5.

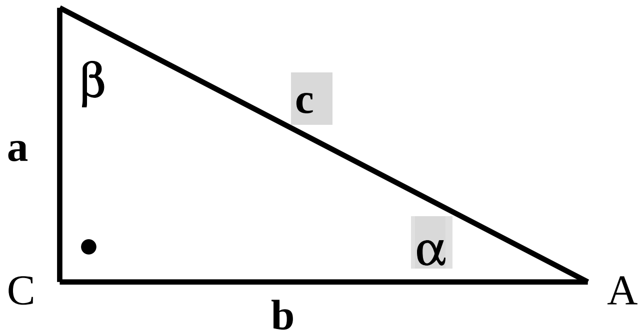

Fig. 5. The right triangle with given hypotenuse c and acute angle α
Source: Elaboration of the Authors

➢ *Data in ΔABC:* The length of the hypotenuse **c** and the measure of angle **α.**

➢ *Unknown in ΔABC:* A length of catheti **a** and **b**, and a measure of angle **β**.

➢ *Solution:*

♦ From the properties of acute angles in a right triangle we obtain:

$$\beta = 90° - \alpha \,. \tag{28}$$

♦ From the definition of the function sine in a right triangle, we have:

$$a = c \cdot \sin(\alpha) \,. \tag{29}$$

♦ From the definition of the function sine in a right triangle, we get:

$$b = c \cdot \cos(\alpha) \,. \tag{30}$$

➢ *Answer:*

The formulae for some sizes which must be calculated are defined as follows:
$\beta = 90° - \alpha$, $a = c \cdot \sin(\alpha)$, $b = c \cdot \cos(\alpha)$.

2.3.2. Numerical algorithms in *MS-Excel, MathCAD* and *Mathematica* programs

For numerical analysis, we assume that in the right triangle some data are: the length of hypotenuse c = 20,224 and measure of the angle α = 40°. Should be determined some length of the sides a and b and measure of the angle β.

♦ Program *MS-Excel 7.0* [3], [9]

Algorithm:	*Commentary:*
`B3=`**`20,224`**	*Given a length of the side* c
`B4=`**`40`**	*Given a measure of the angle* α [°]
`B5=B4*PI()/180`	*Command for the angle* α [rad], α = 0,698
`B6=90-B4`	*Command for the angle* β [°] β = 50°
`B7=B6*PI()/180`	*Command for the angle* β [rad] β = 0,872
`B8=B3*SIN(B5)`	*Calculation of the side* a
`B9=B3*COS(B5)`	*Calculation of the side* b
`50`	*Result: the measure of the angle* β [°]
`13`	*Result: the length of the side* a
`15,492`	*Result: the length of the side* b

♦ Program *MathCAD 15* [11], [14]

Algorithm:	*Commentary:*
c:=**`20.224`**	*Given a length of the side* c
α0:=**`40`**	*Given a measure of the angle* α [°]
$\alpha := \frac{\alpha 0 \cdot \pi}{180} = 0.689$	*Command for the angle* α [rad], α = 0,698
$\beta 0 := 90 - \alpha 0 = 40$	*Command for the angle* β [°] β = 50°
$\beta := \frac{\beta 0 \cdot \pi}{180} = 0.872$	*Command for the angle* β [rad] β = 0,872
$a := c \cdot \sin(\alpha) = 13$	*Calculation of the side* a
$a := c \cdot \cos(\alpha) = 15.492$	*Calculation of the side* b
`50`	*Result: the measure of the angle* β [°]
`13`	*Result: the length of the side* a
`15,492`	*Result: the length of the side* b

♦ Program *Mathematica 7.0* [1], [6]

Algorithm:	*Commentary:*
`c:=`**`20.2244`**	*Given a length of the side* c
`A0:=`**`40`**	*Given a measure of the angle* α [°]
`A=N[A0*Pi/180];`	*Command for the angle* α [rad], α = 0,698
`B0:=90-A0;`	*Command for the angle* β [°] β = 50°
`B=N[B0*Pi/180];`	*Command for the angle* β [rad] β = 0,872
`a=c*Sin[A]`	*Calculation of the side* a
`b=c*Cos[A]`	*Calculation of the side* b
`50`	*Result: the measure of the angle* β [°]
`13`	*Result: the length of the side* a
`15,492`	*Result: the length of the side* b

Numerical results: The length of the side a =13 and b = 15,492.
The measure of the angle β = 50°.

2.4. Two catheti *(Case 4)*

- *Problem:*

  Solve a right triangle with two catheti.

2.4.1. Theoretical analysis

- Let be a triangle as shown in figure 6.

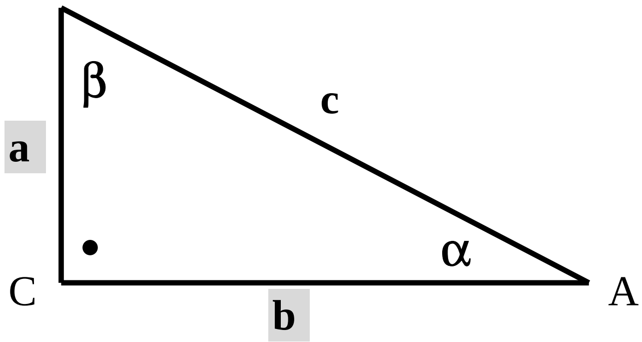

Fig. 6. The right triangle with given catheti a and b
Source: Elaboration of the Authors

- *Data in ΔABC:* The length of catheti **a** and **b.**
- *Unknown in ΔABC:* A length of hypotenuse **c** and a measure of the angles **α** and **β**.
- *Solution:*

♦ From the definition of the tangent (and reverse arctangent) in a right triangle, we have:

$$\alpha = \operatorname{arctg}\left(\frac{a}{b}\right). \tag{31}$$

♦ From the properties of acute angles in a right triangle we obtain:

$$\beta = 90^\circ - \alpha . \tag{32}$$

♦ From the theorem of Pythagoras we determine:

$$c = \sqrt{a^2 + b^2} . \tag{33}$$

- *Answer:*

  The formulae for some sizes which must be calculated are defined as follows:

$$\alpha = \operatorname{arctg}\left(\frac{a}{b}\right), \quad \beta = 90^\circ - \operatorname{arctg}\left(\frac{a}{b}\right), \quad c = \sqrt{a^2 + b^2}.$$

2.4.2. Numerical algorithms in *MS-Excel, MathCAD* and *Mathematica* programs

For numerical analysis, we assume that in the right triangle some data are: the length of catheti a = 13 and b = 15,492. Should be determined some length of hypotenuse c and measure of the angles α and β.

♦ Program *MS-Excel 7.0* [3], [9]

Algorithm:	*Commentary:*
`B3=`**`13`**	*Given a length of the side* a
`B4=`**`15,492`**	*Given a length of the side* b
`B4=ATAN(B3/B4)`	*Command for the angle* α [rad], α = 0,698
`B5=B4*PI()/180`	*Command for the angle* α [°] α = 40
`B6=90-B5`	*Command for the angle* β [°] β = 50°
`B7=B6*PI()/180`	*Command for the angle* β [rad] β = 0,872
`B8=B3*SIN(B5)`	*Calculation of the side* c
40	*Result: the measure of the angle* α [°]
50	*Result: the measure of the angle* β [°]
20,224	*Result: the length of the side* c

♦ Program *MathCAD 15* [11], [14]

Algorithm:	*Commentary:*
a:=**13**	*Given a length of the side* a
b:= **15,492**	*Given a length of the side* b
$\alpha := \frac{\alpha 0 \cdot \pi}{180} = 0.689$	*Command for the angle* α [rad], α = 0,698
$\beta 0 := 90 - \alpha 0 = 40$	*Command for the angle* α [°] α = 40
$\beta := \frac{\beta 0 \cdot \pi}{180} = 0.872$	*Command for the angle* β [°] β = 50°
$a := c \cdot \sin(\alpha) = 13$	*Command for the angle* β [rad] β = 0,872
	Calculation of the side c
40	*Result: the measure of the angle* α [°]
50	*Result: the measure of the angle* β [°]
20,224	*Result: the length of the side* c

♦ Program *Mathematica 7.0* [1], [6]

Algorithm:	*Commentary:*
a:=**13**	*Given a length of the side* a
b:= **15,492**	*Given a length of the side* b
`A0=ArcTan[a/b];`	*Command for the angle* α [rad], α = 0,698
`A=A0*180/Pi`	*Command for the angle* α [°] α ≅ 40°
`B0=90-A`	*Command for the angle* β [°] β ≅ 50°
`B=B0*Pi/180;`	*Command for the angle* β [rad] β = 0,872
`c=Sqrt[a^2+b^2]`	*Calculation of the side* c
40	*Result: the measure of the angle* α [°]
50	*Result: the measure of the angle* β [°]
20,224	*Result: the length of the side* c

Numerical results: The length of the side c = 20,224.
The measure of the angle α = 40° and β = 50°.

3. Conclusions

- Discussed analytical models of solving right triangles allow to perform the other theoretical considerations in each of the four basic cases, taking into account the definition of trigonometric functions and the corresponding properties and patterns.
- Created numerical algorithms of solving the right triangles in *MS-Excel, MathCAD,* and *Mathematica* programs allow for significant execution of faster calculations than the traditional way of using logarithms and logarithmic tables.
- Problem of solving the right triangles finds wide applications in technical sciences and related fields.

Literature

[1] Abel M.L., Braselton J.P.: *Mathematica by example, Revised edition.* Georgia Southern University, Department of Mathematics and Computer Science, Statesboro, Georgia, AP Professional A Division of Harcourt Brace & Company, Boston San Diego New York London Sydney Tokyo Toronto 1993.

[2] Borowska M., Jatczak A.: *Mathematics. Vademecum Matura 2009.* OPERON Publishing House, Gdynia 2006 *(in Polish).*

[3] Bourg D.M.: *Excel in science and technology. Recipes.* HELION Pub. House, Gliwice 2006 *(in Polish).*

[4] Bronsztejn I.N., Siemiendiajew K.A., Musiol G., Mühlig H.: *Modern compendium of mathematics.* Polish Scientific Publishers, Warsaw 2004 *(in Polish).*

[5] Dolciani M.P., Berman S.L., Wooton W., Meder A.E.: *Modern algebra and trigonometry. Structure and Method. Book Two.* Houghton Mifflin Company, Boston New York Atlanta Geneva, Ill. Dallas Palo Alto 1965.

[6] Drwal G., Grzymkowski R., Kapusta A., Słota D.: *Mathematica 4,* Jacek Skalmierski Publishing House, Gliwice 2000 *(in Polish).*

[7] Dziubiński I., Świątkowski T. (Editors): *Mathematical guide.* Polish Scientific Publishers, Warsaw 1978 *(in Polish).*

[8] Gabszewicz Z.: *Trigonometry. Handbook for trainees in the field of secondary schools course. Set of tasks.* Gebethner & Wolff Publishing House, Cracov, Gebethner & Company, Warsaw 1907 *(in Polish).*

[9] Gonet M.: *Excel in scientific computing and engineering.* HELION Publishing House, Gliwice 2010 *(in Polish).*

[10] Gutkowski T.: *Trigonometry with numerous exercises.* Licence of The University of Paris, M. ARCTA Publishing House, Warsaw, 1917 *(in Polish).*

[11] Jakubowski K.: *MathCAD 2000 Professional,* EXIT Publishing House, Warsaw 2000.

[12] Neill Hugh: *Trigonometry. A complete introduction.* Teach Yourself, 2013.

[13] Nowosiłow S.I.: *Special lecture of trigonometry.* Polish Scientific Publishers, Warsaw 1956 *(in Polish).*

[14] Paleczek W.: *MathCAD 12, 11, 2001i, 2001, 2000 in algorithms,* EXIT Publishing House, Warsaw 2005 *(in Polish).*

[15] Pokorny E.J.: *Trigonometry for the self-taught.* Polish Betting School Publishing (PZWS), Warsaw 1962 *(in Polish).*

[16] Wojtowicz Wł.: *Trigonometry. The 5th edition.* Polish Betting School Publishing (PZWS), Warsaw1948, *(in Polish).*

[17] Wojtowicz Wł., Bielecki B., Czyżykowski M.: *Trigonometry for classes X-XI. The 16th edition.* Polish Betting School Publishing (PZWS), Warsaw 1964, *(in Polish).*

[18] Young J.W., Morgan F.M.*: Plane trigonometry and numerical computation,* The MacMillan Company, New York 1919.