Suppression of impulse noise in Track-Before-Detect Algorithms

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Influence of the impulse noise in recurrent Track-Before-Detect Algorithms is considered in this paper. Impulse noise from the object should improve tracking performance but it is not true. This is the SNR paradox that could be explained using Markov matrix theorem. Suppression of the signal value using threshold techniques improves output SNR. Description of this effect is detailed shown in the paper using illustrative examples. Obtained results could be applied for numerous applications of TBD systems.

1. Introduction

Object tracking techniques are used in numerous applications and there are a lot of tracking algorithms currently available [4]. Tracking is the object state estimation using disturbed measurements. The state space could be defined in many domains, but the most typical is the position. Obtained trajectory could be used as output and also as a basis of calculation of next steps. Tracking algorithm uses prediction from previous measurements for estimation of the area where the object could be available. The new measurements descript position of the object. Difference between predicted and measured position is small if the correct motion model is assumed. A small maneuver may occurs because the object trajectory is driven by the numerous factors (intentional or noise) and the difference (an estimation error) is used for update of the predictor state.

There are many tracking algorithms investigated in literature and verified in the practice. The most important are the Benedict-Boerdner [4, 6], the Kalman [4, 6, 9], and the Bayes [4, 14] filters. High quality of the detection process is assumed in the conventional tracking systems. Raw measurements are processed using different signal processing algorithms but binary output is expected. The output value is 1 if the object is detected and corresponding position in the measurement space is available. The output value is 0 otherwise. Such formulation loose signal data what is a source of lower quality of such systems in comparison to the alternative approach where detection is not in the first processing stage. System based on TBD (Track-Before-Detect) approach accumulates signal value so much higher tracking quality could be obtained. The SNR (Signal-to-Noise Ratio) for the first approach must be high (SNR>>1). Signals hidden in the noise floor could be tracked by multiple measurements using the TBD approach [4].

Possibilities of the TBD algorithms are used in the stealth object tracking, especially in the air, space, marine and underwater surveillance applications but non-military applications are available also [4, 5]. The most significant difference between both approaches is depicted in the Fig.1. Conventional approach uses detection first so only selected measurements are used for tracking (some of them could be a noise or true objects position). All possible trajectories are processed by TBD algorithms and signal values over every trajectory separately are accumulated. TBD algorithms process giant amount of trajectories independently on the real number of objects (multitarget tracking). Conventional tracking algorithms support only single object tracking but multitarget scenarios are supported if an additional assignment algorithm is used [3, 4]. Both systems should be selected carefully to the particular applications.

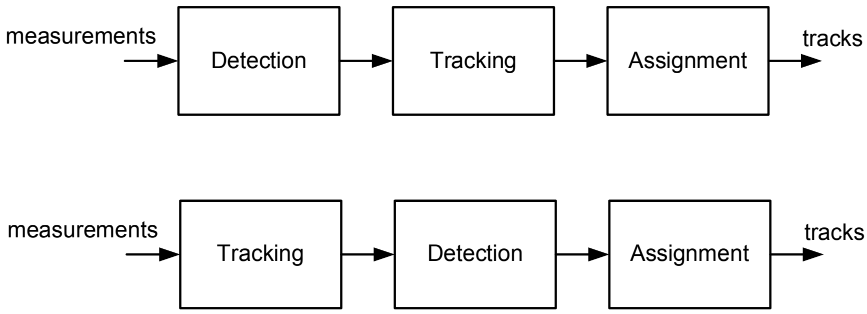

Fig. 1. Conventional and TBD processing schemes

2. Track-Before-Detect Algorithms

There are numerous types of TBD algorithms: recurrent and non-recurrent. Both of them are very important and they behave similarly to the linear IIR and FIR filters respectively. The recurrent algorithms are simpler in implementation with the lower computation and memory costs what is very important for real-time processing. State-space should be carefully designed to the recurrent algorithms for specific set of the possible trajectories. Much more convenient for the trajectory set selection are non-recurrent algorithms. Set of the trajectories define the motion model of the object.

TBD algorithm needs a lot of computations but TBD variants with a limited state-space search are available also. Such algorithms like Particle Filters [7, 13] are used for low reliability applications and they are not optimal. Typical recurrent TBD algorithm like Spatio-Temporal TBD (or Spatial-Temporal TBD) [1, 2, 10-12] uses only a previously computed state space and update it using a new observations what is very useful for the memory limited processing devices. The Spatio-Temporal TBD has following pseudoalgorithm:

Start

$$P(k=0,s)=0 \qquad \text{//initialization} \tag{1a}$$

For $k \geq 1$

$$P^{-}(k,s)=\int_{S} q_k(s \mid s_{k-1})P(k-1,s_{k-1})ds_{k-1} \qquad \text{//motion update:} \tag{1b}$$

$$P(k,s)=\alpha P^{-}(k,s)+(1-\alpha)X(k,s) \qquad \text{//information update} \tag{1c}$$

EndFor

Stop

where: k – iteration number, s – particular space, X – input data, $q_k(s \mid s_{k-1})$ – state transition (Markov matrix), P^{-} – predicted TBD output, P – TBD output, α – weight - smoothing coefficient. $\alpha \in (0,1)$.

The smoothing coefficient is responsible for the balance between dispersion (motion update) and new observations. High values (e.g. 0.9 and more) are used in TBD applications because only a high value gives possibilities of low SNR signal tracking.

Simplified Likelihood Ratio TBD [14] has similar code structure so most results could be extended to this algorithm also. Recurrent algorithms like Spatio-Temporal or Likelihood Ratio TBD uses the Markov matrix for the motion model description. TBD algorithm uses the accumulative approach by multiple measurements. There are two sources of multiple measurements: from the multiple sensors or from the multiple scans. Both sources could be used together for maximal performance. The accumulative approach (calculating of a mean from multiple noised values) increases SNR and it is reason why TBD algorithms restores signals for a low SNR measurements. In this paper a single sensor is assumed for simplification of analyses.

TBD algorithms work very well for high SNR also. Reduction of the object signal reduces a performance of TBD. There is the special class of objects with constant or variable slowly signal level and an additive positive impulses (salt noise). Airplanes, satellites or asteroids have such signal signatures for example. Such positive impulses should improve greatly a tracking process because a high value will be accumulated and output SNR should be improved.

In a few papers [10-12] computer simulations for such cases are shown – the results are different from the expectations. Positive impulse should improve tracking process but an overall tracking performance is reduced. Reduction of measurement values (by the saturation function) improves tracking performance (this is paradox, because results should be opposite). In this paper paradox will be explained also using one-dimensional model of infinite grid. Simple Markov matrix is assumed: there is a dispersion of particular value over the three grid cells

(Fig.2) in every processing step k. The information update formula is based on the exponential filter.

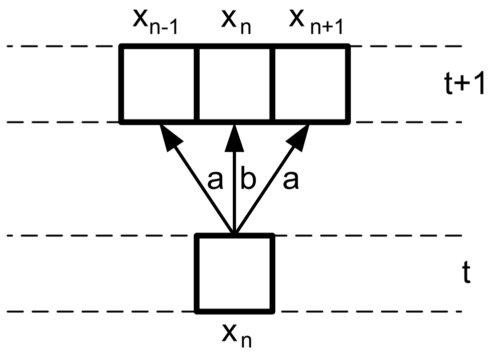

Fig. 2. Markov matrix based values evolution

The sum of the Markov matrix transitions should be equal to the unity (sum of row) because information can not be generated using the recurrent formula. Response of the Markov matrix after N-scans could be calculated [8] using following formula:

$$r_N = r_0 \cdot P^N \tag{2}$$

where: $r_0 = [\ldots \quad 0 \quad 0 \quad C \quad 0 \quad 0 \quad \ldots]$ - starting vector, with the single object and without a noise, C − pulse magnitude, e.g. $C = 1$.

Symmetrical values of transitions are assumed in the Markov matrix what is typical in typical tracking scenarios and is assumed for simplification of the analysis:

$$a + b + a = 1 \tag{3}$$

so the Markov matrix has following form:

$$Q = \begin{bmatrix} \ddots & \vdots & & & & & \\ \cdots & a & & & & & \\ & b & a & & & & \\ & a & b & a & & & \\ & & a & b & a & & \\ & & & a & b & a & \\ & & & & a & b & \\ & & & & & a & \cdots \\ & & & & & \vdots & \ddots \end{bmatrix}$$

Following formula could be used for calculations of the real response of TBD for assumed existence of the impulse only in the input measurements:

$$r_N = (1-\alpha)\cdot\alpha^{(N-1)}\cdot r_0 \cdot P^N \quad (4)$$

The first part of this formula $(1-\alpha)$ represent of the input measurement mixing with existing state space. The second part of formula $\alpha^{(N-1)}$ represents a recurrent calculations and the exponential values reduction.

3. Evolution of Impulse Responses

Distortions generated by impulses are the Markov matrix values dependent. Two examples are shown because they depicts a possible results and lack of the impulse noise reduction (impulse has a magnitude 30, but the expected measurement values are from the range $\langle 0,1\rangle$. The smoothing coefficient is set to the $\alpha = 0.9$ value. Measured impulse is reduced to the $30\cdot(\alpha-1)$ value by the information update formula in the first processing step.

The first one example assumes transitions with the three equal values $(a=1/3, b=1/3, a=1/3)$ - uniform values of this kernel. After a few iterations a Gaussian shape is obtained what is the result of the central limit theorem (similar technique is used often for the recurrent approximation of the Gaussian filter).

Obtained results shown in Fig.3 and Fig.4 are unimodal with the single maximum. In the center of value set (b -value position) is this maximum located. This impulse exists in next scans (with decaying values) due to recurrent calculations of Spatio-Temporal Recurrent TBD. Central elements (b) of Markov matrix has quite often a higher value in comparison to the surrounding (a) and the simulation result is shown in the next example.

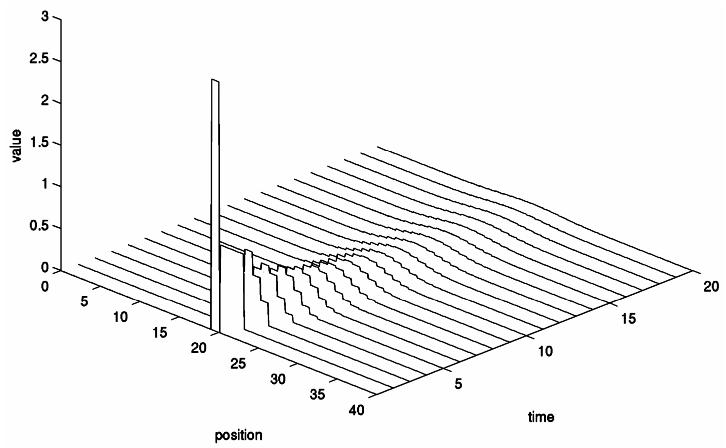

Fig. 3. Time evolution of impulse - transition set $(a=1/3, b=1/3, a=1/3)$

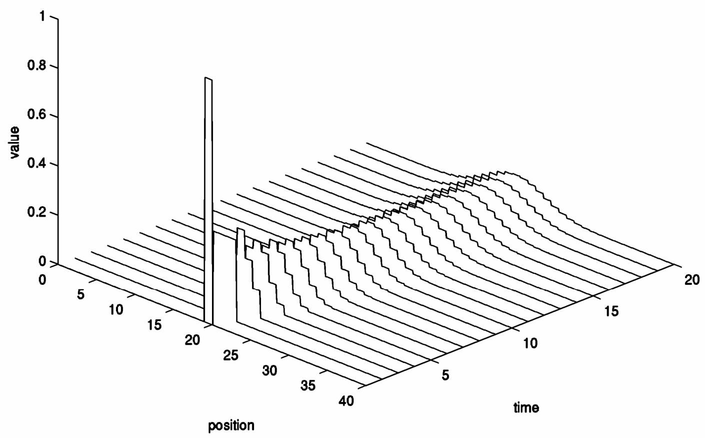

Fig. 4. Impulse response of TBD system - transition set $(a = 1/3, b = 1/3, a = 1/3)$

In Fig.5 and Fig.6 single modal response are shows (they are less dispersed in comparison to the previous example). The impulse in this example is long-time living and disturbs the proper trajectory of moving object. This disturbance occurs if the proper trajectory is different from direction of defined by the impulse response maxima. Multiple maxima (mods) are possible for special cases (if the central element has lower value than surrounding) and for example the set $(a = 0.45, b = 0.1, a = 0.45)$ has such property.

There is a special case of the Markov matrix where impulse noise behaves according to the expectation (improves SNR). Complete signal value is passed to the next scan and is reduced only by the information update formula for $a = 0; b > 1$ (so Markov matrix has only ones and zeros). Such case is interesting for the applications with the fixed object motion. Measurements limiting by the saturation algorithm for this special case is not so good idea.

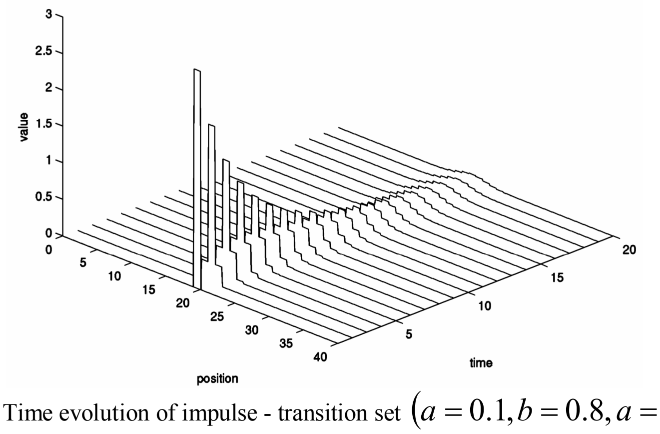

Fig. 5. Time evolution of impulse - transition set $(a = 0.1, b = 0.8, a = 0.1)$

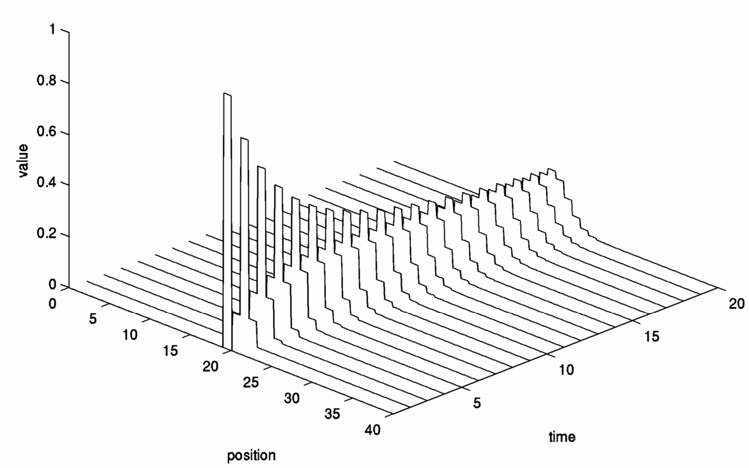

Fig. 6. Impulse response of TBD system - transition set $(a = 0.1, b = 0.8, a = 0.1)$

4. Single and Parallel Track-Before-Detect Analysis

Spatio-Temporal TBD is the linear signal processing algorithm so the linear systems theorem could be used for the impulse response and the object signal analysis. The single Spatio-Temporal TBD block depicted in Fig.7 could be replaced by the two parallel TBD processing blocks with separated inputs. The TBD1 block processes the signal and noise. The TBD2 block process impulse noise only. Such separation is not possible in the real case but is very convenient for the explanation of paradox - observed results is the sum of responses of both TBD blocks.

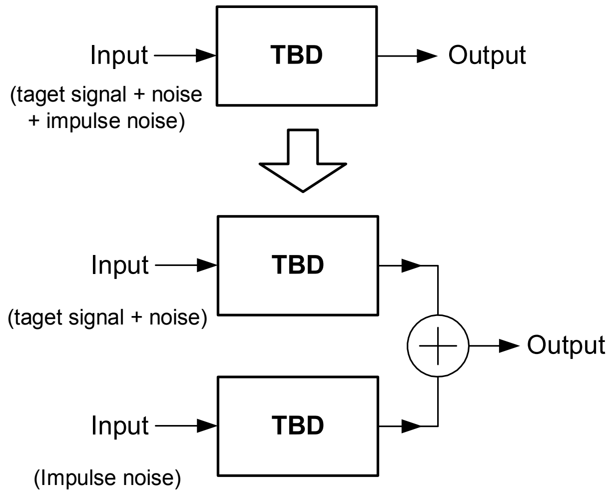

Fig. 7. Decomposition of the linear Track-Before-Detect systems

The constant object signal and zero mean Gaussian noise are assumed. The TBD1 give response related to the object with a suppressed noise. The TBD2 block give the impulse response and the output of this block behaves according to the description from the previous section and create the long-time decaying response (with direction defined by the central value b).

The output value of TBD1 does not exceed the 1.0 magnitude for noise-less case. The maximal response value for some time moment for TBD2 could be larger then 1.0. The maximal value is obtained from the impulse not from the real object so it is a source of tracking errors and the paradox is explained finally.

5. Suppression of impulse noise with fixed threshold

There are available different techniques for the reduction of this noise related to the object signal [10-12]. The first technique is based on the saturation of the upper value using fixed and arbitrary set threshold value T , incorporated into the information update formula:

$$P(k,s) = \alpha P^{-}(k,s) + (1-\alpha) Sat(X(k,s),T) \tag{5}$$

High value of threshold gives abilities of higher influence of the impulse noise on final results. The second technique is based on the switching between two update formulas:

$$P(k,s) = \begin{cases} P^{-}(k,s) & : X(k,s) > T \\ \alpha P^{-}(k,s) + (1-\alpha) X(k,s) & : otherwise \end{cases} \tag{6}$$

Predicted value is used only if the input signal value exceeds threshold value. The input measurement is set to the zero for high value of the smoothing coefficient ($\alpha \approx 1.0$).This technique could be considered also from another point-of-view as a replacement of the input value by the predicted one. Values below threshold are used like in the original information update formula.

Recurrent behaviors of considered Spatio-Temporal TBD lock ability of the direct application of the median filter. Non-recurrent TBD algorithms are much better for incorporating median filtering together with an accumulation.

6. Examples of Suppression of Impulse Noise with Fixed Threshold

In the following test examples the smoothing coefficient is equal to the 0.95. There are 313 iterations for full circle of the target and threshold values: 2, 3 and 5 are tested. Signal strength is equal to the 1 and measurements are disturbed by an additive noise with uniform distribution and values from 0 to 0.5 range.

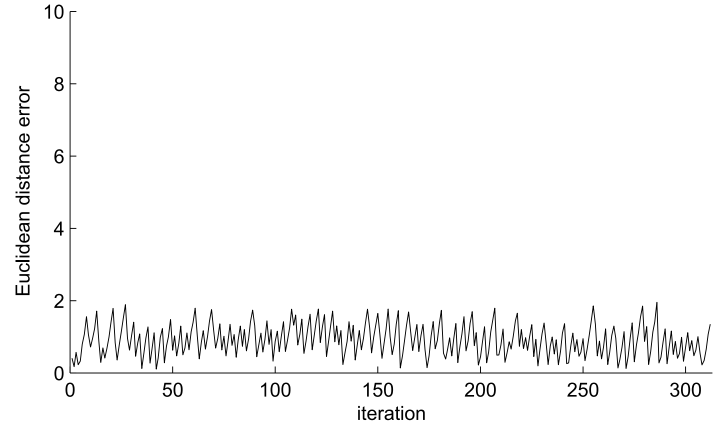

Fig. 8. Euclidean errors for threshold value 2 and algorithm (5), 5% impulse rate

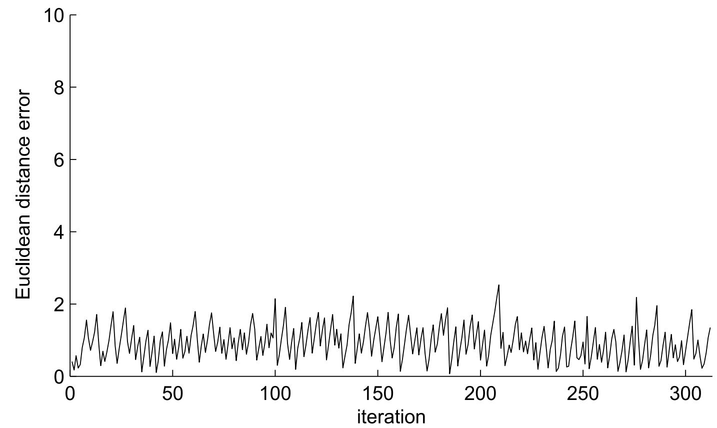

Fig. 9. Euclidean errors for threshold value 2 and algorithm (6), 5% impulse rate

Saturation algorithm according to (5) preserves additive pulse strength what is not recommended and occurs if threshold value is high. Alternative algorithm (6) remove pulse what is much more efficient and smaller error are obtained. Further lowering of the threshold level for the formula (5) will reduce errors and introduce some nonlinear effect to the overall signal if the level will be to near the upper boundary defined as a sum of constant signal strength and the positive noise values. In Fig. 8 and Fig.9 are shown example where the errors are comparable for low threshold value 2.

Formula (6) is preferred if the impulse rate is low and the threshold value is high, otherwise information update works as a data proxy between steps and integration process is not working well (Fig. 11).

For the high ratio pulses threshold basic threshold (5) work better what is shown in Fig. 10 and Fig. 11.

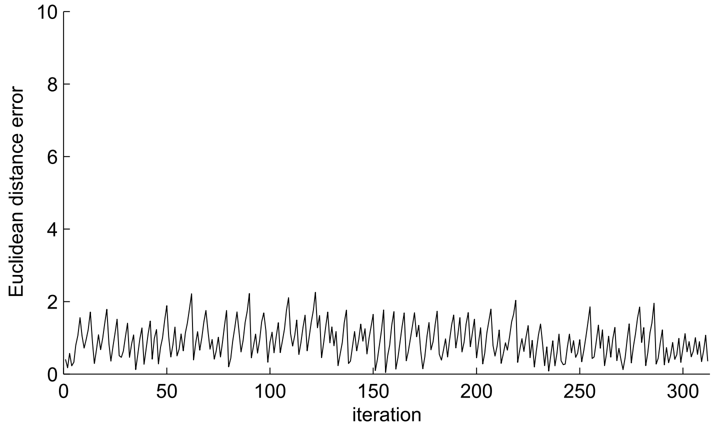

Fig. 10. Euclidean errors for threshold value 2 and algorithm (5), 30% impulse rate

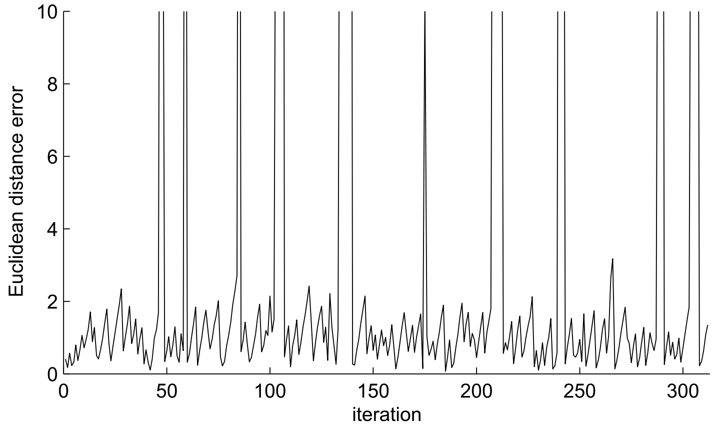

Fig. 11. Euclidean errors for threshold value 2 and algorithm (6), 30% impulse rate

7. Conclusions

Presented analysis of the SNR paradox in TBD systems shows reasons why the positive impulse (or impulses) reduces tracking performance. High-peak impulse value force the trajectory according to the maxima of impulse response. Reduction of peaks even by simple saturation functions improves tracking. Such assumption is correct for a non-trivial Markov matrix. A trivial case of Markov matrix where in every row is the single 1.0 value has not such behavior and the saturation function reduce a performance. Obtained results are correct for TBD systems based on Spatio-Temporal and Likelihood Ratio TBD. Analysis of this paradox is very important for the real systems. An improvement by saturation function is important for implementations based on the fixed-point arithmetic.

Selection of the correct formula and the threshold value is non trivial task, because depending on the threshold level results are different. Large threshold value is not recommended for formula (5). High impulse ratio reduces possibility of application of formula (6). Selection between them will be considered in the further works.

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