Monte Carlo Simulation Approach to Determination of Oil Spill Domains at Port and Sea Water Areas

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ABSTRACT: Monte Carlo simulation method of oil spill domains determination based on the probabilistic approach to the solution of this problem is proposed. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed and its parameters are defined. The general stochastic model of oil spill domain movement for various hydro-meteorological conditions is described. Monte Carlo simulation procedure is created and applied to generating the process of changing hydro-meteorological conditions and the prediction of the oil spill domain movement impacted by these changes conditions.

1 INTRODUCTION

The probabilistic approach to determination of oil spill domains at port and sea water areas is proposed in (Dąbrowska & Kołowrocki 2019B). In this paper, the stochastic approach is supplemented by the Monte Carlo simulation approach (Dąbrowska 2019, Law & Kelton 2000, Zio & Marseguerra 2002) to the oil spill domain movement in changing hydro-meteorological conditions (Dąbrowska & Kołowrocki 2019A, 2019B). First, the model of the process of changing hydro-meteorological conditions is defined and its parameters are introduced. The identification methods of the unknown parameters of process of changing hydro-meteorological conditions is described in (Dąbrowska & Kołowrocki 2019A). Next, a bit modified probabilistic model of oil spill domains, considered in (Dąbrowska & Kołowrocki 2019B) is introduced. After that, Monte Carlo simulation approach general procedure is created and applied to generating the process of changing hydro-meteorological conditions at oil spill area and to the prediction of oil spill domain in varying hydro-meteorological conditions.

2 MODELLING PROCESS OF CHANGING HYDRO-METEOROLOGICAL CONDITIONS AT OIL SPILL AREA

We denote by $A(t)$ the process of changing hydro-meteorological conditions at the sea water areas where the oil spill happened and distinguish m its states from the set $A = \{1,2,...,m\}$ in which it may stay at the moment t, $t \in <0,T>$, where $T > 0$. Further, we assume a semi-Markov model of the process $A(t)$ and denote by θ_{ij} its conditional sojourn time in the state i while its next transition will be done to the state j, where i, $j = 1,2,...,m$, $i \neq j$ (Dąbrowska & Soszyńska-Budny 2018, 2019A). Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters (Dąbrowska & Soszyńska-Budny 2018, Kołowrocki & Soszyńska-Budny 2011):

– the vector of probabilities of its initial states at the moment t = 0

$$[p(0)] = [p_1(0), p_2(0),..., p_m(0)], \qquad (1)$$

– the matrix of probabilities of its transitions between the particular states

$$[p_{ij}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \qquad (2)$$

where

$p_{ii} = 0,\ i = 1,2,...,m;$

– the matrix of distribution functions of its conditional sojourn times θ_{ij} at the particular states

$$[W_{ij}(t)] = \begin{bmatrix} W_{11}(t) & W_{12}(t) & \cdots & W_{1m}(t) \\ W_{21}(t) & W_{22}(t) & \cdots & W_{2m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1}(t) & W_{m2}(t) & \cdots & W_{mm}(t) \end{bmatrix}, \qquad (3)$$

where

$W_{ii}(t) = 0,\ i = 1,2,...,m.$

3 MODELLING OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

We assume that the process of changing hydro-meteorological conditions $A(t)$ in succession takes the states

$k_1,\ k_2,\ ...,\ k_{n+1},\ k_i \in \{1,2,...,m\},\ i = 1,2,...,n+1.$

For a fixed step of time Δt, after multiple applying sequentially the procedure from Section 4.1 in (Dąbrowska & Kołowrocki 2019B):

– for

$$t = 1\Delta t, 2\Delta t, \cdots, s_1\Delta t, \qquad (4)$$

– at the process $A(t)$ state k_1;
– for

$$t = (s_1 + 1)\Delta t, (s_1 + 2)\Delta t, \cdots, s_2\Delta t, \qquad (5)$$

– at the process $A(t)$ state k_2;
…
– for

$$t = (s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t, \cdots, s_n\Delta t, \qquad (6)$$

– at the process $A(t)$ state k_n;

we receive the following sequence of oil spill domains:

$$\overline{D}^{k_1}(1\Delta t),\ \overline{D}^{k_1}(2\Delta t),\ ...,\ \overline{D}^{k_1}(s_1\Delta t), \qquad (7)$$

$$\overline{D}^{k_2}((s_1 + 1)\Delta t),\ \overline{D}^{k_2}((s_1 + 2)\Delta t),\ ...,\ \overline{D}^{k_2}(s_2\Delta t), \qquad (8)$$

…

$$\overline{D}^{k_n}((s_{n-1} + 1)\Delta t),\ \overline{D}^{k_n}((s_{n-1} + 2)\Delta t),\ ...,\ \overline{D}^{k_n}(s_n\Delta t), \qquad (9)$$

where $s_i,\ i = 1,2,...,n,$ are such that

$$(s_i{-}1)\Delta t < \sum_{j=1}^{i} \overline{\theta}_{k_j\,k_{j+1}},\ \le s_i\Delta t,\ i = 1,2,...,n,$$

$$s_n\Delta t \le T, \qquad (10)$$

and

$$\overline{\theta}_{k_j k_{j+1}},\quad j = 1,2...,n, \qquad (11)$$

are the realizations of the process $A(t)$, $t \in <0,T>$, conditional sojourn times $\theta_{k_j k_{j+1}}$, $j = 1,2...,n$ at the states k_j, upon the next state is k_{j+1}, $j = 1,2...,n$, $k_i \in \{1,2,...,m\}$, $i = 1,2...,n$, introduced in Section 2 and in (Dąbrowska & Kołowrocki 2019B).

Hence, the oil spill domain

$$\overline{D}^{k_1,k_2,...,k_n},\quad k_1,\ k_2,\ ...,\ k_n \in \{1,2,...,m\},$$

is described by the sum of determined domains of the sequences (7)-(9), given by

$$\overline{D}^{k_1,k_2,...,k_n} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{s_i - s_{i-1}} \overline{D}^{k_i}((s_{i-1} + j)\Delta t)$$

$$= [\overline{D}^{k_1}(1\Delta t) \cup \overline{D}^{k_1}(2\Delta t) \cup ... \cup \overline{D}^{k_1}(s_1\Delta t)]$$

$$\cup [\overline{D}^{k_2}((s_1 + 1)\Delta t) \cup \overline{D}^{k_2}((s_1 + 2)\Delta t) \cup ... \cup \overline{D}^{k_2}(s_2\Delta t)]$$

…

$$\cup [\overline{D}^{k_n}((s_{n-1} + 1)\Delta t) \cup \overline{D}^{k_n}((s_{n-1} + 2)\Delta t) \cup ... \cup \overline{D}^{k_n}(s_n\Delta t)]$$

for k1, k2, ..., kn $\in \{1,2,...,m\}$, $s_0 = 0$ (12)

The oil spill domain $\overline{D}^{k_1,k_2,...,k_n}$ defined by (12) is determined for constant radiuses

$$r^{k_i}(t) = r^{k_i},\quad t \in <0,T>, k_i \in \{1,2,...,m\}, i = 1,2,...,n.$$

If the radiuses are not constant, we define the sequence of domains for each state k_i, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$, in a way similar to that described in Remark 1 in (Dąbrowska & Kołowrocki 2019B), i.e. we define the sequence of domains

$$\overline{\overline{D}}^{k_1,k_2,...,k_n}(b_i\Delta t) = \bigcup_{i=1}^{n} \bigcup_{a_i=1}^{b_i} \overline{\overline{D}}^{k_i}(s_{i-1} + a_i\Delta t)$$

$$= [\overline{\overline{D}}^{k_1}(1\Delta t) \cup \overline{\overline{D}}^{k_1}(2\Delta t) \cup ... \cup \overline{\overline{D}}^{k_1}(s_1\Delta t)]$$

$$\cup [\overline{\overline{D}}^{k_2}((s_1 + 1)\Delta t) \cup \overline{\overline{D}}^{k_2}((s_1 + 2)\Delta t) \cup ... \cup \overline{\overline{D}}^{k_2}(s_2\Delta t)]$$

…

$\cup [\overline{\overline{D}}^{k_n}((s_{n-1}+1)\Delta t) \cup \overline{\overline{D}}^{k_n}((s_{n-1}+2)\Delta t) \cup ... \cup \overline{\overline{D}}^{k_n}(s_n\Delta t)]$

for $b_i = 1,2,...,s_i - s_{i-1}$, $k_i \in \{1,2,...,m\}$,

$i = 1,2,...,n,$ \hfill (13)

where

$\overline{\overline{D}}^{k_i}(s_{i-1}+a_i\Delta t) := \overline{D}^{k}(s_{i-1}+a_i\Delta t),$

$a_i = 1,2,...,b_i,\ b_i = 1,2,...,s_i - s_{i-1},\quad k_i \in \{1,2,...,m\},$

$i = 1,2,...,n,$

with the following, modified slightly in comparison that defined in (Dąbrowska & Kołowrocki 2019B), substitutions:

$m_X^k(t) := m_X^{k_{i-1}}(s_{i-1}\Delta t) + m_X^{k_i}(a_i\Delta t),$

$m_Y^k(t) := m_Y^{k_{i-1}}(s_{i-1}\Delta t) + m_Y^{k_i}(a_i\Delta t),$

$\overline{\sigma}_X^k(t) := \overline{\overline{\sigma}}_X^{k_i}(s_{i-1}+a_i\Delta t)$

$= \sigma_X^{k_i}(s_{i-1}+a_i\Delta t) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t),$

$\overline{\sigma}_Y^k(t) := \overline{\overline{\sigma}}_Y^{k_i}(s_{i-1}+a_i\Delta t)$

$= \sigma_Y^{k_i}(s_{i-1}+a_i\Delta t) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t),$

where $\quad m_X^{k_0}(s_0\Delta t) = 0, \qquad m_Y^{k_0}(s_0\Delta t) = 0,\quad$ for

$a_i = 1,2,...,b_i,\ b_i = 1,2,...,\ s_i - s_{i-1},\ k_i \in \{1,2,...,m\},\ i = 1,2,...,n,$ $s_0 = 0.$

4 MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

4.1 *Generating process of changing hydro-meteorological conditions at oil spill area*

We denote by $k_i = k_i(q)$, $i \in \{1,2,...,m\}$, the realization of the process' $A(t)$ initial state at the moment $t = 0$. Further, we select this initial state by generating realizations from the distribution defined by the vector $[p(0)]_{1 \times m}$, according to the formula

$k_i(q) = k_\xi,\quad \sum_{\xi=1}^{\psi} p_{\xi-1}(0) \le q < \sum_{\xi=1}^{\psi} p_\xi(0),$

$\psi \in \{1,2,...,m\},$ \hfill (14)

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$ and $p(0)$ for $\xi = 0$ equals 0.

After selecting the initial state k_i, $i \in \{1,2,...,m\}$, we can fix the next operation state of the process of changing hydro-meteorological conditions at oil spill area. We denote by $k_j = k_j(g)$, $j \in \{1,2,...,m\}$, $i \neq j$, the sequence of the realizations of the operation process'

consecutive states generated from the distribution defined by the matrix $[p_{ij}]_{m \times m}$. Those realizations are generated for a fixed i, $i \in \{1,2,...,m\}$, according to the formula

$k_j(g) = k_\xi,\quad \sum_{\xi=1}^{\psi} p_{i\,\xi-1} \le g < \sum_{\xi=1}^{\psi} p_{i\,\xi},$

$\psi \in \{1,2,...,m\},\ \psi \neq i,$ \hfill (15)

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1 \rangle$ and $p_{i0} = 0$.

We can use several methods generating draws from a given probability distribution, e.g. an inverse transform method, a Box-Muller transform method, Marsaglia and Tsang's rejection sampling method (Dąbrowska 2019). The inverse transform method (also known as inversion sampling method) is convenient if it is possible to determine the inverse distribution function (Grabski & Jaźwiński 2009). This section will consider only this one sampling method, but the other methods are discussed in (Law & Kelton 2000, Rao & Naikan 2016, Zio & Marseguerra 2002).

We denote by $t_{ij}^{(v)}$, $i,j \in \{1,2,...,m\}$, $i \neq j$, $v = 1,2,...,n$, the realization of the conditional sojourn times θ_{ij} of the process $A(t)$, $t \in \langle 0,T \rangle$, generated from the distribution function $W_{ij}(t)$, where v denotes the subsequent number of the sojourn times realizations and n is the number of those sojourn time realizations during the experiment time T. Thus, using the inverse transform method, the realization $t_{ij}^{(v)}$ is generated from

$t_{ij} = W_{ij}^{-1}(h),\ i,j \in \{1,2,...,m\},\ i \neq j,$ \hfill (16)

where $W_{ij}^{-1}(h)$ is the inverse function of the conditional distribution function $W_{ij}(t)$ and h is a randomly generated number from the interval $\langle 0,1 \rangle$;

Having the realizations $t_{ij}^{(v)}$, $i,j \in \{1,2,...,m\}$, $i \neq j$, $v = 1,2,...,n$, of the process $A(t)$, it is possible to determine approximately the entire sojourn time as the sum of all sojourn time realizations during the experiment time T, applying the formula

$\tau_n = \sum_{v=1}^{n} t_{ij}^{(v)},\ i,j \in \{1,2,...,m\},\ n = 1,2,....$ \hfill (17)

The exemplary realisation of the process $A(t)$ and the entire sojourn time is presented in a figure below.

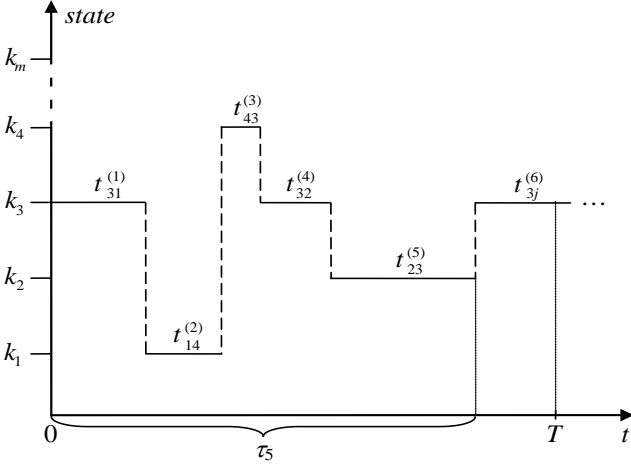Figure 1. Exemplary realization of the process $A(t)$.

4.2 General procedure of Monte Carlo simulation application to determine the oil spill domain in varying hydro-meteorological conditions

The procedure of generating and estimating the parameters of the process of changing hydro-meteorological conditions at oil spill area characteristics is formed as follows.

First, we have to draw a randomly generated number g from the uniform distribution on the interval $\langle 0,1 \rangle$. Next, we can select the initial state k_i, $i \in \{1,2,...,m\}$, according to (14). Further, we draw another randomly generated number g from the uniform distribution on the interval $\langle 0,1 \rangle$. For the fixed i, $i \in \{1,2,...,m\}$, we select the next state k_j, $j \in \{1,2,...,m\}$, $j \neq i$, according to (15). Subsequently, we draw a randomly generated number h from the uniform distribution on the interval $\langle 0,1 \rangle$. For the fixed i and j, we generate a realization t_{ij} of the conditional sojourn time θ_{ij} from a given probability distribution, according to (16). Then, we compare the realization t_{ij} of the conditional sojourn time with the experiment time T. If the realisation t_{ij} of the conditional sojourn time is less than the experiment time T, we draw the sequence of domains

$$\overline{\overline{\boldsymbol{D}}}^{\,k_1,k_2,...,k_n}(b_i\Delta t),$$

for $b_i = 1,2,...,s_i - s_{i-1}$, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$, using formula (13).

As the realisation t_{ij} is the first one, we put $v = 1$ and consequently

$$\tau_1 = t_{ij}^{(1)}.$$

Further, we substitute $i := j$ and repeat drawing another randomly generated numbers g and h (selecting the states k_j and generating another realization $t_{ij}^{(v)}$, $v = 2$, of the conditional sojourn time. Having the realizations $t_{ij}^{(v)}$, $i,j \in \{1,2,...,m\}$, $i \neq j$, $v = 1,2$, of the process $A(t)$, we calculate the entire sojourn time τ_n, $n = 1,2,...$, applying the formula (17), i.e. we have

$$\tau_2 = t_{ij}^{(1)} + t_{ij}^{(2)}.$$

Further, we compare it with time T. If the sum τ_2 is less than the experiment time T, we draw the sequence of domains using formula (13).

We repeat the procedure above until the sum τ_n of all generated realizations $t_{ij}^{(v)}$, $v = 1,2, ..., n$, reach a fixed experiment time T. Consequently, we calculate the entire sojourn time τ_n, according to (17) and draw the sequence of domains using formula (13).

Finally, we put together all the sequences of domains draw before and we get the oil spill domain movement (Figure 2). In the interval $\langle 0, \tau_1 \rangle$ the number of ellipses is $s_1 - s_0 = s_1$, in the next intervals $\langle \tau_2 - \tau_1, \tau_2 \rangle$, ..., $\langle \tau_n - \tau_{n-1}, \tau_n \rangle$ the number of ellipses are respectively $s_2 - s_1$, $s_3 - s_2, ..., s_n - s_{n-1}$, where s_i, $I = 1,2, ..., n$, are defined by (10).

The general Monte Carlo simulation flowchart for generating and determination of a process of changing hydro-meteorological conditions at oil spill area is illustrated in Figure 3.

4.3 Monte Carlo simulation prediction of the oil spill domain in varying hydro-meteorological conditions

Using the procedures of the process of changing hydro-meteorological conditions at oil spill area prediction described in Sections 4.1-4.2 and the modified method of the domain of oil spill determination presented in Section 4.3 in (Dąbrowska & Kołowrocki 2019B) the Monte Carlo simulation oil spill domain prediction can be done.

The modified method of the domain of oil spill determination presented in Section 4.3 in (Dąbrowska & Kołowrocki 2019B) depends on changing the procedure (4)-(12) by replacing the conditions (10)-(12) by conditions:

The s_i, $i = 1,2,...,n$, existing in (4)-(9), according to (10), are such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{i} t_{k_j k_{j+1}} = s_i \Delta t,\ i = 1,2,...,n,\ s_n \Delta t \leq T, \qquad (18)$$

and

$$t_{k_j k_{j+1}},\ j = 1,2...,n-1,\ (19)$$

are the realizations of the process $A(t)$, $t \in <0,T>$, conditional sojourn times

$$\theta_{k_j k_{j+1}},\ j = 1,2...,n-1$$

at the states k_j, upon the next state is k_{j+1}, $j = 1,2...,n-1$, k_j, k_{j+1}, $\in \{1,2,...,m\}$, $j = 1,2...,n-1$, defined in Section 4.1.

Figure 2. Oil spill domain for changing hydro-meteorological conditions.
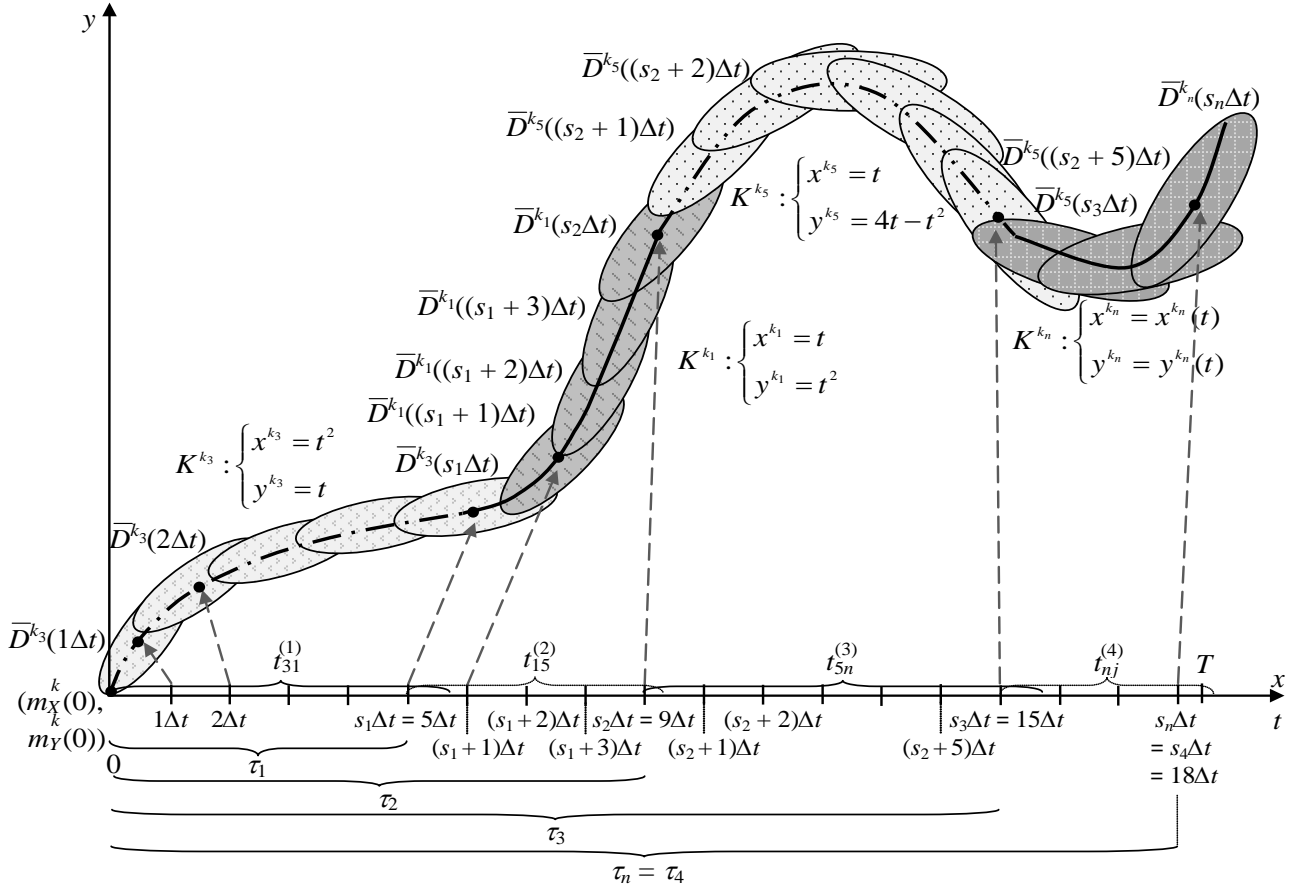

Figure 3. General Monte Carlo flowchart for prediction of the oil spill domain in varying hydro-meteorological conditions.

5 CONCLUSIONS

The proposed Monte Carlo simulation approach allows us for the determination of oil spill domains at port and sea water areas in changing hydro-meteorological conditions. It is a new supplementary method to the probabilistic methods of the oil spill domains determination in the varying hydro-meteorological states presented in (Dąbrowska & Kołowrocki 2019A, 2019B), (Kim et.al. 2013) and (Chen, Li & Li 2007). The comparison of results of these methods' applications in real conditions should lead to the selection one of them with the best accuracy and develop in the future research.

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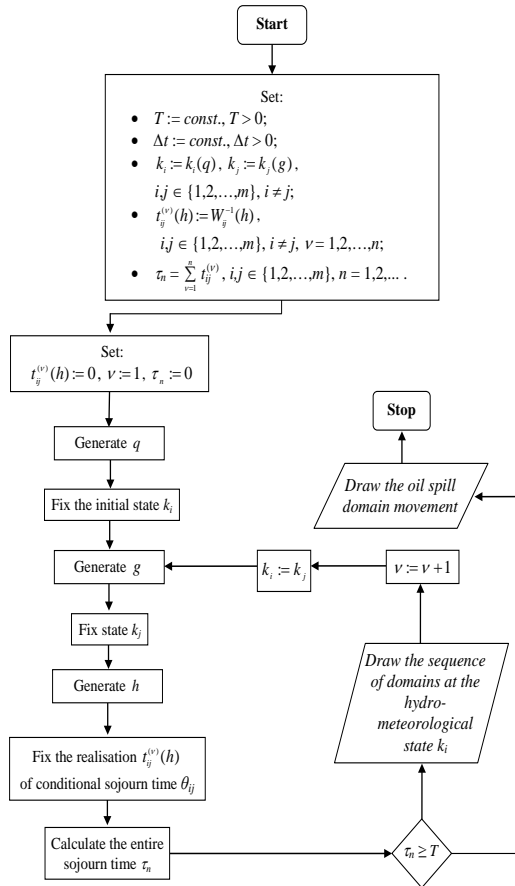
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