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DETERMINATION OF BOLT FORCES FOR THE OPERATIONAL STATE OF A BOLTED FLANGE CONNECTION

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Summary

In the paper modelling and calculations of an asymmetrical bolted flange connection at the operational stage are presented. The physical model of the joint is based on a flexible flange element that is connected with a rigid support by means of hybrid elements, which substitute bolts. Between the flange element and the support, the nonlinear Winkler model of a contact layer is taken into consideration.

A computational model of the system is proposed, which makes it possible to analyze any preloaded bolted flange connection subjected to an eccentric normal load. Results obtained from the calculations are compared with experimental research described in [1].

Keywords: bolted flange connection, operational state, bolt force

Wyznaczanie sił w śrubach w kołnierzowym połączeniu śrubowym w stanie eksploatacyjnym

Streszczenie

W pracy przedstawiono modelowanie i obliczenia asymetrycznego kołnierzowego połączenia śrubowego w stanie eksploatacyjnym. Model fizyczny połączenia jest zbudowany z odkształcalnego kołnierza mocowanego do nieodkształcalnej ostoi za pomocą elementów hybrydowych, modelujących śruby. Pomiędzy elementami łączonymi wprowadzono nieliniową warstwę sprężystą typu winkle-rowskiego. Zaproponowano model połączenia umożliwiającego analizę dowolnego kołnierzowego połączenia śrubowego, wstępnie naprężonego i obciążonego zewnętrznie mimośrodowo siłą normalną. Uzyskane wyniki obliczeń porównano z wynikami wykonanych badań doświadczalnych [1].

Słowa kluczowe: kołnierzowe połączenie śrubowe, stan eksploatacyjny, siła w śrubie

Nomenclature

A_j – j-th elementary contact area, mm^2
c_{yi} – stiffness coefficient of the i-th bolt's model, kN/mm
F_{mi} – preload of the i-th bolt, kN
F_e – external normal load, kN
F_{si} – force in the i-th bolt, kN
h – thickness of the joined element, mm

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i – number of the bolt (for $i = 1, 2, \ldots, k$)
j – number of the nonlinear spring (for $j = 1, 2, \ldots, l$)
$\boldsymbol{K}$ – stiffness matrix
$\boldsymbol{p}$ – loads vector
$\boldsymbol{q}$ – displacements vector
R_j – force in the centre of the j-th elementary contact area, kN
u_j – deformation of the j-th nonlinear spring element (for $j = 1, 2, \ldots, l$), mm
W – accuracy index, %

1. Introduction

Preloaded bolted flange connections are usually designed for carrying very differential external loads. Among them, there are asymmetrical joints with an arbitrary arrangement of bolts widely applied in machine and machine tool building. Such a connection very often should be treated as a nonlinear system. This nonlinearity is caused by both contact phenomena between joined elements and stiffness characteristics of washers or gaskets [1-3].

Elaborations on modelling and calculations of the bolted flange connection occurring in the literature mostly deal with symmetrical joints. The authors assume that bolts can be modeled as:

- cylindrical 3D elements [4-6],
- rod elements [7],
- linear springs [8, 9].

In some other works, calculations of bolted flange connections are based only on the forces balance for the joint without including bolts in the joint model [10, 11].

The following methods are used for modelling of the connections in these works:

- the finite element method (FEM) [12] implemented in commercial computer programs [4-9],
- the theory of elasticity [13] in theoretical papers [8, 9],
- the Koves method [10] and the Taylor-Forge method [11].

The main aim of this paper is to verify a new model of an asymmetrical bolted flange connection. In the previous paper [14] some results of theoretical investigations of an asymmetrical preloaded bolted connection of a flange and a rigid support, subjected to an external normal load, were released. In that model of the joint, bolt holes were not taken into consideration and bolts were treated as linear springs. In the current paper some new results of investigations of an analogical model of the joint are presented. In the new model, bolt holes are taken into account and bolts are treated as hybrid elements consisted of a flexible plain part of the bolt and a rigid bolt head [15]. For modelling and calculations of the bolted flange connection the finite element method (FEM) [12] is used.

2. Physical model of the bolted flange connection

A general structure of the bolted flange connection model results from an idea presented in article [14]. The model of the joint is based on a flexible flange element that is fastened to a rigid support by means of *k* hybrid elements [15], which substitute bolts (Fig. 1b). Spring properties of the *i*-th bolt's model (for $i = 1, 2, \ldots, k$) are determined from the relation [16]

$$c_{yi} = \frac{1}{\sum_{n} \frac{1}{c_n}} \tag{1}$$

where c_n denotes the stiffness coefficient of the *n*-th bolt's fragment.

A contact layer between the flange element and the support is modeled as the nonlinear Winkler model, which is described by means of *l* one-sided nonlinear spring elements, defined by the following relationship

$$R_j = A_j \cdot \mathrm{f}(u_j) \tag{2}$$

where: R_j is the force in the centre of the *j*-th elementary contact area, A_j is the *j*-th elementary contact area and u_j is deformation of the *j*-th nonlinear spring element (for $j = 1, 2, \ldots, l$).

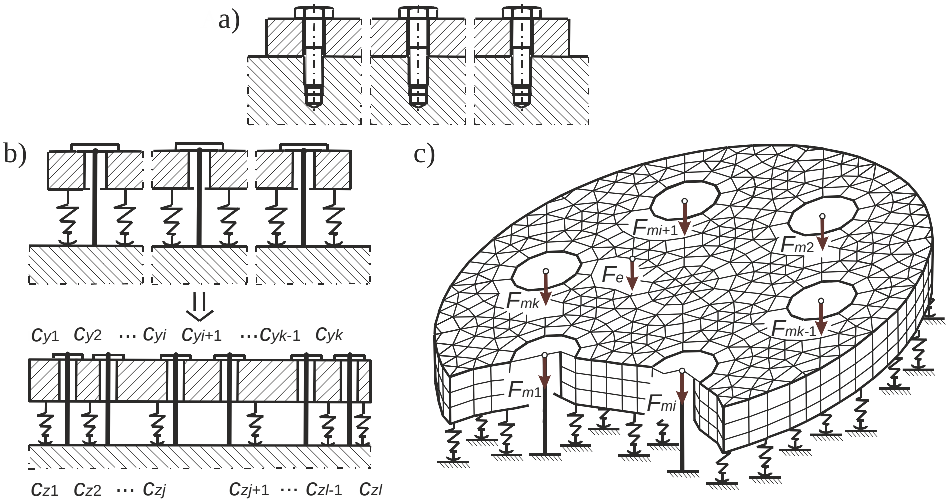

Fig. 1. Bolted flange connection: a) diagram of the joint, b) description of system spring properties, c) FEM-model of the joint

The equation of system equilibrium (Fig. 1c) can be written in the form

$$\boldsymbol{K} \cdot \boldsymbol{q} = \boldsymbol{p} \tag{3}$$

where: $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{q}$ is the displacements vector and $\boldsymbol{p}$ is the loads vector.

In the operational state, the loads vector $\boldsymbol{p}$ is composed of external normal loads F_e (Fig. 1c).

The generating procedure of the stiffness matrix $\boldsymbol{K}$ is presented in works [14, 15]. Adopting the division of the joint into three subsystems ($\boldsymbol{B}$ – the set of bolts, $\boldsymbol{F}$ – the flange element model, $\boldsymbol{C}$ – the nonlinear Winkler model of the contact layer), formula (3) can be rewritten as follows

$$\begin{bmatrix} \boldsymbol{K}_{BB} & \boldsymbol{K}_{BF} & \boldsymbol{0} \\ \boldsymbol{K}_{FB} & \boldsymbol{K}_{FF} & \boldsymbol{K}_{FC} \\ \boldsymbol{0} & \boldsymbol{K}_{CF} & \boldsymbol{K}_{CC} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{q}_B \\ \boldsymbol{q}_F \\ \boldsymbol{q}_C \end{bmatrix} = \boldsymbol{p} \tag{4}$$

where: $\boldsymbol{K}_{BB}$, $\boldsymbol{K}_{FF}$, $\boldsymbol{K}_{CC}$ are the stiffness matrices of subsystems $\boldsymbol{B}$, $\boldsymbol{F}$, $\boldsymbol{C}$ and $\boldsymbol{K}_{BF}$, $\boldsymbol{K}_{FB}$, $\boldsymbol{K}_{FC}$, $\boldsymbol{K}_{CF}$ are the matrices of elastic couplings among subsystems $\boldsymbol{B}$, $\boldsymbol{F}$, $\boldsymbol{C}$.

On the grounds of so defined model of the bolted flange connection, displacements of bolts and bolt forces after the operational state has been completed can be evaluated.

Calculations of the bolted flange connection are accomplished in an iterative process. In the first step of this process, in the equation (4) the stiffness matrix of the bolts subsystem $\boldsymbol{K}_{BB}$ received at the end of the assembly process [17] is taken into account. As a result of solving the equation (4) one obtains the displacements vector of bolts $\boldsymbol{q}_B$

$$\boldsymbol{q}_B = \mathrm{col}\left(q_{B1}, q_{B2}, \ldots, q_{Bi}, \ldots, q_{Bk}\right) \tag{5}$$

Final displacements of bolts q_{Bi} are measured from the working points $W_i^{'}$, which determine tension of bolts in the previous step of calculations (Fig. 2a). On the basis of so defined displacements q_{Bi}, forces in bolts F_{si} can be computed from the relation

$$F_{si} = c_{yi} \cdot q_{Bi} \tag{6}$$

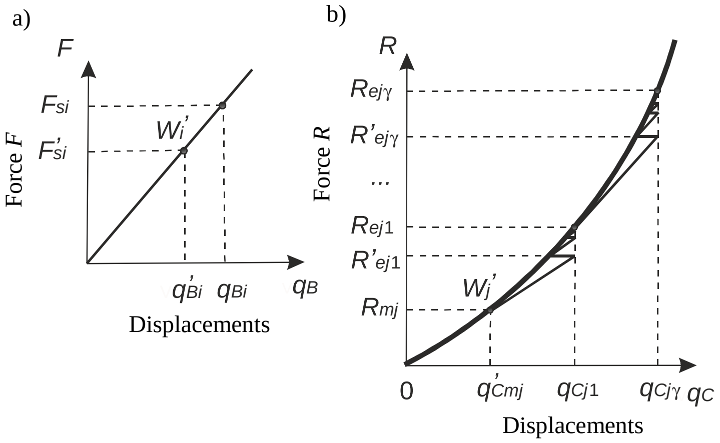

Fig. 2. Determining of the working load: a) in the case of a linear hybrid element, b) in the case of a nonlinear spring

As a result of solving the equation (4) one obtains the displacements vector of nonlinear springs $\boldsymbol{q}_C$ too, which can be evaluated using the formula

$$\boldsymbol{q}_C = \mathrm{col}\left(q_{C1}, q_{C2}, \ldots, q_{Cj}, \ldots, q_{Cl}\right) \tag{7}$$

In order to determine displacements of nonlinear springs q_{Cj} one achieves a linearization by means of the incremental-iterative method [15, 18]. It runs according to the way shown in Fig. 2b starting from the working points $W_j^{'}$, which define tension of nonlinear springs in the previous step of calculations. The linearization process is kept running in Γ iterations (for Γ= 1, 2, 3, ..., γ) in which one looks for so load values for which the following condition has been qualified

$$\left| \frac{\Delta R_{j\Gamma}^{'} - \Delta R_{j\Gamma}}{\Delta R_{j\Gamma}} \right| \le \varepsilon \tag{8}$$

where

$$\Delta R_{j\Gamma} = \begin{cases} R_{ej1} & \text{for} \quad \Gamma = 1 \\ R_{ej\Gamma} - R_{ej(\Gamma-1)} & \text{for} \quad \Gamma = 2, 3, 4, \ldots, \gamma \end{cases} \tag{9}$$

$$\Delta R'_{j\Gamma} = \begin{cases} R'_{ej1} & \text{for} \quad \Gamma = 1 \\ R'_{ej\Gamma} - R_{ej(\Gamma-1)} & \text{for} \quad \Gamma = 2, 3, 4, \ldots, \gamma \end{cases} \tag{10}$$

where: $R'_{j\Gamma}$ is the force in the nonlinear spring No. j and in the step No. Γ obtained from the linearization, $R_{j\Gamma}$ is the real force in the nonlinear spring No. j and in the step No. Γ and ε is the admissible error of the linearization.

Final displacements of nonlinear springs q_{Cj} are equal to $q_{Cj\gamma}$. On the basis of so defined displacements q_{Cj}, the forces R_{ej} can be determined from the relation (2) for u_j equal to q_{Cj}.

The diagram of iterative calculations of the bolted flange connection for the operational state is shown in Fig. 3.

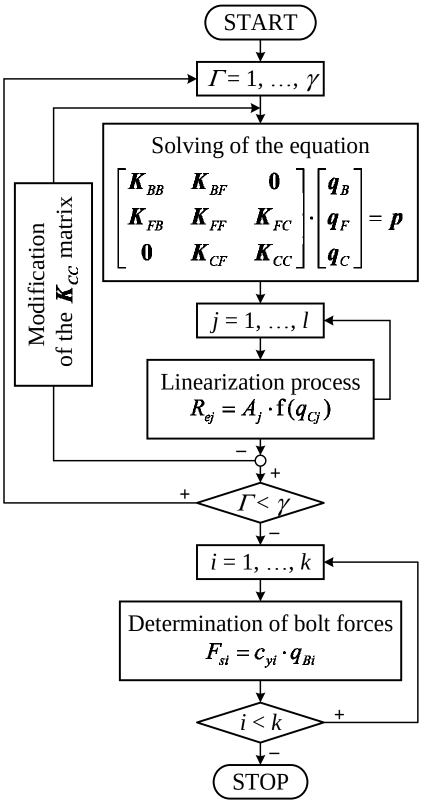

Fig. 3. Block diagram of iterative calculations
of the bolted flange connection

3. Calculations of the bolted flange connection at the operational stage

According to the presented method, computations of an asymmetrical bolted flange connection were performed (Fig. 4). The joint, assumed here, is an element of the special test stand designed to measurement of bolt forces in such a connection [1]. A simplified FEM-based model of the joint is shown in Fig. 4a. A fragment of the cross-section of the joint with models of bolts and the contact layer is illustrated in Fig. 4b. A contact surface between joined elements as well as the bolt's arrangement and their numeration are shown in Fig. 4c. Calculations were carried out for three values of the joined element's thickness h (for $h \in \{20\text{ mm},\ 40\text{ mm},\ 80\text{ mm}\}$). Characteristics of nonlinear springs are described as the following power function [15]

$$R_j = A_j \cdot (3{,}428 \cdot u_j^{1{,}657}) \tag{11}$$

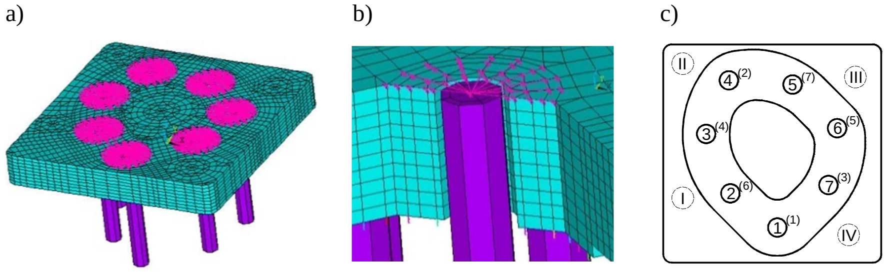

Fig. 4. Considered bolted flange connection: a) simplified FEM-based model, b) fragment of the cross-section, c) contact surface

To fastening of the joint, the bolts M10×1,25 are used. The preload of bolts F_{mi} is equal to 20 kN and it is set down on the base of Polish Standard [19]. The tightening sequence taken here on [15], is parenthesized in Fig. 4c. After the preloading process, the bolted flange connection is subjected to an eccentric normal load F_e equal to 50 kN acting consecutively at four points (I, II, III, IV) shown in Fig. 4c.

Results of calculations were put together in graphs illustrated in Fig. 5 and 6 according as both the individual thickness of the flange element h and four points of the load F_e acting. In the respective figures, values of bolt forces F_{si} related to preloads F_{mi} [17] are presented.

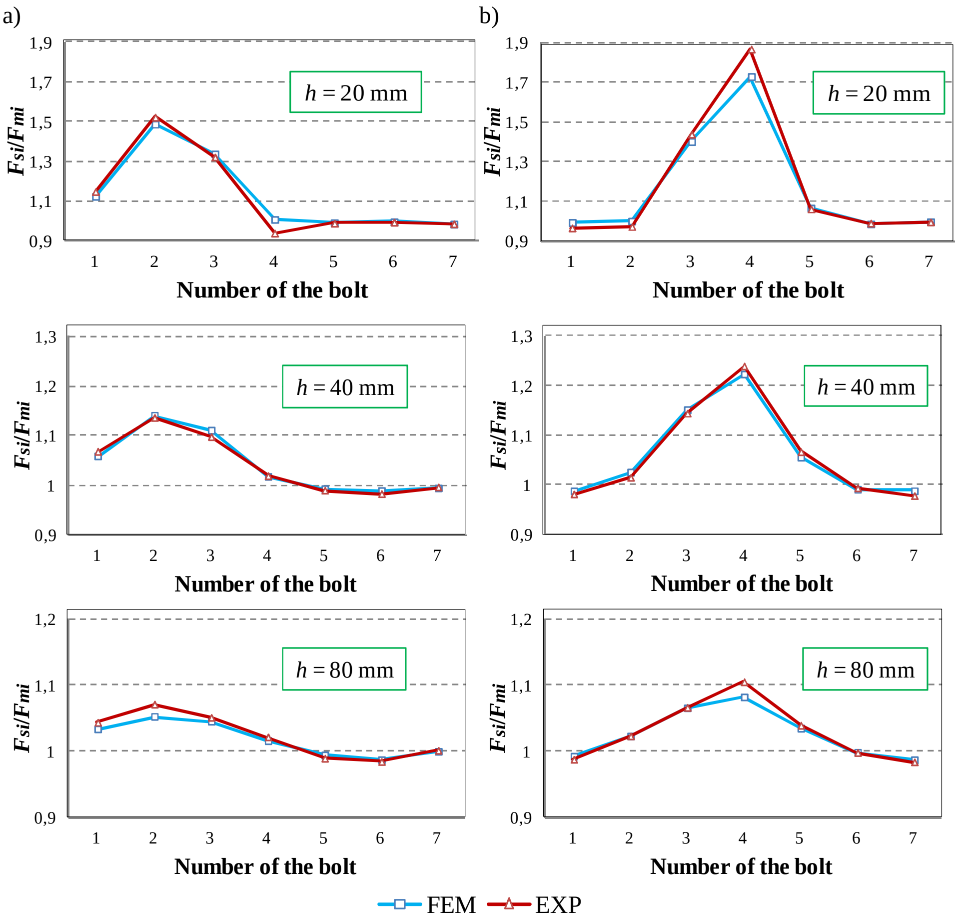

Fig. 5. Bolt load values in the joint loaded external at the point: a) I, b) II

In most of the cases, computed values of forces in individual bolts are smaller than their experimental values [1]. Analysis of relative difference between obtained bolts forces is done on the basis of the *W* index

$$W = \left| \frac{F_{\mathrm{s\,max}}^{FEM} - F_{\mathrm{s\,max}}^{EXP}}{F_{\mathrm{s\,max}}^{EXP}} \right| \cdot 100\% \tag{12}$$

where: $F_{s\,max}^{FEM}$ is the maximal force in bolts according to the FEM-model of the joint and $F_{s\,max}^{EXP}$ is the maximal force in bolts according to experimental research [1].

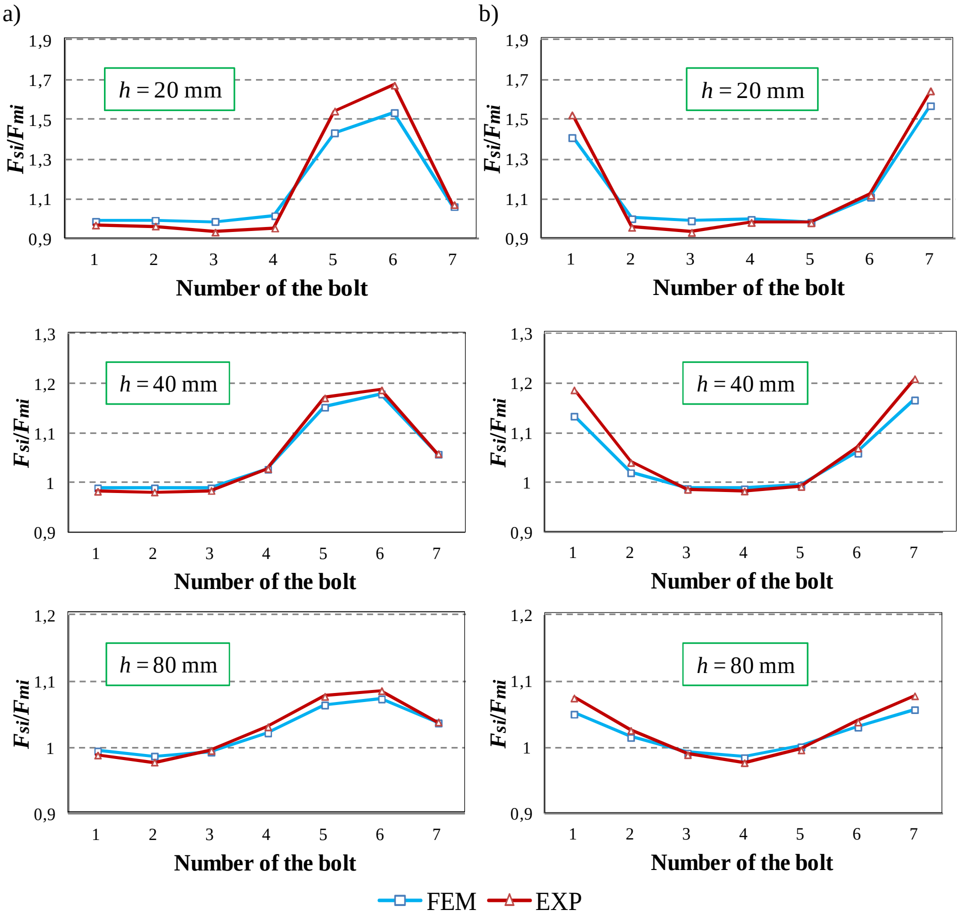

Fig. 6. Bolt load values in the joint loaded external at the point: a) III, b) IV

W index values as a function of the thickness of the joined element are set up in Table 1. On the grounds of comparisons, it can be noted that the accuracy of results of calculations increases with reduction of joined element's flexibility. Depending on the joined element's thickness, the error of estimation may range from 8.24 to 2.07%.

4. Conclusions

The presented model of the preloaded bolted flange connection subjected to an external normal load can be successfully used in load analysis of any joint in which a flexible flange element is connected with a rigid support. The model can be modified to carry out analysis of the bolted flange connection loaded by an arbitrary external force. It is practicable by applying a contact model including also the tangential stiffness to the proposed multi-bolted system.

Table 1. *W* index values as a function of the joined element's thickness

Joined element's thickness			
Accuracy index W, %	***h* = 20 mm**	***h* = 40 mm**	***h* = 80 mm**
	Point No. I		
	2.31	0.40	1.73
	Point No. II		
	7.44	1.30	2.07
	Point No. III		
	8.24	0.66	1.15
	Point No. IV		
	4.56	3.55	1.93

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