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FACE GEAR WITH AN INVOLUTE TEETH LINE IN APPLICATIONS FOR CLUTCH CONNECTIONS

Summary: The paper presents methods of forming a face gear with an involute teeth line in application for clutch connections. The method is based on application of a disc tool instead of application of a hob. The solution is possible after application of the CNC milling machine with five axis numeric system control. The method of forming a face gear proposed in this article is based on application of the disc tool with a single blade edge and CNC milling machine with a special program. The special program is used for control technology of cutting face gear. The algorithm and geometrical model for calculation of a modification of an involute teeth line of a face gear have been shown in the paper.

Key words: face-gear, involute teeth line

1. INTRODUCTION

Development of the face gear forming technology used in clutch connections is focused on decreasing production costs while ensuring a high quality of the connection. Apart from traditional applications of the face gear, that have the task of transferring torque in a disjunctive clutch connection, they are more and more often used in precise indexing devices.

Face gear can be classified according to the type of the teeth line that was cut in the gear. The most popular clutch connections have gears with straight modified teeth lines [3,7,10,11,12]. There are also clutch connections with involute and arc-circular teeth lines [1, 2, 4-6, 8, 10, 13, 14].

Face gear with an involute teeth line can be cut with conical or cylindrical hob cutter [12-14]. There is also a possibility of cutting an involute teeth line

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with the use of a single blade tool. The authors of the paper [14] point out that there are big problems with sharpening hobbing cutters with small radius blades used for cutting face gear and hob scale error that are transferred into the gear.

The development tendency of cutting face gear with an involute teeth line is to use numerically controlled machine tools and tools with carbide blades. First studies of forming possibilities of cutting teeth with an involute line were carried out by the author [10] with the application for spiroid gear.

Clutch connections may have different type of an involute teeth line and are used (normal, elongated, shortened) with a modification of the teeth line and profile, in order to avoid edge contacts. All those involute teeth lines in face gear variations can be cut with the use of a disc tool on CNC machine tools.

2. TOOL ORIENTATION DURING FORMING NORMAL AND ELONGATED INVOLUTE IN FACE GEAR

In order to avoid edge contact in the clutch with an involute teeth line (fig. 1), proper setting of the tool and the direction of its movement is used during teeth line formation. The tool and its movement should be selected in such way, to achieve the required teeth line modification, allowing to avoid edge contact.

Avoiding edge contact in a clutch with an involute teeth line of face gears is possible only in the cases of face gear with involute line modification.

The general principle of modified involute teeth line of a face gear is to set the conformity of the track described by the edges of blade's curve radius and the teeth line of the face gear in the calculate point and with rolling from a ring with a smaller diameter [5,6]. The selection of the radius of the ring, that circumscribes the involute, has influence on the modification size of the line of the face gear (the smaller the rolling ring, the bigger the modification).

Exemplary orientation of the disc tool on the space of machine tool in relation to the face gear installed on the rotation table were presented in fig. 1 [2,6].

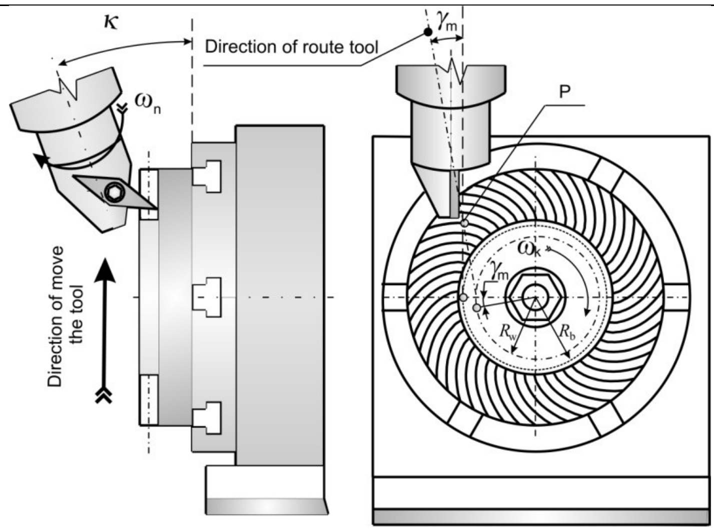

Fig. 1. Tool orientation during chasing of gearing with a left direction of inclination resulting from the requirement of obtaining a common normal of the trace defined by the cutting blade's edges and a gearing teeth line [2,6], R_b - *theoretical rolling radius,* R_w - *radius of generation a circle tooth line of face-gear,* ω_k *– rotational speed of face gea,* ω_n *– rotational speed of tools ,* γ_m *–twist angle of the spindle,* κ *– inclination angle of the spindle to face gear,* P *– settings point tools*

The formation of the other cooperating face gear of the clutch connection is executed by cutting the involute teeth line that is a mirror reflection of the face gear. Figure 2 shows the way of cutting face gear with a right direction of teeth line inclination with a twisted tool head (fig. 2a) and with skew movement, without a twisted tool head (fig. 2b).

During formation, the machine tool's control system couples the spindle rotations with the table's rotations with the face gear in order to ensure continuous index. The roll movement (forming of the involute teeth line) is implemented through connecting the SN rotation table movement with the movement of the tool along the axis of the face gear in a rotated coordinate system by an angle γ_m in the plane of the face gear. The rotation of the machine tool's coordinates, with the use of standard machining cycles, is carried out in order to ensure a common normal of the trace marked by the cutting blade's edges and the face gear teeth line. The angle γ_m depends on the type of the assumed involute (γ_m – positive values for elongated involute, negative for shortened involute and zero in the case of normal involute). A universal, numerically controlled milling machine, equipped with a twist tool head, allows any inclination of the tool axis towards the machined rim.

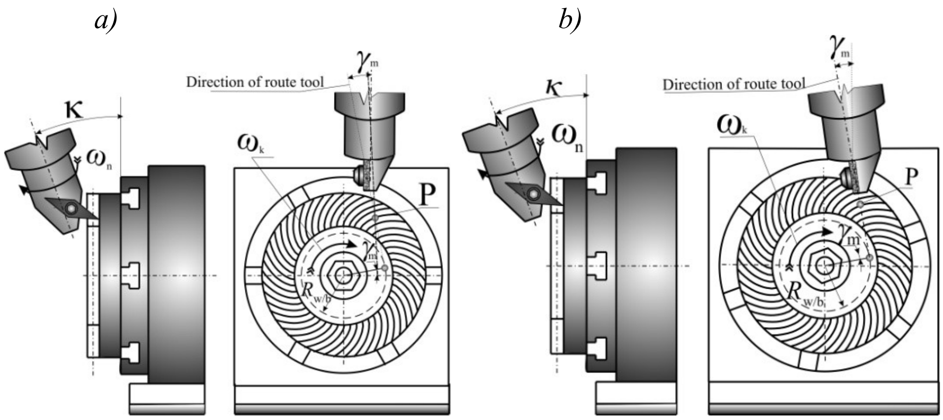

Fig. 2. Tool orientation during cutting gearing with right teeth line inclination,
R_b - theoretical rolling radius, R_w - radius of generation a circle tooth line of face-gear, ω_k – rotational speed of face gea, ω_n – rotational speed of tools , γ_m – twist angle of the spindle, κ – inclination angle of the spindle to face gear, P – settings point tools

During forming the involute teeth line, it is possible to select: tool movement track, inclination direction of the teeth line and the tool axis twist in the system connected with the machined rim on the tooth tip plane. Possible ways of cutting the involute line in a face gear were listed in figure 3.

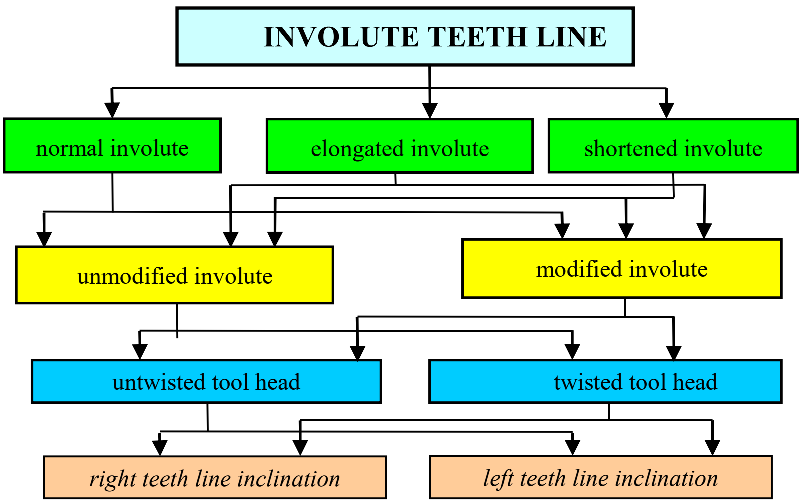

Fig. 3. Possibilities of forming face gear with involute teeth line with the use of a disc tool

3. DESIGNING FACE GEARS WITH AN INVOLUTE GEAR LINE

Geometrical models were developed and mathematical relations were set with the matrix-vector method for calculating surfaces of face gear teeth working together in a clutch with an involute teeth line.

The equations of the face gear's side teeth surface have been derived on the basis of a geometrical model with a part independent of the teeth line (calculative profile, cutting blade's edge trace) and model dependent of the teeth line [3]. Geometric dependencies have been presented in the form of parametric equations. The models have a cylindrical coordinate system (R_v, Φ) connected with the formed teeth-line and an unmovable in regard to the tool axis rectangular coordinate system (x, z). Face gear rotation axis was assumed as the centre of the coordinate system [3].

It was assumed that the forming of the face gear will be executed on a numerically controlled milling machine with the use of a single blade disc tool with a continuous indexing method. In this case, the profile of the gearing tooth's groove and the trace of the tool's cutting edge depends only on the shape of the blade which the teeth will be formed. These features allow to determine a common geometric model independent of the teeth line [3].

3.1. Calculation profile of face gear done by disc tool

The complex movement during formation of a face gear with the continuous indexing method, causes a widening of the face gear grooves. The groove depth in the case of teeth formed blade disc tool can be determined on the basis of figure 4 [3,5]:

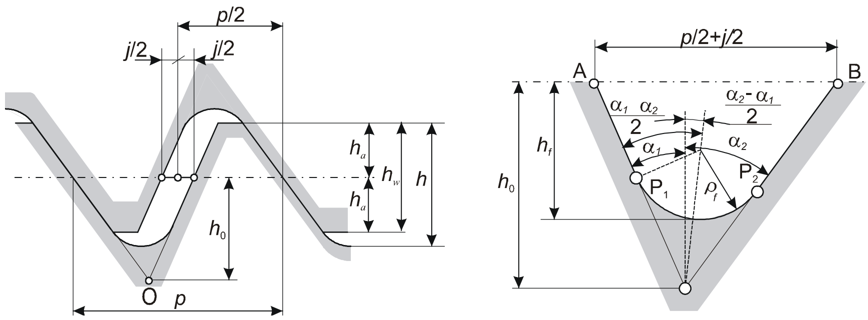

Fig. 4. A geometrical model of the calculation profile of the face-gear done with a disc tool, a) calculation profiles of two face gear in nominal meshing b) increased profile of the tooth root [3]

There are 3 dependencies resulting from the figure:

– nominal pitch

$$p = \frac{2 \cdot \pi \cdot R}{z}, \tag{1}$$

– AB distance on the split line

$$AB = h_0 \cdot tg\alpha_1 + h_0 \cdot tg\alpha_2 = \frac{p}{2} + \frac{j}{2}, \tag{2}$$

– the theoretical height of the tooth root

$$h_0 = \frac{\pi \cdot R}{z} \cdot \frac{1 + \frac{j}{p}}{tg\alpha_1 + tg\alpha_2} \tag{3}$$

Profiles of the clutch coupling may be in contact only in the area formed by the straight edge of the blade, i.e. limited by points P_1 and P_2 (fig. 4).

For meshing with symmetrical profiles $\alpha_1 = \alpha_2 = \alpha$ and for face gear without slack take the form of:

– height of the tooth root

$$h_0 = \frac{\pi \cdot R}{2 \cdot z \cdot tg\alpha}, \tag{4}$$

– height of the tooth tip

$$h_a \leq h_f - \rho_f \cdot (1 - \sin\alpha), \tag{5}$$

– tooth height (groove depth)

$$h = h_a + h_f. \tag{6}$$

3.2. Tool forming an involute line

An important issue during cutting involute face gear is to get a correct shape of the trace marked by the disc tool's cutting edge.

The structure of the tool and its setting/movement in regard to the face gear should ensure a common normal surface of formed teeth and a trace marked by the cutting tool's edges on the split plane of the face-gear. Figure 4 shows shape variants of traces marked by the cutting edge that have been placed in the groove of a face-gear with an involute line on a split plane. The gearing's teeth line is formed by points on the outer profile of the trace in its central part (fig. 5b, points P_1, P_2).

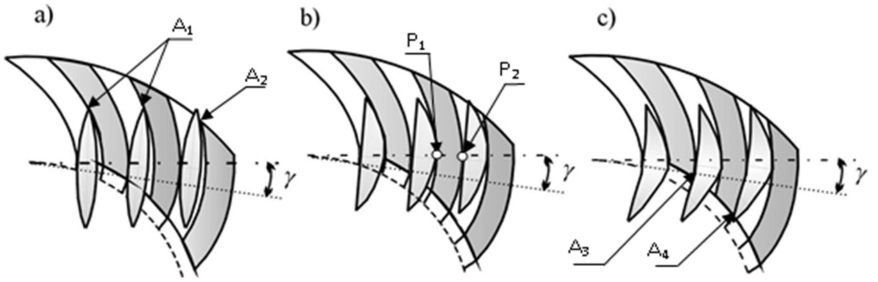

Fig. 5. Possible types of traces marked by cutting tool edges with blade profile angles: a) symmetrical, b), c) skew

It can be noticed in figure 5a, that the trace of the cutting edge contacts the tip of the blade with a concave tooth surface (point A_1). There is also a contact between the outer edge of the face gear diameter and the tool trace in point A_2. The tool trace should contact the teeth line only in its centre part (have a common normal), what ensures a correctly shaped teeth line. Keeping the same gearing geometry and using an unsymmetrical edge trace of the cutting blade, as presented in figure 5b and 5c, allows to obtain a contact area between the trace centre of the blade determined by the edges with a concave side of the face gear. Using a faulty design of the tool may lead to the convex side of face gear's tooth be formed by the blade's edges, what was shown in figure 5c. In point A_3 in figure 5c, the inner diameter of the face gear edge contacts the trace of the blade that will cut it during the process of chasing the teeth line. In point A_4 the blade edge contacts the convex gearing side, which is the start of tooth chasing. Further movement of the tool from the middle of the face gear will cause an increase cut in the teeth line, thus creation of a teeth line execution error.

3.3. Tool trace coordinates described by the cutting edge

The coordinates of the cutting edge trace of the face gear with an involute line, formed with a disc tool can be calculated by the model presented in figure 6 [3].

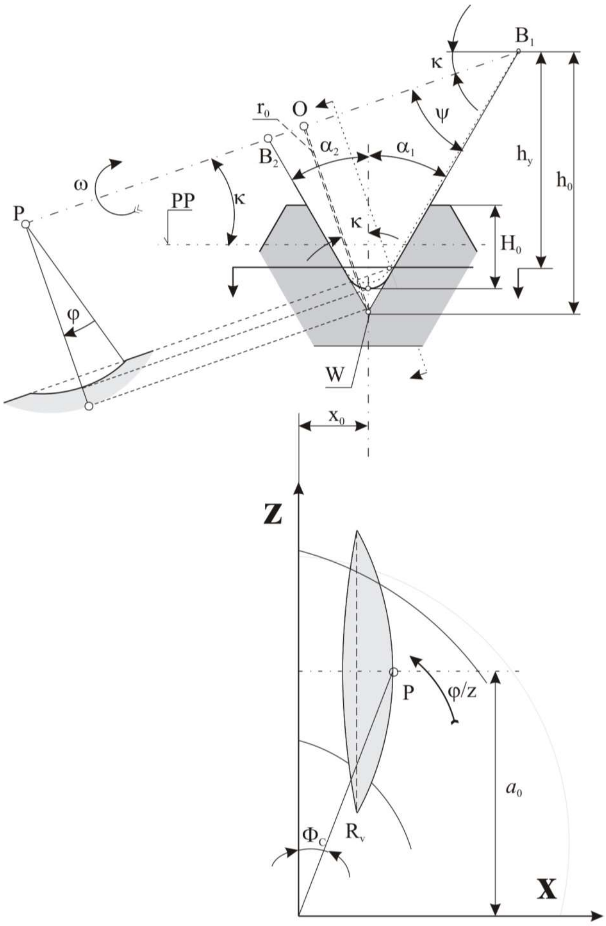

Fig. 6. The geometrical model for calculating trace coordinates of the blade's cutting edge in a fixed system (x, z) or (R_v,Φ_C) – connected with the turning table plate

The figure shows a dependency:

$$h_0 = r_0 \cdot \frac{\cos\alpha}{\cos(\alpha+\kappa)}. \tag{7}$$

In order to set a point coordinate of the formed edge trace, an auxiliary variable was introduced:

$$\psi = arc\,tg\frac{\cos\varphi}{tg(\alpha+\kappa)}, \tag{8}$$

and then, in a fixed coordinate system:

$$\left.\begin{aligned} x &= x_0 + h_0 \cdot tg\alpha - \frac{h_y}{tg(\kappa+\psi)} \\ z &= a_0 - \frac{h_y \cdot tg\varphi \cdot \sin\psi}{\sin(\kappa+\psi)} \end{aligned}\right\} . \tag{9}$$

In a fixed polar system (R_v, Φ_C) (connected with a turning table plate) the trace coordinates are derived from a system of equations:

$$\left.\begin{aligned} R_v &= \sqrt{x^2+z^2} \\ \Phi_c &= arc\,tg\frac{x}{z} \end{aligned}\right\} . \tag{10}$$

To set the equations for envelopes of a cutting edge trace family, emerging during face gear formation, function derivatives were set, for which the equations are presented below:

$$\frac{\partial\psi}{\partial\varphi} = -tg\varphi \cdot \sin\psi \cdot \cos\psi\,, \tag{11}$$

and

$$\left.\begin{aligned} \frac{\partial x}{\partial\varphi} &= \frac{h_y}{\sin^2(\kappa+\psi)} \cdot \frac{\partial\psi}{\partial\varphi} \\ \frac{\partial z}{\partial\varphi} &= -\frac{h_y}{\sin(\kappa+\psi)} \cdot \left(\frac{\sin\psi}{\cos^2\varphi} + \frac{\partial\psi}{\partial\varphi} \cdot \frac{\sin\kappa}{\sin(\kappa+\psi)}\right) \end{aligned}\right\} . \tag{12}$$

Figure 7 shows a geometrical model for calculating rectangular and polar coordinates of cross-sections of a face gear with an involute line. It is a modified model based on the work [3].
The figure shows a relation for a momentary radius vector:

$$r_x \cdot \cos\varphi \cdot \cos\kappa - r_x \cdot \cos\kappa \cdot tg(\alpha_1+\kappa) = h_y\,. \tag{13}$$

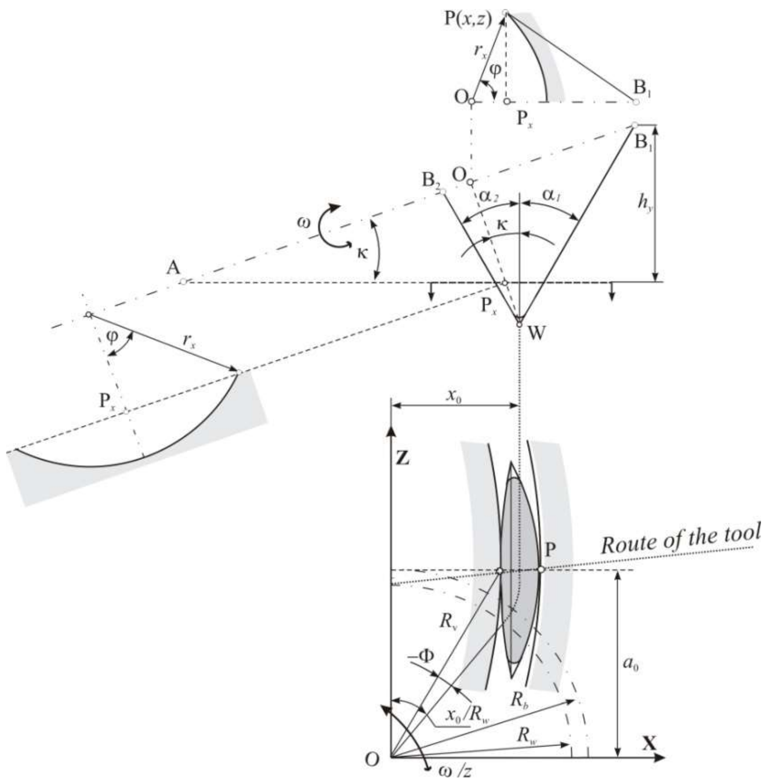

Fig. 7. The geometrical model for calculating modification of rectangular and polar coordinates of the cross-section in of face gear with involute line

In a system of coordinates (R_v,Φ_c) after considering a continuous indexing and system of equations (10) take the form:

$$\left.\begin{aligned} R_v &= \sqrt{x^2 + z^2} \\ \Phi &= \operatorname{arc\,tg}\frac{x}{z} - \frac{k}{z_k}\varphi - \frac{x_0}{R_w} \end{aligned}\right\}, \tag{14}$$

where:

k - number specifying involute type (k = +1 in case of shortened involute and k = -1 in case of elongated involute),

z_k - number of teeth of the face gear

The equations (14) after variation of the x_0 parameter describe a family of curves, where the envelope of the teeth line is obtained by adding the following condition:

$$\begin{vmatrix} \dfrac{\partial R_v}{\partial \varphi} & \dfrac{\partial \Phi}{\partial \varphi} \\ \dfrac{\partial R_v}{\partial x_0} & \dfrac{\partial \Phi}{\partial x_0} \end{vmatrix} = 0\,. \tag{15}$$

Calculation of derivatives in (15) gives determinant elements (16)

$$\left.\begin{aligned} &\frac{\partial R_v}{\partial \varphi} = \frac{1}{R_v} \cdot \left(\frac{\partial x}{\partial \varphi} \cdot x + \frac{\partial z}{\partial \varphi} \cdot z \right) \\ &\frac{\partial \Phi}{\partial \varphi} = \frac{1}{R_v^2} \cdot \left(\frac{\partial x}{\partial \varphi} \cdot z - \frac{\partial z}{\partial \varphi} \cdot x \right) - \frac{k}{\mathrm{z_k}} \\ &\frac{\partial R_v}{\partial x_0} = \frac{x}{R_v}; \quad \frac{\partial \Phi}{\partial x_0} = \frac{z}{R_v^2} - \frac{1}{R_w} \end{aligned}\right\}. \tag{16}$$

After conversion, cancellation and simplification the teeth line equation takes the form:

$$\left(\frac{\partial z}{\partial \varphi} + \frac{k}{\mathrm{z_k}} \cdot x \right) \cdot R_w - \frac{\partial x}{\partial \varphi} \cdot x - \frac{\partial z}{\partial \varphi} \cdot z = 0\,. \tag{14}$$

Modification Δ was defined as the distance point of the curve obtained during cutting the teeth of the teeth line from the involute line (fig. 8).

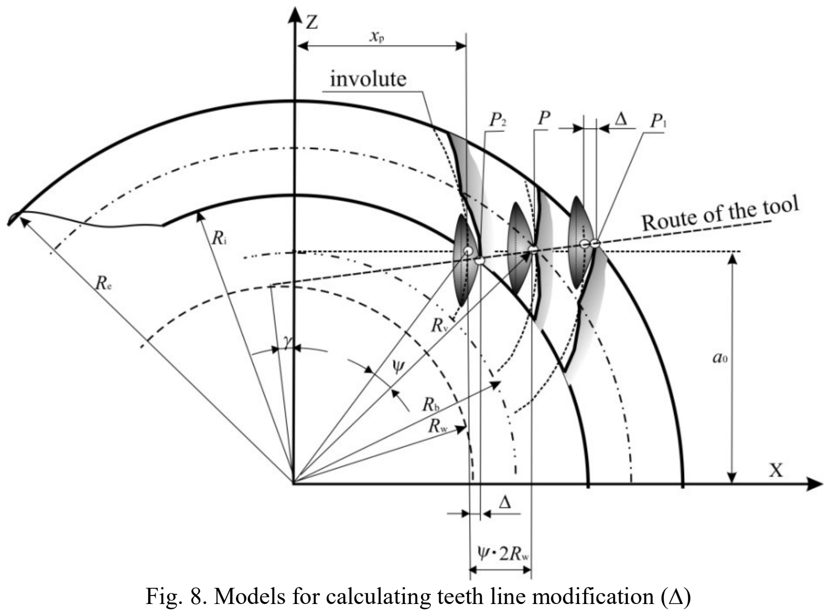

Fig. 8. Models for calculating teeth line modification (Δ)

According to the above relations and on the basis of figure 8 we can write:

$$\Delta = \sqrt{R_v^2 - R_0^2} - x_p - \frac{k}{z_k} \cdot R_w \cdot \varphi . \tag{18}$$

The depth of the modification takes the value Δ = 0 at the calculation point in the middle of the face gear; the tool rotation angle at this point φ = 0 and the envelope condition is met, which can be written as:

$$x_{\Delta=0} = r_0 \cdot \frac{z_u}{k} \cdot \frac{h_y}{h_0} \cdot \left(\frac{R_0}{R_w} - 1 \right). \tag{19}$$

The teeth line is formed as an envelope of cutting edges' traces of disc tool. Designing too small inner diameter of the face gear may cause it to be formed by blade tips, what is illustrated in figure 5. In order to avoid shearing the concave side of the tooth, a limit radius of the tool was set (on the side forming the concave side).

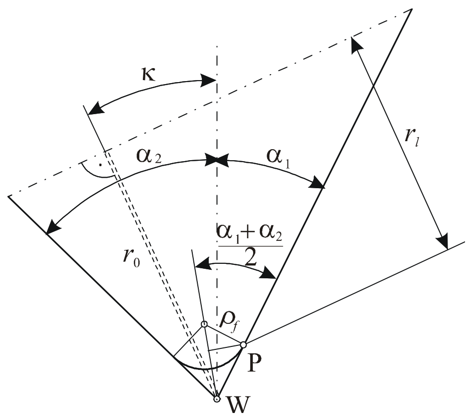

Fig. 9. Geometric model for determining the limit radius r_l of the rectilinear part of the cutting edge [3]

On the basis of fig 9, we can write:

$$r_l = r_0 - \rho_f \frac{\cos(\alpha_1 + \kappa)}{tg \frac{\alpha_1 + \alpha_2}{2}} \tag{20}$$

After substituting formula 13 with the r_l radius (instead of r_x) we get a relation for the rotation limiting angle of the tool φ_l, for which the cutting edge will not be chasing the tooth's concave side:

$$\varphi_l = arc\cos\left[\frac{h_y}{r_l \cdot \cos\kappa} - tg(\alpha_1 + \kappa)\right]. \tag{21}$$

Calculating the modification is possible only for specific numeric date. The algorithm of these calculations is presented in fig. 10 and 11 on the basis of elaborations [4]. The iterative loop predicates the examination of the volatility of certain sizes (e.g. x = var – YES/NO).

Start

Dane: R_0, z, α_1, α_2, κ, $t_{\alpha\kappa}$= tg($\alpha_{1/2} + \kappa$), x_0, x_B, k, h_y, φ

CONDITION

$$\psi = \mathrm{arc\,tg}\frac{\cos\varphi}{t_{\alpha\kappa}};\; S_{\alpha\kappa} = \sin(\kappa+\psi);\; d_{\psi\varphi} = tg\cdot\sin\psi\cdot\cos\psi;$$

$$x = x_0 - \frac{h_y}{tg(\kappa+\psi)};\; z = R_0 - \frac{h_y\cdot tg\varphi\cdot\sin\psi}{S_{\kappa\psi}};\; R_z + \sqrt{x^2+z^2};$$

$$\frac{\partial x}{\partial\varphi} = \frac{h_y}{S_{\kappa\psi}^2}\cdot d_{\psi\varphi};\qquad \frac{\partial y}{\partial\varphi} = -\frac{h_y}{S_{\kappa\psi}}\cdot\left(\frac{\sin\psi}{\cos^2\varphi} + d_{\kappa\varphi}\cdot\frac{\sin\kappa}{S_{\kappa\varphi}}\right)$$

Result: $F = \left(\frac{\partial y}{\partial\varphi} + \frac{k}{z}\cdot x\right)\cdot R_w - \frac{\partial x}{\partial\varphi}\cdot x - \frac{\partial y}{\partial\varphi}\cdot y$; ($x$, z, R_z)

Fig. 10. Algorithm for calculating the envelope condition (theoretical teeth line)

Fig. 10 presents an algorithm for calculating the theoretical teeth line as a procedure repeated at several points of an algorithm calculating the teeth line modification. The calculation results are: value of the left side of the equation (3.17), coordinates (x, z) of the blade trace point and the R_v distance from that point to the gear centre.

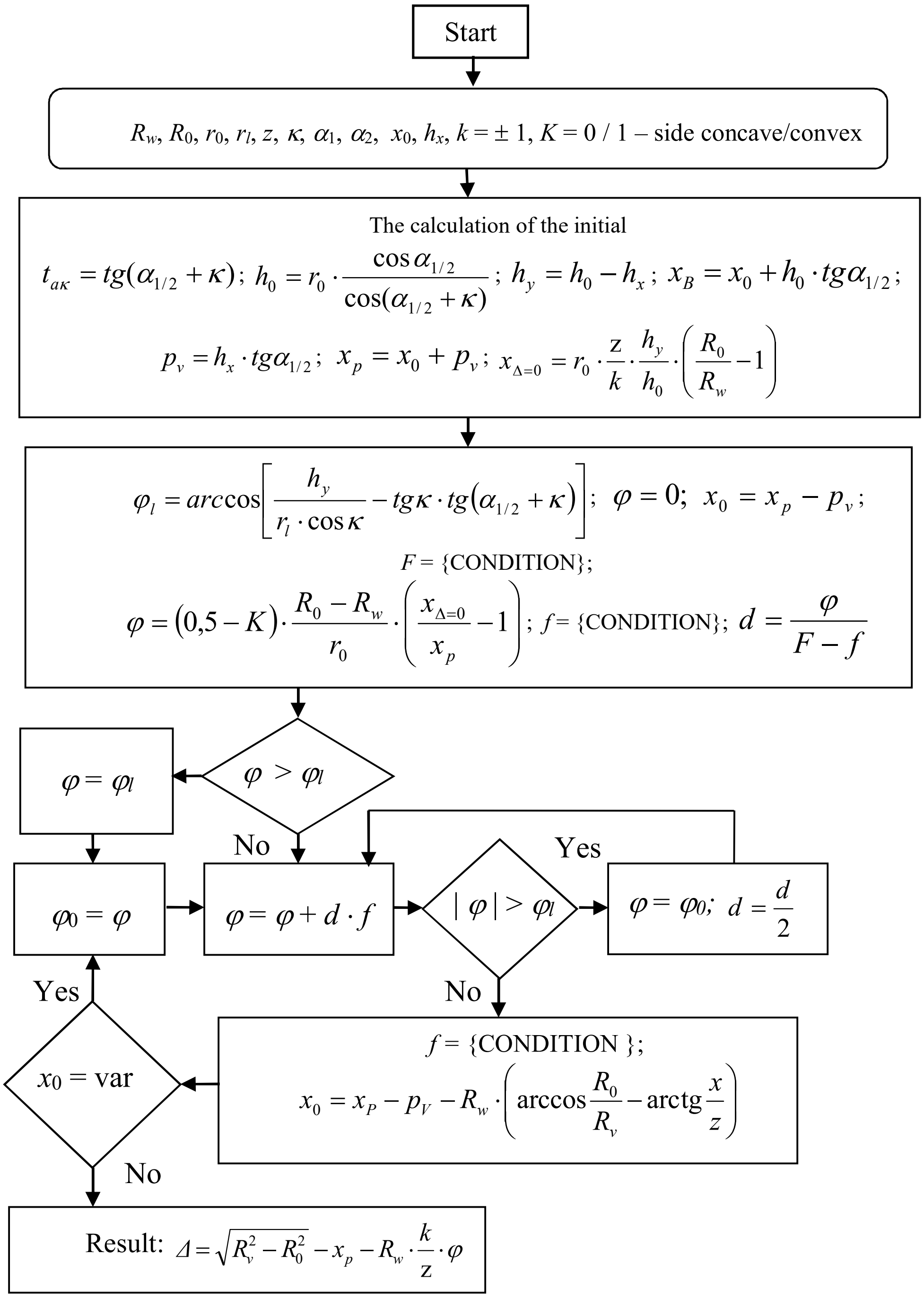

Fig. 11. Algorithm for calculating face gear teeth line modification

4. EXAMPLES OF CUTTING FACE GEARS WITH AN INVOLUTE GEAR LINE

Attempts to shape a face gear were conducted on the milling machine FYN – 50Nd type, equipped with numerically controlled rotary table. The milling machine has been equipped with the control system of the TNC 407 type of the Heidenhain.

Heidenhain 407 controller enables simultaneous interpolation in three axes (linear or circular in three dimensional space). Steering of processing of the outline is held with digital speed control. Servo systems in each axis are in position of regulated type, controlled by deviation signals. The feed axis X, Y, Z and A are carried out by four independent pulse-controlled AC motors. The drive of the spindle is equipped with a system for continuous variable speed transmission. In the axis of the spindle of the milling machine a rotational-pulse sensor was fastened, which signals are transmitted to the control system of the machine tool what allows to control spindle as a rotational axis (C).

Theoretical principles of a teeth forming, which are presented in this paper have been confirmed by experimental researches. The view of the exemplary face gear after cutting on CNC milling machine is presented in fig. 12.

a) *b)*

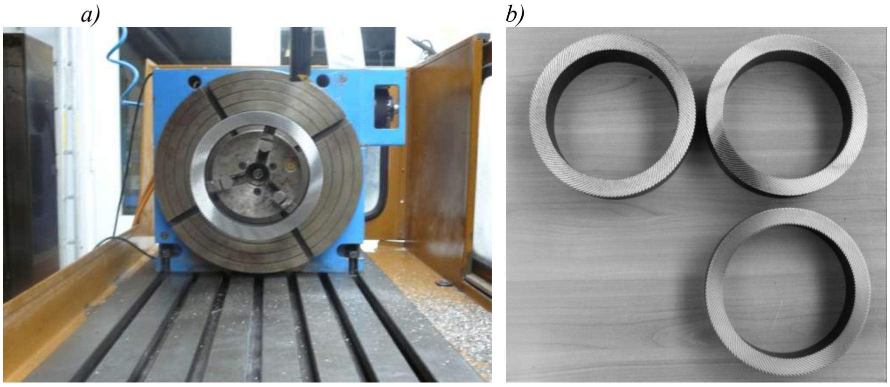

Fig. 12. View: a) investigations stand for cutting a face gear, b) example cutting teeth line in the face gear

Design parameters of a face gear

Inner radius of the face gear R_i	– 99,5 mm
Outer radius of the face gear R_e	– 136 mm
Number of teeth of the face gear z_k	– 90
Tooth height (groove depth) h	– 4,37 mm
Twist angle of the spindle γ_m	– 2,339°

concave profiles angle of the face gear α_1 – 5 °
convex profiles angle of the face gear α_2 – 30°
distance from the axis of the tool axis of face gear – ±78 mm

5. CONCLUSIONS

The presented algorithm for calculation of the modification of a face gear teeth line allows to calculate only one half of the clutch connection. Designing a clutch connection basing on two face gears, we need to consider the sum of modification of the teeth lines working together in the connection.

Presented possibilities of cutting face gear with an involute teeth line allow to execute them on milling machines with different kinematics.

Discussed methods of forming gear teeth are based on contemporary machining offered by modern CNC machine tools. Technological solutions of cutting teeth of gear take advantage of parametrised control program for moving working units of the CNC machine tool.

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UZĘBIENIA CZOŁOWE O EWOLWENTOWEJ LINII ZĘBÓW W APLIKACJACH DO POŁĄCZEŃ SPRZĘGŁOWYCH

S t r e s z c z e n i e

W artykule przedstawiono metody kształtowania uzębień czołowych o ewolwentowej linii zębów z przeznaczeniem do połączeń sprzęgłowych. Przedstawiona metoda bazuje na zastąpieniu frezu ślimakowego narzędziem krążkowym. Kształtowanie tych uzębień jest możliwe na pięcio osiowej frezarce CNC, przy użyciu specjalnego programu. Program sterujący pracą obrabiarki nadzoruje technologię nacinania uzębienia czołowego. W pracy przedstawiono algorytm i model geometryczny do obliczania modyfikacji linii ewolwentowej uzębienia czołowego.

Słowa kluczowe: uzębienie czołowe, ewolwentowa linia zęba