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METHOD OF IDENTIFICATION OF EQUIVALENT PARAMETERS OF THE FRACTIONAL-ORDER TRANSFORMER

The paper presents a method for identifying the parameters $L_{\beta 1}$, β_1, $L_{\beta 2}$, β_2, M_γ, γ of a fractional-order transformer. This method is based on the measurement of the phase resonance frequency in a few systems containing: the investigated fractional-order transformer and two standard capacitors. The dependencies allowing the determination of the fractional-order transformer parameters have been given, on the basis of the described measurements. The obtained results are illustrated by an example.

Keywords: fractional-order mutual inductance, fractional-order parameters identification, phase resonance.

1. INTRODUCTION

There are many works devoted to the analysis of systems with fractional-order elements, eg [1–5], whereas only a few works on the realization of the fractional-order elements L_β, C_α have appeared yet. There are two main methods of the realization of fractional-order elements.

The first method bases on the physical realization of these elements, using the physicochemical properties of the materials used in their construction:
– electrolytes and dielectrics, in case of supercapacitors [6, 7],
– soft ferromagnetic materials, in case of coils [8].

Due to the limited choice of materials, the values of fractional-order coefficients α and β (see formulas (1) and (2)) are not arbitrary, their values are within the range $\alpha, \beta \in \langle 0,1 \rangle$.

The second method is based on the realization of electronic circuits implementing impedance transformations of C_α, L_β elements, obtained by the previous method. Electronic active systems, such as gyrator or the generalized GIC impedance converter [9], are used for this purpose.

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There is also a separate group of methods of the realization of fractional-order elements, which uses approximations of time or frequency models with *RC*-ladder circuit structures [10].

The subject of this paper concerns the first group of the original fractional-order elements and the problem of their models parameters identification.

The simplest models of the fractional-order capacitor and fractional-order coil, in frequency domain, are given by the formulae:

$$Z_C(s) = \frac{U(s)}{I(s)} = \frac{1}{s^{\alpha} C_{\alpha}}, \tag{1}$$

$$Z_L(s) = \frac{U(s)}{I(s)} = s^{\beta} L_{\beta}, \qquad \alpha, \beta \in \mathrm{R}, \tag{2}$$

where: L_β – pseudoinductance, C_α – pseudocapacitance, α, β – fractional-order parameters (dimensionless). The names: pseudoinductance and pseudocapacitance, result from the units of these quantities. It is not henr and farad as in the case of classic inductance and capacitance, but $\mathrm{Hs}^{(1-\beta)}$ and $\mathrm{F/s}^{(1-\alpha)}$.

The existence of fractional-order elements creates the need and necessity to identify their parameters L_β, β, C_α, α.

Most often, these parameters are determined using the frequency characteristics of fractional-order elements [11–12]. The modified method of determining these parameters relies on the determination of voltage transmittance amplitude for three different values of the high-pass filter frequency, realized using supercapacitor [13]. It is also possible to determine these parameters based on the analysis of time waveforms in transient states [14], eg. in supercapacitor charging process, or its response to a unit voltage step [15–16].

To determine the frequency characteristics of supercapacitors and the parameters of their fractional-order models, the electrochemical impedance spectroscopy (EIS) method is often used [11–12]. The advantage of this method is the high accuracy of the impedance module and phase determination, also for very low frequencies ($f < 0.01$ Hz). The disadvantage, unfortunately, is the high cost of the apparatus of this method. While there are quite a lot of works on the fractional-order L_β, C_α elements, there are still few works on the fractional-order mutual inductance [17–19]. In [17] the concept of fractional-order mutual inductance has been introduced, its main assumptions and properties have been described. In [18] the electromagnetic Maxwell equations of the fractional-order mutual inductance have been analyzed and presented. The wireless power transmission system has been modeled as a fractional-order mutual inductance system in [19]. The existence of the fractional-order mutual inductance (fractional-order transformer) implies the need and necessity to determine its model parameters. In [20] a method for identifying the parameters of a fractional-order inductance L_β with an iron core was proposed, which involves the approximation of the time response to the unit voltage step using the least squares method.

This paper presents a proposition of different method of the fractional-order parameters $L_{\beta 1}$, β_1, $L_{\beta 2}$, β_2, M_γ, γ identification of the mutual inductances, based on the phenomenon of phase resonance in a series circuit of the class $RL_\beta C_\alpha$.

2. FRACTIONAL-ORDER MUTUAL INDUCTANCE MODEL

It is similar to the model of classic magnetically-coupled coils [21]. The system of fractional-order mutual inductance is shown in Fig. 1.

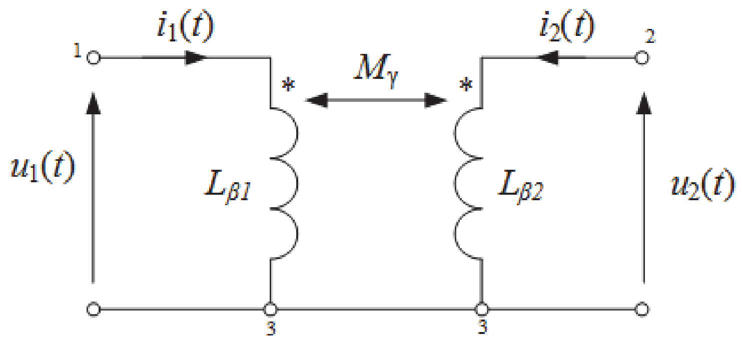

Fig. 1. Model of a fractional-order magnetically-coupled coil system connected in a common node of series-aiding connection

In frequency domain, this model can be represented as (Fig. 2):

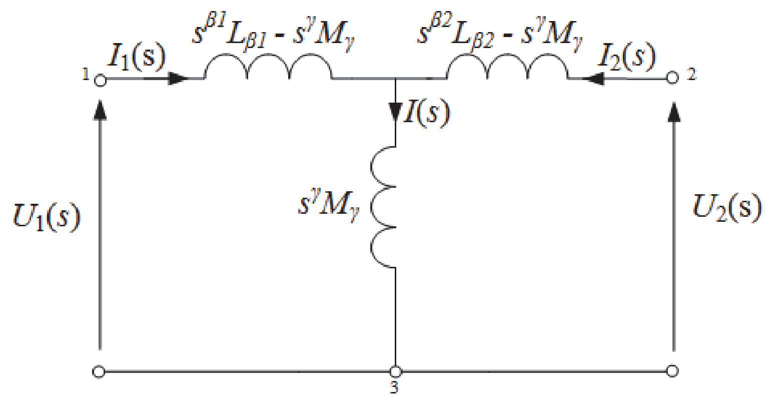

Fig. 2. Equivalent model of fractional-order transformer without magnetic couplings

The method proposed in the paper is based on the properties of the $RL_\beta C_\alpha$ series circuit, which is discussed below.

It should be noted that the unit of both pseudoinductance $L_{\beta 1}$, $L_{\beta 2}$ and mutual pseudoinductance M_γ is not henr, as in the case of classic inductance, but $H\cdot s^{(1-\beta)}$.

3. PHASE RESONANCE IN SERIES $RL_\beta C_\alpha$ CIRCUIT

For the circuit, as in Fig. 3, the input impedance is defined by a formula:

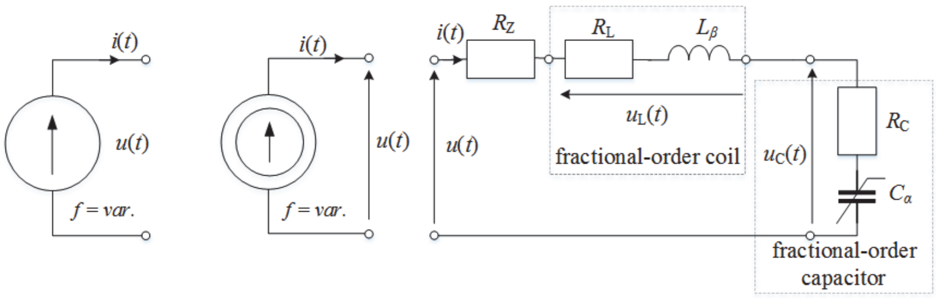

Fig. 3. Series circuit of the class $RL_\beta C_\alpha$

$$Z(\mathrm{j}\omega) = R + (\mathrm{j}\omega)^{\beta} L_\beta + (\mathrm{j}\omega)^{-\alpha} C_\alpha^{-1}, \tag{1}$$

which can be presented in a form:

$$\begin{aligned} Z(\mathrm{j}\omega) = &\left(R + \omega^{\beta} L_\beta \cos\left(\frac{\beta\pi}{2}\right) + \omega^{-\alpha} C_\alpha^{-1} \cos\left(\frac{\alpha\pi}{2}\right)\right) + \\ &+\mathrm{j}\left(\omega^{\beta} L_\beta \sin\left(\frac{\beta\pi}{2}\right) - \omega^{-\alpha} C_\alpha^{-1} \sin\left(\frac{\alpha\pi}{2}\right)\right) \end{aligned} . \tag{2}$$

where:

$$R = R_Z + R_L + R_C . \tag{3}$$

From the general phase resonance conditions:

$$\mathrm{Im}\{Z(\mathrm{j}\omega)\} = 0 , \tag{4}$$

and:

$$\mathrm{Im}\{Y(\mathrm{j}\omega)\} = 0 , \tag{5}$$

where: $Y(\mathrm{j}\omega)$ – the input admittance of the system, the identical relation describing the resonance frequency results, which can be defined as [22]:

$$f_{\mathrm{rezf}} = \frac{1}{2\pi} \sqrt[\alpha+\beta]{\frac{1}{L_\beta C_\alpha} \frac{\sin\left(\frac{\pi}{2}\alpha\right)}{\sin\left(\frac{\pi}{2}\beta\right)}} . \tag{6}$$

The analysis of the formula (6) shows, that not for all the parameters α and β the phase resonance exists, which has been illustrated in Fig. 4, for exemplary parameters values $L_\beta = 1\ \mathrm{Hs}^{(1-\beta)}$, $C_\alpha = 0.1\ \mathrm{F/s}^{(1-\alpha)}$.

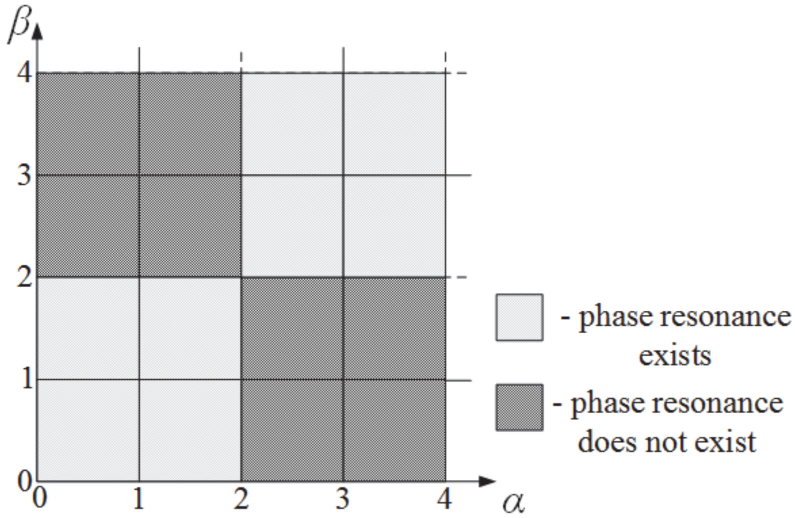

Fig. 4. Conditions of the phase resonance existence for the sets of α and β parameters

It can be noticed, that in specific cases:

1. $\alpha = \beta$:

$$f_{\mathrm{rezf}} = \frac{1}{2\pi}\sqrt[2\alpha]{\frac{1}{L_\alpha C_\alpha}}\,. \tag{7}$$

2. $\alpha = \beta = 1$:

$$f_{\mathrm{rezf}} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}\,. \tag{8}$$

In the specific case of $\alpha = \beta = 1$ the equation (6) is reduced to the classic case of an integer-order *RLC* circuit.

4. IDENTIFICATION METHOD FOR THE PARAMETERS OF FRACTIONAL-ORDER MUTUAL INDUCTANCE

The proposed method consists of several stages:

I. For supplying the system, shown schematically in Fig. 5, from a sinusoidal voltage source and with the secondary terminals open, its equivalent model can be represented as follows:

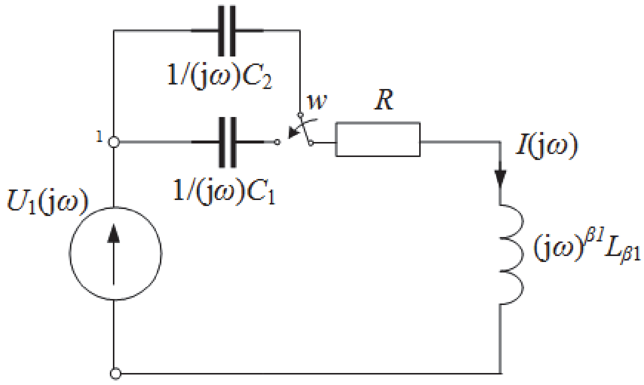

Fig. 5. The equivalent circuit of the system for measuring the parameters $L_{\beta 1}$ and β_1 of the transformer primary winding

The circuit from Fig. 5 is supplied from a source of adjustable sinusoidal voltage. Then it should be brought to phase resonance state, which will occur when the phase shift between the voltage measured on the series connection of L_β, C_1 elements and the flowing current will equal zero. The next step is to repeat the same measurement for a different capacitance C_2 value. From the phase resonance condition for the frequencies f'_{01}, f'_{02} (radial frequencies ω'_{01}, ω'_{02}) the following equations result [7–8]:

$$\omega'^{\beta}_{01} L_\beta \sin\left(\frac{\beta\pi}{2}\right) = \frac{1}{\omega'_{01} C_1}, \tag{9}$$

$$\omega'^{\beta}_{02} L_\beta \sin\left(\frac{\beta\pi}{2}\right) = \frac{1}{\omega'_{02} C_2}. \tag{10}$$

from which, after transformations, the parameters β_1, $L_{\beta 1}$ can be determined:

$$\beta_1 = \ln\left(\frac{\omega'_{02} C_2}{\omega'_{01} C_1}\right) \Big/ \ln\left(\frac{\omega'_{01}}{\omega'_{02}}\right), \tag{11}$$

$$L_{\beta 1} = \left(\omega'^{\,1+\frac{\ln(\omega'_{02} C_2/\omega'_{01} C_1)}{\ln(\omega'_{01}/\omega'_{02})}}_{01} C_1 \sin\left(\frac{\ln(\omega'_{02} C_2/\omega'_{01} C_1)}{\ln(\omega'_{01}/\omega'_{02})} \frac{\pi}{2} \right) \right)^{-1}. \tag{12}$$

The symbol ω' indicates the resonance frequencies in the system from Fig. 5 and the symbol ω'' the frequencies of the system from Fig. 6.

II. Acting identically for the secondary winding of the transformer, the equivalent circuit for the measurement of the fractional-order parameters is as follows:

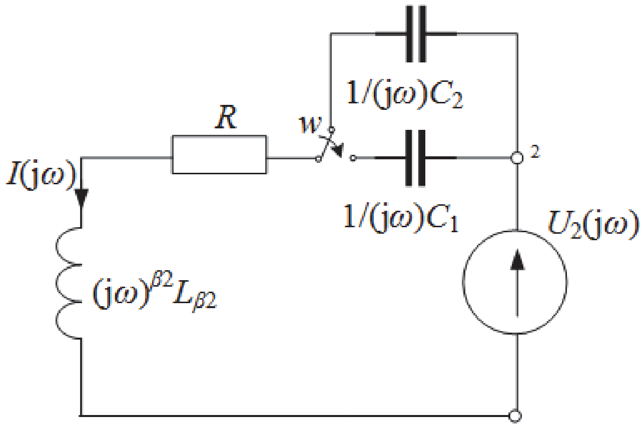

Fig. 6. The equivalent circuit of the system for measuring the parameters $L_{\beta 2}$ and β_2 of the transformer secondary winding

Also in this case, the circuit from Fig. 6 should be brought into a phase resonance state in two connections with capacitances C_1, C_2. Similarly, by writing two equations describing the phase resonance frequencies (9) and (10), for the switching circuit from Fig. 6, the searched fractional-order parameters values can be determined for the secondary winding of the transformer.

Parameters β_2, $L_{\beta 2}$ are determined according to the following formulae:

$$\beta_2 = \ln\left(\frac{\omega''_{02} C_2}{\omega''_{01} C_1}\right) \Big/ \ln\left(\frac{\omega''_{01}}{\omega''_{02}}\right), \tag{13}$$

as well as:

$$L_{\beta 2} = \left(\omega''^{\,1+\frac{\ln\left(\omega''_{02} C_2 / \omega''_{01} C_1\right)}{\ln\left(\omega''_{01}/\omega''_{02}\right)}}_{01} \, C_1 \sin\left(\frac{\ln\left(\omega''_{02} C_2 / \omega''_{01} C_1\right)}{\ln\left(\omega''_{01}/\omega''_{02}\right)} \frac{\pi}{2} \right) \right)^{-1}. \tag{14}$$

III. The third step is the determination of the fractional-order parameters γ, M_γ of the mutual inductance. It is possible to determine them by performing two measurements of the input impedance at the series-aiding and opposite-aiding connection of the fractional-order coupled coils. The equivalent circuits of the measuring systems are shown in Figs. 7 and 8.

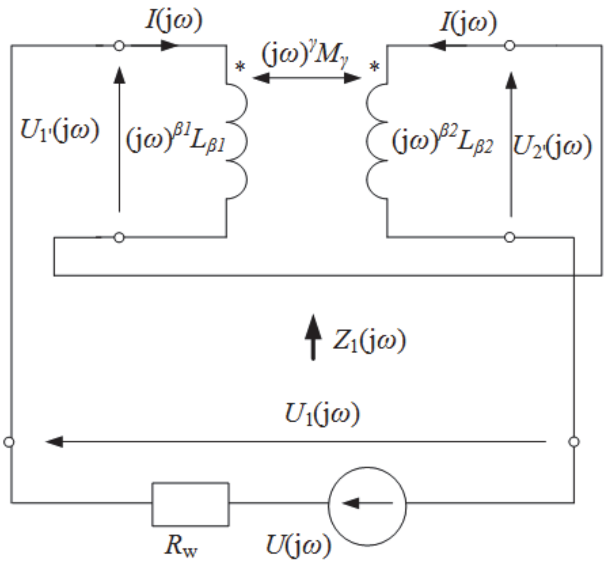

Fig. 7. The model of the circuit for the equivalent input impedance measurement of the series-aiding connection of the fractional-order mutual inductance

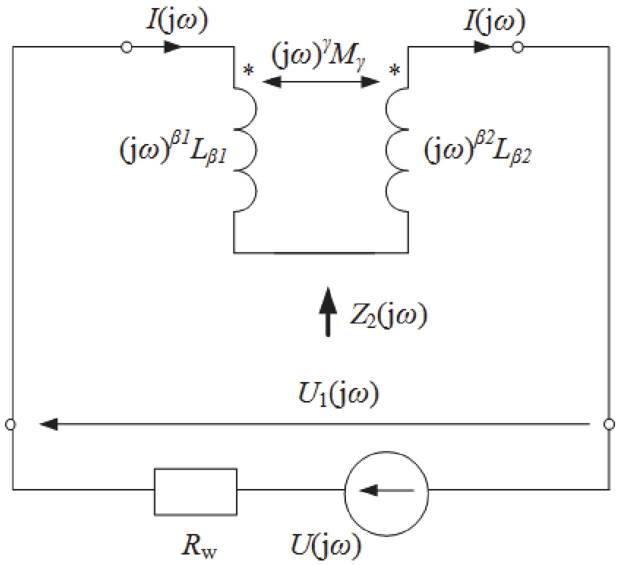

Fig. 8. The model of the circuit for the equivalent input impedance measurement of the opposite-aiding connection of the fractional-order mutual inductance

In both cases, the RMS values of the voltage $U_1(\mathrm{j}\omega)$ and the current $I(\mathrm{j}\omega)$ in the system should be measured (see Fig. 7, 8). It is also necessary to measure the phase shift between the voltage $U_1(\mathrm{j}\omega)$ and the current $I(\mathrm{j}\omega)$. These measurements in the system from Fig. 7, 8, should be performed for the same value of the radial frequency. The impedances $Z_1(j\omega)$ and $Z_2(j\omega)$ can be determined from these measurements, as:

$$Z_1(j\omega) = R + (j\omega)^{\beta 1} L_{\beta 1} + (j\omega)^{\beta 2} L_{\beta 2} + 2(j\omega)^{\gamma} M_{\gamma}, \tag{15}$$

while in the opposite-aiding connection of the coils:

$$Z_2(j\omega) = R + (j\omega)^{\beta 1} L_{\beta 1} + (j\omega)^{\beta 2} L_{\beta 2} - 2(j\omega)^{\gamma} M_{\gamma}, \tag{16}$$

where: R – the equivalent resistance of the series connection of the coil resistances.

The measurements described above should be performed twice, for two radial frequencies ω_1 and ω_2.

Then from dependencies:

$$\frac{Z_1(j\omega_1) - Z_2(j\omega_1)}{Z_1(j\omega_2) - Z_2(j\omega_2)} = \left(\frac{\omega_1}{\omega_2}\right)^{\gamma}, \tag{17}$$

the fractional-order parameter γ can be determined:

$$\gamma = \log_{\left(\frac{\omega_1}{\omega_2}\right)} \left(\frac{Z_1(j\omega_1) - Z_2(j\omega_1)}{Z_1(j\omega_2) - Z_2(j\omega_2)} \right), \tag{18}$$

while, the value of mutual pseudoinductance M_γ can be determined by substituting the determined value of the coefficient γ to one of the impedance differences for one of the frequency cases and written as:

$$M_{\gamma} = \frac{\left| Z_1(j\omega_1) - Z_2(j\omega_1) \right|}{4\omega_1^{\gamma}}. \tag{19}$$

As a result, by performing three measurement steps, it is possible to determine all the six fractional-order parameters $L_{\beta 1}$, β_1, $L_{\beta 2}$, β_2, M_γ, γ. The full model of the fractional-order transformer contains the resistance of the primary and secondary windings too. In order to simplify the considerations, these resistances were omitted, because they do not affect the presented measurement algorithm. These resistances can be determined by a DC bridge.

The described algorithm has been illustrated with a simulation example.

5. EXAMPLE

On the basis of the proposed method, a case of the fractional-order mutual inductance (fractional-order transformer) has been analyzed. Three measurement steps have been performed to determine the searched values of fractional-order parameters.

I. Parameters $L_{\beta 1}$, β_1 of the transformer primary windings:

For two capacitors of known capacitances C_1 = 10 mF, C_2 = 3,53 mF in the examined circuit, as in Fig. 5, two resonance frequency values f'_{01} = 100 Hz, f'_{02} = 200 Hz were recorded. From relations (11) and (12), the searched values of fractional-order parameters result:

$$\beta_1 = 0{,}503\,, \tag{20}$$

$$L_{\beta 1} = 8{,}813\,\mathrm{mH}\cdot \mathrm{s}^{(1-\beta)}\,. \tag{21}$$

II. Parameters $L_{\beta 2}$, β_2 of the transformer secondary windings:

Again, for the same values of capacitors, with the same capacitances of C_1 = 10 mF, C_2 = 3,53 mF, measurements were taken, bringing the circuit from Fig. 6 to a phase resonance state. The frequency phase resonance detector recorded frequencies: f''_{01} = 200 Hz and f''_{02} = 400 Hz.

From dependencies (13) and (14), the searched values of the fractional order parameters were determined:

$$\beta_2 = 0{,}502\,, \tag{22}$$

$$L_{\beta 1} = 3{,}113\,\mathrm{mH}\cdot \mathrm{s}^{(1-\beta)}\,. \tag{23}$$

III. Parameters M_γ, γ of the magnetic coupling:

Input impedance measurements for two connections of fractional-order mutual inductance were performed for two frequencies: f_1 = 100 Hz, and f_2 = 200 Hz.

The input voltage of the system has been set to $U_1(\mathrm{j}\omega)$ = 1 V.

The obtained values of currents and calculated values of input impedances of the series- and opposite-aiding connection of magnetically-coupled coils $Z_{\mathrm{in}}(\omega)$ are summarized in Table 1.

Table 1. Measured and determined parameters in the fractional-order mutual inductance M_γ measurements.

Magnetic coupling connection	$f_{1,2}$	U_1	I	Z_1 (jω)	Z_2(jω)
	Hz	V	mA	Ω	Ω
Series-aiding	100	1 $e^{j0°}$	220,1 $e^{-j67,3°}$	4,54 $e^{j67,3°}$	-
	200		130,9 $e^{-j67,5°}$	7,64 $e^{j67,5°}$	-
Opposite-aiding	100	1 $e^{j0°}$	251,5 $e^{j109,4°}$	-	3,98 $e^{-j109,4°}$
	200		146,3 $e^{j109,4°}$	-	6,84 $e^{-j109,7°}$

From dependencies (18) and (19), the searched values of fractional-order parameters have been determined:

$$\gamma = 0{,}765\,, \tag{24}$$

$$L_{\beta 1} = 1{,}540\,\mathrm{mH}\cdot \mathrm{s}^{(1-\beta)}\,. \tag{25}$$

6. SUMMARY

The paper proposes a method for identifying $L_{\beta 1}$, β_1, $L_{\beta 2}$, β_2, M_γ, γ parameters of a fractional-order transformer. This method is based on the measurement of the phase resonance frequency in a measurement stand containing the analyzed transformer and two switchable standard capacitors. The dependencies allowing the determination of the fractional-order transformer parameters have been given, on the basis of the described measurements. The obtained results are illustrated by a theoretical example of a magnetically-coupled coils system.

REFERENCES

[1] Elwakil A.S., Fractional-order circuits and systems: an emerging disciplinary research area, IEEE Circuits Syst. Mag, vol. 10, no. 4, 2010, pp. 40–50.

[2] Bošković M. Č., Šekara T. B., Lutovac B., Daković M., Mandić P. D., Lazarević M. P., Analysis of electrical circuits including fractional order elements, 6th Mediterranean Conference on Embedded Computing (MECO), Bar, 2017, pp. 1–6.

[3] Jakubowska-Ciszek A., Walczak, J., Analysis of the transient state in a parallel circuit of the class $RL_\beta C_\alpha$, Applied Mathematics and Computation, vol. 319, 2018, pp. 287–300.

[4] Sowa M., A harmonic balance methodology for circuits with fractional and nonlinear elements, Circuits, Systems & Signal Processing, vol. 37, no. 11, 2018, pp. 4695–4727.

[5] Majka Ł., Applying a fractional coil model for power system ferroresonance analysis, Bul. Pol. Acad. Sci., Tech. Sci., vol. 66, no 4, 2018, pp. 467–474.

[6] Rafik F., Gualous H., Gallay Y., Frequency, thermal and voltage supercapacitor characterisation and modelling, Journal of Power Sources, Vol. 165, March 2007, pp. 928–934

[7] Burke A., Ultracapacitors, why, how and where is the technology, Journal of Power Sources, Vol. 91, 2000, pp. 37–50.

[8] Schafer J., Kruger K., Modeling of coils using fractional derivatives, Journal of Magnetism and Magnetic Materials, Vol. 307, 2006, pp. 91–98.

[9] Tripathy M.Ch., Mondal D., Biswak K., Sen S., Experimental studies on realization of fractional inductors and fractional-order bandpass filters, International Journal on Circuit Theory and Application, no. 43, 2015, pp. 1183–1196.

[10] Shi L., Comparison of ultracapacitor electric circuit models, IEEE Power and Energy Society General Meeting, Pittsburgh, 20-24 July, 2008, pp. 1–6.

[11] Barsali S., Ceraolo M., Frequency dependent parameter model of supercapacitors, Measurement, Vol. 43, 2010, pp. 1683–1689.

[12] Cuadras A., Ovejas V.A., Supercapacitor impedance in time and frequency domains, Proc. 9th Int. Multi-Conf. on Systems, Signals and Devices, 2012, pp. 1–6.

[13] Maundy B.J., Elwakil A., Freeborn T., Allagui A., Improved method to determine supercapacitor metrics from highpass filter response, 28th International Conference on Microelectronics (ICM), 2016, pp. 25–28.

[14] Freeborn T.J., Maundy B., Elwakil A.S., Measurement of supercapacitor fractional-order model parameters from voltage-excited step response, IEEE Journal on Emerging and Selected Topics in Circuits and Systems, vol. 3, no. 3, 2013, pp. 367–376.

[15] Freeborn T.J., Maundy B.J., Elwakil, A.S., Accurate time domain extraction of supercapacitor fractional-order model parameters, IEEE International Symposium on Circuits and Systems ISCAS2013, 2013, pp. 2259–2262.

[16] Maundy, B.J., Elwakil, A., Freeborn, T., Allagui, A., Determination of supercapacitor metrics using a magnitude-only method, IEEE Int. Symp. on Circuits and Systems (ISCAS), 2016, pp. 1186–1189.

[17] Soltan, A., Radwan, A.G., Soliman A.M., Fractional-order mutual inductance: analysis and design, International Journal on Circuit Theory and Applications, Vol. 44, No. 1, 2015, pp. 85–97.

[18] Liang G., Hao J., Shan D., Electromagnetic interpretation of fractional-order elements, Journal of Modern Physics, vol. 8, 2017, pp. 2209–2018.

[19] Shu X., Zhang B., The effect of fractional orders on the transmission power and efficiency of fractional-order wireless transmission system, Energies, Vol. 11(1774), Issue 7, 2018, pp. 1–9.

[20] Xia P., Liang S., A parameter identification method for fractional-order inductance of iron core reactor, 1st International Conference on Information Technology, Information Systems and Electrical Engineering (ICITISEE), 2016, Yogyakarta, Indonesia, pp. 31–36.

[21] Hasegawa K., Wada K., Omura I., Mutual inductance measurement for power device package using time domain reflectometry, IEEE Energy Conversion Congress and Exposition (ECCE), WI, 2016, pp. 1–6.

[22] Walczak J., Jakubowska A., Phase resonance in series $RL_{\beta}C_{\alpha}$ circuit, Przegląd Elektrotechniczny, ISSN 0033 - 2097, r. 90, vol. 4/2014, pp. 210–213.

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