Electrical system elements aggregation for parallel computing in real-time digital simulators

Sławomir Cieślik
University of Technology and Life Sciences
85-796 Bydgoszcz, ul. Kaliskiego 7, e-mail: slavcies@utp.edu.pl

Above article is about the issues of constructing digital real-time simulators of electrical systems. A characteristic thing of this type of simulators is the possibility of direct cooperation with physical devices, such as regulators etc. It is expected to obtain the results of numerical computations for a given level of adequacy in a particular time. To allow perform parallel calculations, it is appropriate mathematical models of electric system have to be used. We can achieve this using decomposition of the mathematical models of electrical systems at the simulator design stage. To accelerate calculations it is recommended to use mathematical models of aggregated electrical system elements. In this article we derived the mathematical model of the aggregated linear sample of three-phase element, which is used afterwards in the simulator of linear electrical system sample. Experiment results with the use of digital platform with 6-core processor have been also covered in this paper.

KEYWORDS: digital simulation of electrical systems, real-time simulation, mathematical model decomposition, parallel computation

1. Introduction

Continuous development of microprocessor and computer technology is conducive to the evolution of the simulation techniques and tools used in the electrical engineering (for example science research, electrical engineer education process, industry and operation). For the last several years, reduction of costs of computer technology has been observed, while at the same time computing processor systems performance has increased. We have also noticed rapid development in the field of numerical computation accelerators. For example using processors dedicated for graphics processing are performed numerical computations [4]. These are just some of the aspects, which allow to applying simulation tools (in particular based on personal computers classic platforms) for analyzing of complex problems in short time. Among these tools are digital simulators of electrical systems, which are operate in real time (ESRTDS). This type of simulators can be used in cooperation with the physical device (e.g. digital regulator) in real time. These issues are discussed in book [3] on the basis of which has been written this paper.

Due to the extensive use of personal computers, digital platforms based on them are important group of ESRTDS. The main advantages of use general purpose processors [8], compared with the use of specialized processors are lower costs and faster improvement of their technology.

In modern personal computers use of multi-core processors is a standard. Paper [9] describes the parallelization of an optimal power flow with is solved with a multi-core processor. An important aspect of parallel computations is algorithm scalability, which is the ability to increase the acceleration as the number of processors (cores) increases. Scalability reflects algorithm ability for effective use of resources consisting of great number of CPUs (cores) [6, 11]. According to the authors of the paper [9], the scalability can be assumed to be good in the case of 2 cores and acceptable in 4 cores, with the efficiency use of cores respectively 70% and 44% (2011 publication). It is not reasonable to use parallel computations for mathematical models of small sized electrical systems. In the book [3] there is an example of mathematical models of electric systems with a relatively small size, used in the ESRTDS, where it is appropriate to apply parallel computations upon prior decomposition of the model.

In the paper [10], it is determined that implementation of graphics processor unit (GPU) for parallel integration of differential system of ordinary equations allows to increase the speed of calculations even 115 times in comparison with sequential calculations and a 15 times faster compared to calculations on multi-core processors. The problem of variable length tasks in threads (execution on the GPU) of the same block is highlighted. In the consequence of that we can propose the specific way to connect multiple data allocation for each thread in order to reduce frequency of unwanted calculations.

It is intentional to search for new methods of computer simulation of electrical systems operation (electromagnetic and electromechanical processes) that will generate results for a certain level of precision and in precisely defined period of time. The main subject of our research is mathematical model of electrical systems, which allow to performing parallel calculation. This is achieved at the constructing stage of simulator using a decomposition of the mathematical models of electrical systems. To accelerate the computations (especially in the algorithm part, where the systems of algebraic equations are solved), it is suggested to apply mathematical models of the aggregated elements of this system. In this paper we derived the mathematical model of the aggregated linear three-phase element sample, which is used in the simulation of linear electrical system sample. We can find there also results of the experiment with the use of the six-core CPU digital platform.

2. Operation mode of the real-time digital simulator of electrical systems

We can describe a system, which operate in real-time as a computer digital platform, where the results of the calculations depend on the data generated by the environment, as well as on the time. Basing on the IEEE standard [5, 12], we can define the term of real-time simulator as a digital platform, where computations are concurrent with external process (the environment) in order to control and supervision or to react for process events.

Figure 1 shows a time-scale sample of three computational tasks implementation in single step of integration with the use of three different methods: 1 – calculations of all tasks are performed sequentially, 2 – calculations of one task which involve system of equations solving are performed in parallel, and the calculation of the other two tasks are carried out sequentially and 3 – calculations of all tasks are performed in parallel.

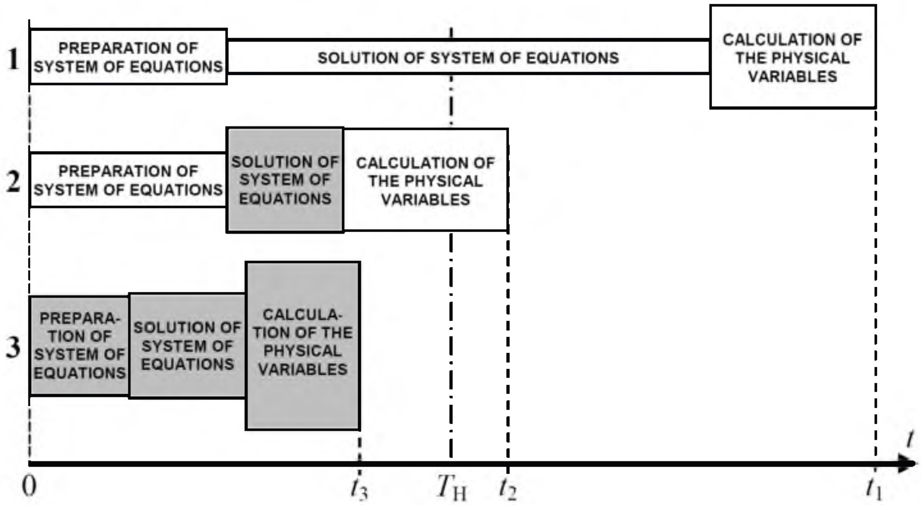

Fig. 1. The time-scale for implementation of three computations in single step of integration with three different methods (this is described on the text)

Specific quality of real-time simulators is the data exchange with the environment, which takes place in a strictly determined time slots. It is assumed that an input and output signals values are „locked" in memory units in determined time slots. Data exchange with the environment is carried out with the specified constant clock frequency, which is called data exchange frequency between simulator and environment. We can directly deduce T_H simulator operational time slice (Fig. 1). To allow the deadline events reaction, simulator's numerical calculations have to be performed in shorter amount of time than simulator operational time slice. In respect of that, the software step integration of differential equations h was introduced. The differential equations occurred in mathematical model are solved with this software step h. Obviously the software step integration of differential equations has to be equal to simulator operational time slice ($h = T_H$). Simulator operates effectively in terms of events on time response, when mathematical model of integrated equations with determined software step h are solved in amount of time t_R, which has to be shorter than simulator's operational time slice ($t_R < T_H$).

According to the time-scale showed on Fig.1 we can deduct, that only 3rd method allow effective simulator operation in terms of events on time response ($t_R = t_3 < T_H$, $t_R = t_1 > T_H$, $t_R = t_2 > T_H$). Important aspect of the calculations is the precision of the results. These ESRTDS issues are discussed in book [3].

This paper presents a mathematical model of aggregated element of electrical system, which together with the decomposition of the model of that system (for parallel computation) allows to reducing the number of equations included in mathematical system. From the implementation of computational processes point of view certain electrical system nodes are moved to the interior of aggregated element, so it decreases the amount of equations in the mathematical system, which is solved on every step of integration (The task of „solution of system of equations" showed in Figure 1). A measurable effect of that is reduction of time required for solving system of equations.

3. Simulator of the electrical system operation

3.1. Example of the electrical system

Figure 2 shows a scheme of an electrical system as a part of a medium-voltage electric power distribution network, which was chosen as an example. This system was analyzed by the author in his earlier works [1, 3]. Examined system consists following elements: electrical power system as a substitute generator with transformer, load at medium voltage, electrical power cables and overhead lines, switches and capacitors as reactive power compensation.

Figure 3 shows an equivalent circuit for an example the electrical system. Modelled electrical system consists of twenty-five structural elements SE1-SE25. For such selected structural elements in analyzed electric system there are 39 explicit electric nodes.

Each six-pole has six external nodes and three external currents (corresponding pairs of external nodes of multipoles are connected one branch). Theoreticaly, each node has a different value of the electric potential. In two structural elements (SE24 and the SE25) two variables per one phase occur, and in the remaining 23 elements only one variable per one phase occur. A detailed description of the structural elements is provided in [3]. There are 81 variables in electrical system, and each one of them has to be calculated in every step of integration.

Our task is to construction a simulator, which can test the transient and steady state in electrical system sample (Fig. 2). Designed simulator has to cooperate also with physical external devices. Therefore the simulator operational time slice has to be equal to 0,2 ms.

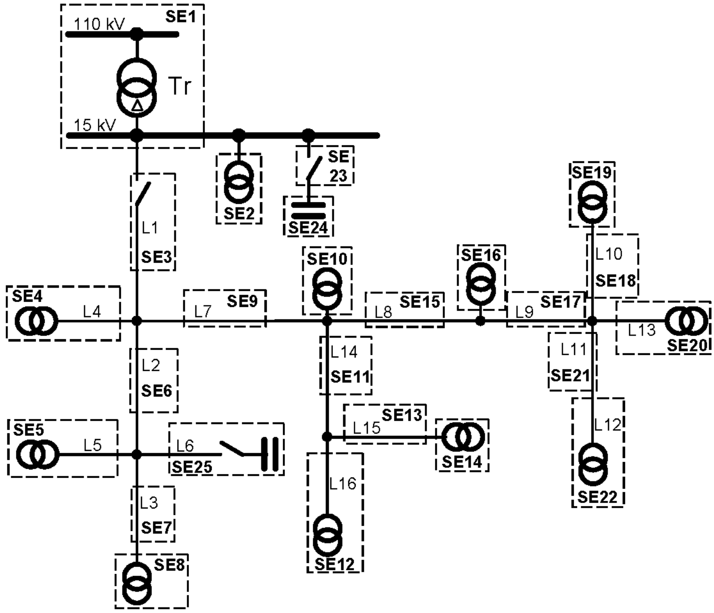

Fig. 2. Scheme of electrical system sample as a part of medium-voltage electric power distribution network [1]

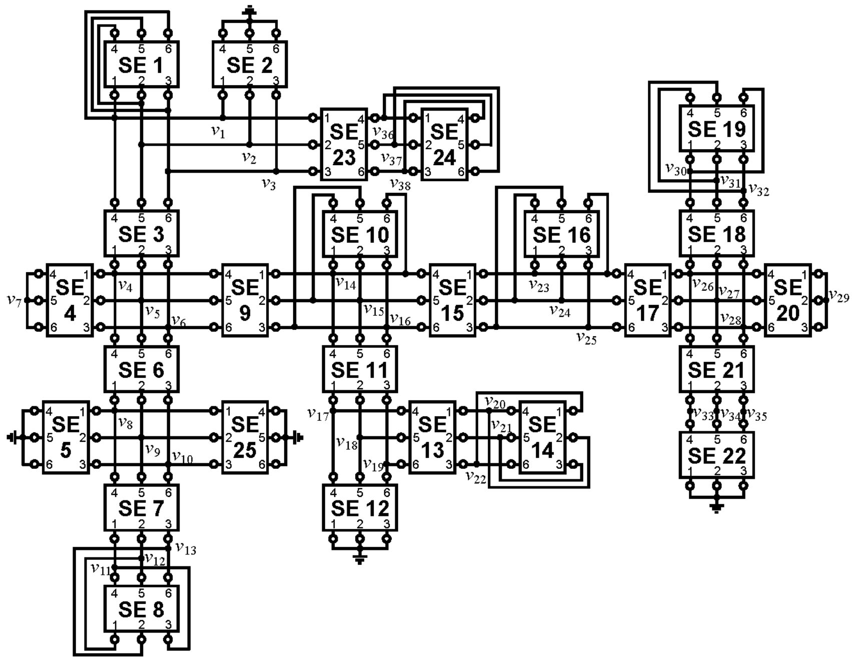

Fig. 3. Equivalent circuit for an example of the electrical system [3]

That means the software step of integration of the mathematical system has to be equal 0,2 ms as well. Although it can be assumed that it is possible to perform calculations with the software step of integration in less than a simulator operational time slice. Then we can perform k-integration steps with h equal software operational time slice divided by k, within one simulator operational time slice. In practice, if there is possibility we can reduce simulator operational time slice.

3.2. Algorithm of calculation

Problems concerning mathematical modeling of electrical systems belong to the group of numerical issues characterized by intense use of arithmetic calculations. It is defined [7] as a ratio of time required to perform arithmetic operation to the time required to transfer data with internal storage. For this type of issues the graphic processors (GPU) can be used [4]. Therefore, it is appropriate to find the efficient division of calculation, which can be performed in parallel.

Mathematical model of electrical systems and structural elements are described in details in [3].

Figure 4 shows the algorithm of computer simulation of electrical system (Fig. 2), where are task blocks with parallel calculations. The first task block where it is possible to perform parallel calculations contains as many tasks as the amount of structural elements included. For each extracted task, the following computation elements can be highlighted: calculation of external nodes potential value of structural element, calculation of currents (derivatives of currents) in the external branches of structural element, calculation of other internal physical quantities of structural element, calculation of the matrix elements value in the external equation of structural element, calculation of the products value with the use of the incidence matrix and matrix in external equation of structural element as well as a matrix determination of electric system nodes potential equation, which can be splitted evenly for all of tasks.

The second block of calculations which can be performed in parallel begins with the preparation of remain elements of electrical system nodes potentials equation and system of equation solving relatively to considered electrical system nodes potentials. The aggregation of electrical system elements is intentional in system of equations solving process acceleration.

3.3. Mathematical model of the six-pole with RL branches

Figure 5 shows a diagram of a three-phase linear structural RL type element as the six-pole. The six-pole consists of three independent branches, which contain serial connection of two ideal linear electric system elements (coil and resistor).

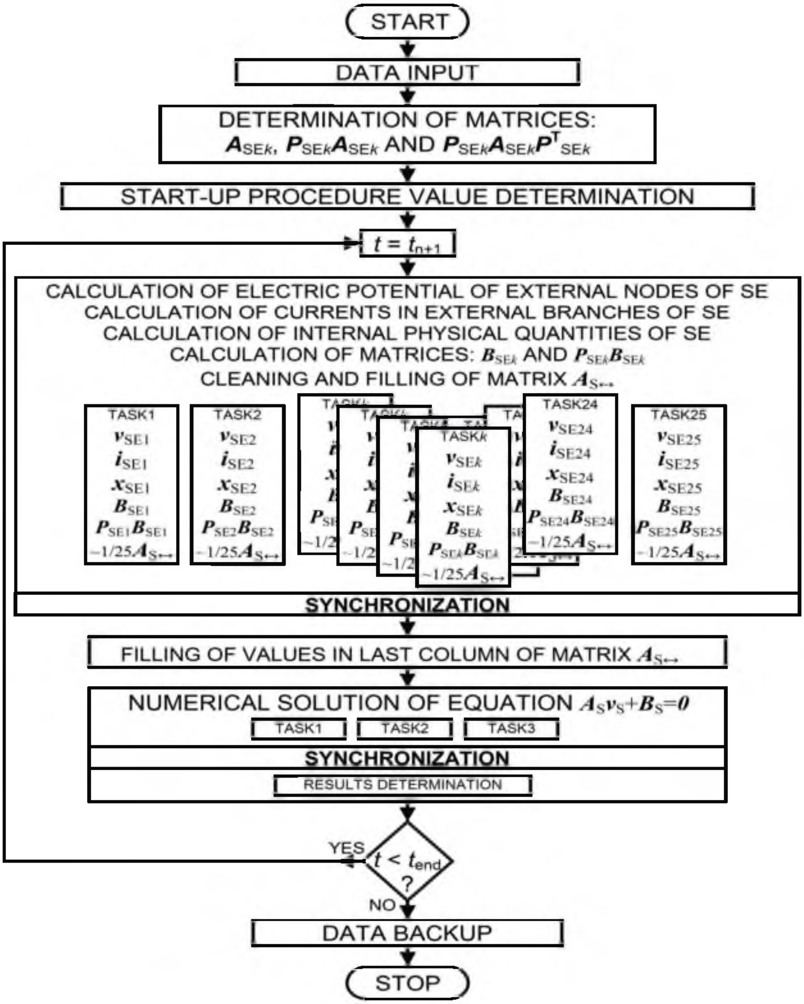

Fig. 4. Computer simulation algorithm of electric system operational states (Fig. 2) with parallel computations [2]

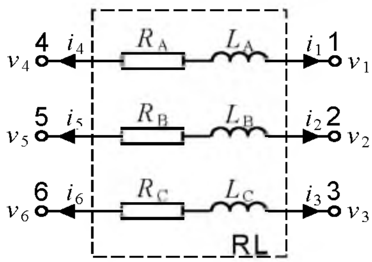

Fig. 5. Diagram of the three-phase structural RL element as an electric six-pole

For any value of the independent t variable, the three-phase electric state of linear structural element (six-pole showed on Fig. 5) can be presented as system of differential equations

$$\begin{cases} v_4(t) - v_1(t) - R_A i_1(t) - L_A \dfrac{\mathrm{d}i_1(t)}{\mathrm{d}t} = 0 \\ v_5(t) - v_2(t) - R_B i_2(t) - L_B \dfrac{\mathrm{d}i_2(t)}{\mathrm{d}t} = 0 \\ v_6(t) - v_3(t) - R_C i_3(t) - L_C \dfrac{\mathrm{d}i_3(t)}{\mathrm{d}t} = 0 \end{cases}. \tag{1}$$

From numerical integration point of view equations (1) can be approximated with many different functions. That is all depends of integration algorithm method. Going this way, you can construct mathematical models of structural elements associated with specified integration algorithm. In the remaining part of this article associated model with Euler's interpolation algorithm will be derived.

Euler's interpolation algorithm is implemented in following formula

$$x(t_{n+1}) = x(t_n) + h\frac{\mathrm{d}x(t_{n+1})}{\mathrm{d}t}, \tag{2}$$

where: x – independent variable, h – integration step.

After substitute of the Euler's interpolation formula (2) to the equations (1) following external equation for RL multipole (Fig. 5) is obtained:

$$\boldsymbol{i}_{SE} + \boldsymbol{A}_{SE}\boldsymbol{v}_{SE} + \boldsymbol{B}_{SE} = \boldsymbol{0}, \tag{3}$$

where: $\boldsymbol{i}_{SE} = [i_1 \quad i_2 \quad i_3 \quad i_4 \quad i_5 \quad i_6]^T$, $\boldsymbol{v}_{SE} = [v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6]^T$, $\boldsymbol{A}_{SE} \overset{ozn.}{=} \begin{bmatrix} \boldsymbol{a}_{SE} & -\boldsymbol{a}_{SE} \\ -\boldsymbol{a}_{SE} & \boldsymbol{a}_{SE} \end{bmatrix}$, $\boldsymbol{a}_{SE} = \mathrm{diag}(\alpha_A, \alpha_B, \alpha_C)$, $\boldsymbol{B}_{SE} \overset{ozn.}{=} \begin{bmatrix} \boldsymbol{b}_{SE} \\ -\boldsymbol{b}_{SE} \end{bmatrix}$, $\boldsymbol{b}_{SE} = [\beta_A \quad \beta_B \quad \beta_C]^T$. And after transformation:

$$\alpha_\zeta = \left(R_\zeta + h^{-1}L_\zeta\right)^{-1}, \beta_\zeta = \alpha_\zeta\left(-h^{-1}L_\zeta i_k(t_n)\right), \tag{4}$$

where: ζ – a phase of structural element (ζ := A,B,C), k – index, 1 for phase A (ζ := A), 2 for phase B and 3 for phase C.

4. Mathematical model of aggregated linear three-phase element of electrical system [2]

Aggregated linear three-phase structural element is actually a fusion of at least two primary linear three-phase structural elements, where internal nodes of electrical system are highlighted (node as a common point of at least three branches).

Figure 6 shows the diagram of aggregated three-phase linear RL + RL type structural element as the three-pole.

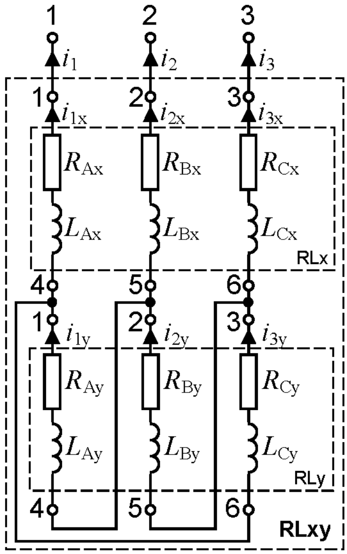

Fig. 6. Scheme of aggregated linear three-phase RL+RL type structural element as three-pole [2]

Aggregated linear three-phase RL+RL type structural element consists of two linear three-phase RL type structural elements (Fig. 5), as a fusion with SE7 and SE8 structure in Fig. 3. To distinguish elements, their parameters and physical quantities associated with them the following designations applied: RLx and Rly. The parameters and physical quantities of aggregated element are designated as RLz. Interior of RLz element there are three nodes distinguished: 1w, 2w and 3w.

Because of external (in respect of RLy element) nodes connection (triangle system), the equation for RLy element is represented as follows:

$$\boldsymbol{i}_{\mathrm{RLy}} + \boldsymbol{A}_{\mathrm{RLy}} \boldsymbol{v}_{\mathrm{w}} + \boldsymbol{B}_{\mathrm{RLy}} = \boldsymbol{0}, \tag{5}$$

where: $\boldsymbol{i}_{\mathrm{RLy}} = \begin{bmatrix} i_{1\mathrm{RLy}} & i_{2\mathrm{RLy}} & i_{3\mathrm{RLy}} \end{bmatrix}^{\mathrm{T}}$ – vector of currents in RLy element branches,

$\boldsymbol{v}_{\mathrm{w}} = \begin{bmatrix} v_{1\mathrm{w}} & v_{2\mathrm{w}} & v_{3\mathrm{w}} \end{bmatrix}^{\mathrm{T}}$ – vector of electric potentials interial nodes of element RLz,

$\boldsymbol{A}_{\mathrm{RLy}} = \begin{bmatrix} \alpha_{\mathrm{ARLy}} & -\alpha_{\mathrm{ARLy}} & 0 \\ 0 & \alpha_{\mathrm{BRLy}} & -\alpha_{\mathrm{BRLy}} \\ -\alpha_{\mathrm{CRLy}} & 0 & \alpha_{\mathrm{CRLy}} \end{bmatrix}$ – matrix, elements of that matrix are determined using 1st formula included in (4),

$\boldsymbol{B}_{\mathrm{RLy}} = \begin{bmatrix} \beta_{\mathrm{ARLy}} & \beta_{\mathrm{BRLy}} & \beta_{\mathrm{CRLy}} \end{bmatrix}^{\mathrm{T}}$ – vector, elements of that vector are determined using 2nd formula included in (4).

Due to direct branch transfer to the outside of aggregated RLz $(i_{1\mathrm{RLz}} = i_{1\mathrm{RLx}}$, $i_{2\mathrm{RLz}} = i_{2\mathrm{RLx}}$, $i_{3\mathrm{RLz}} = i_{3\mathrm{RLx}})$ element, the vector equation for Rlx element is represented as follows:

$$\boldsymbol{i}_{\mathrm{RLz}} + \boldsymbol{a}_{\mathrm{RLx}}\boldsymbol{v}_{\mathrm{RLz}} - \boldsymbol{a}_{\mathrm{RLx}}\boldsymbol{v}_{\mathrm{w}} + \boldsymbol{B}_{\mathrm{RLx}} = \mathbf{0}, \tag{6}$$

where: $\boldsymbol{i}_{\mathrm{RLz}} = [i_{1\mathrm{RLz}} \quad i_{2\mathrm{RLz}} \quad i_{3\mathrm{RLz}}]^{\mathrm{T}}$ – vector of currents in external branches of aggregated RLz element,

$\boldsymbol{v}_{\mathrm{RLz}} = [v_{1\mathrm{RLz}} \quad v_{2\mathrm{RLz}} \quad v_{3\mathrm{RLz}}]^{\mathrm{T}}$ – vector of external nodes electric potential for aggregated RLz element,

$\boldsymbol{a}_{\mathrm{RLx}} = \mathrm{diag}(\alpha_{\mathrm{ARLx}}, \alpha_{\mathrm{BRLx}}, \alpha_{\mathrm{CRLx}})$ – matrix, elements of that matrix are determined using 1st formula included in (4),

$\boldsymbol{B}_{\mathrm{RLx}} = [\beta_{\mathrm{ARLx}} \quad \beta_{\mathrm{BRLx}} \quad \beta_{\mathrm{CRLx}}]^{\mathrm{T}}$ – vector, elements of that vector are determined using 2nd formula included in (4), with taking into consideration following equation $\boldsymbol{i}_{\mathrm{RLz}} = \boldsymbol{i}_{\mathrm{RLx}}$.

The 1st Kirchhoffs law can present the following equation:

$$\boldsymbol{i}_{\mathrm{RLz}} + \boldsymbol{P}_{\mathrm{wRLz}}\boldsymbol{i}_{\mathrm{RLy}} = \mathbf{0}, \tag{7}$$

where $\boldsymbol{P}_{\mathrm{wRLz}} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ – incidence matrix of internal connections for aggregated RLz element.

After equations (5), (6) and (7) transformation we achieved following dependencies used to determine matrix $\boldsymbol{A}_{\mathrm{RLz}}$ and vector $\boldsymbol{B}_{\mathrm{RLz}}$ values in external linear three-phase equation $\boldsymbol{i}_{\mathrm{RLz}} + \boldsymbol{A}_{\mathrm{RLz}}\boldsymbol{v}_{\mathrm{RLz}} + \boldsymbol{B}_{\mathrm{RLz}} = \mathbf{0}$ of aggregated structural element (Fig. 6):

$$\begin{aligned} \boldsymbol{A}_{\mathrm{RLz}} &= \boldsymbol{H}\boldsymbol{P}_{\mathrm{wRLz}}\boldsymbol{A}_{\mathrm{RLy}}, \\ \boldsymbol{B}_{\mathrm{RLz}} &= \boldsymbol{H}\boldsymbol{P}_{\mathrm{wRLz}}\left(\boldsymbol{A}_{\mathrm{RLy}}\boldsymbol{a}_{\mathrm{RLx}}^{-1}\boldsymbol{B}_{\mathrm{RLx}} + \boldsymbol{B}_{\mathrm{RLy}}\right), \end{aligned} \tag{8}$$

where $\boldsymbol{H} = \left(\boldsymbol{P}_{\mathrm{wRLz}}\boldsymbol{A}_{\mathrm{RLy}}\boldsymbol{a}_{\mathrm{RLx}}^{-1} - \mathbf{1}\right)^{-1}$.

The currents value of the structural RLy element is given by the following formula:

$$\boldsymbol{i}_{\mathrm{RLy}} = -\boldsymbol{A}_{\mathrm{RLy}}\left(\boldsymbol{a}_{\mathrm{RLx}}^{-1}(\boldsymbol{i}_{\mathrm{RLz}} + \boldsymbol{B}_{\mathrm{RLx}}) + \boldsymbol{v}_{\mathrm{RLz}}\right) - \boldsymbol{B}_{\mathrm{RLy}}. \tag{9}$$

Internal nodes potential vector of the aggregated RLz element is determined by the formula (6) transformation, as following formula:

$$\boldsymbol{v}_{\mathrm{w}} = \boldsymbol{a}_{\mathrm{RLx}}^{-1}(\boldsymbol{i}_{\mathrm{RLz}} + \boldsymbol{B}_{\mathrm{RLx}}) + \boldsymbol{v}_{\mathrm{RLz}}. \tag{10}$$

Another example of three-phase aggregated structural element (electric three-pole) is shown in [2], as a mathematical model of the transistor converter with capacitor and three-phase choke.

5. Experimental results of simulation with aggregated element and conclusion

Experimental researches of the mathematical model of the electrical system sample (Fig. 2) were carried out on the digital platform (Platform no 1) based on personal computer with the six-core Intel® Core™ i7 970 @ 3.20 GHz, 1597 MHz (6.00 GB of physical memory, 4.51 GB of available space of physical memory, 12.0 GB of total size of virtual memory and 10.3 GB of available virtual memory). Standard Microsoft Windows 7 Professional operating system has been used (6.1.7601 Service Pack 1).

The results of experiments using a mathematical model of the aggregated structural element, which replaces two structural elements - ES7 and ES8 (Fig. 3) are shown in Fig. 7.

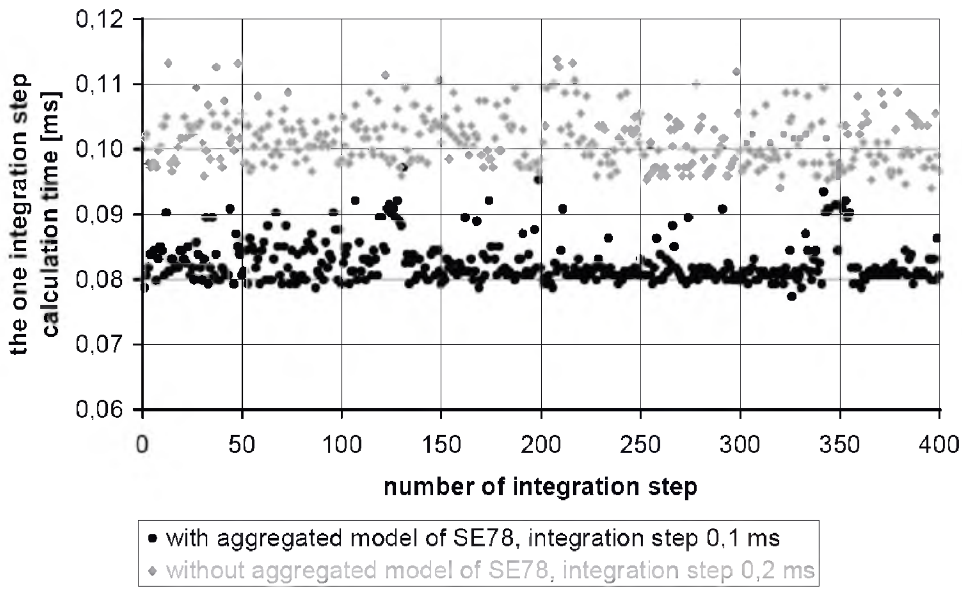

Fig. 7. The temporary effect of calculations in a single step of integration after aggregated model has been applied [2]

We were using aggregated structural element in order to move specific electric system nodes to the interior of aggregated structural element. As a result of that the amount of equations decreases, which allow accelerating calculations. This is confirmed by the results of experiment provided on Fig. 7.

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