Dynamics Analysis of a Truck-Mounted Crane with the LuGre Friction Model in the Joints

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Abstract

A dynamics analysis of a selected truck-mounted crane is presented in this article. A mathematical model of the crane, considered in a form of an open-loop kinematic chain, allows to take into account flexibility of its support system, hoist rope and drives of particular links, and also friction in the joints. The geometry of the crane model is described using the Denavit-Hartenberg notation based on joint coordinates and homogeneous transformation matrices. Its equations of motion were derived on basis of the Lagrange formalism. The LuGre model was used to describe friction in the joints.

1. Introduction

In the today's era of computer systems a development of virtual complex models of mechanical systems is achieved using commercial or proprietary calculation programs. Computerization of a designing process of these systems shortens time significantly from determining design assumptions to making a final product.

In the literature there are lots of publications devoted to the dynamics analysis of different types of cranes. Unfortunately, there are hardly any publications devoted strictly to issues of the dynamics analysis of the truck-mounted cranes. This scope of investigations can be deemed – according to the authors of this publication – as poorly advanced. While making an overview of the literature the authors want to draw attention to a few selected works devoted to the dynamics analysis of cranes taking into account flexibility of their support systems (e.g. [1, 2, 3, 4]), flexibility of a hoist rope (e.g. [2, 3]), and also occurring of friction in joints (e.g. [5, 6]). All the issues mentioned are also a subject of the analysis presented in this work.

In the work it is assumed that the links of the modeled crane are driven directly by torques, whereas the retractable link by a force. It is a simplification, because in the real system the links are driven by three hydraulic cylinders (two of them are jib cylinders, and the third one is a telescopic cylinder). The assumed crane model is an open-loop kinematic chain, like models of robot manipulators. For this reason the Denavit-Hartenberg notation [7], taken from robotics, based on use of joint coordinates and homogeneous transformation matrices was applied to describe its geometry. Equations of the model motion were assumed using the Lagrange formalism [8]. Transported load was modeled in a form of a material point. In all the revolute joints, and also in the prismatic joint of the crane between the links moving on each other friction is taken into account. The friction phenomenon is described by the advanced LuGre model [9] based on bristle interpretation of friction [10]. This model allows to take into account the both phases of friction in the joints, that is the static and kinetic friction, and more precisely such phenomena as: a preliminary displacement, the Stribeck effect and a frictional lag.

2. Mathematical model of the crane

The model of the crane in question is presented in Fig. 1. This model consists of five links $\left(n_l = 5\right)$. First of them constitutes a truck chassis on which the crane is mounted. This chassis is settled flexibly by six supports $\left(n_s = 6\right)$, out of which four are the wheels of the truck.

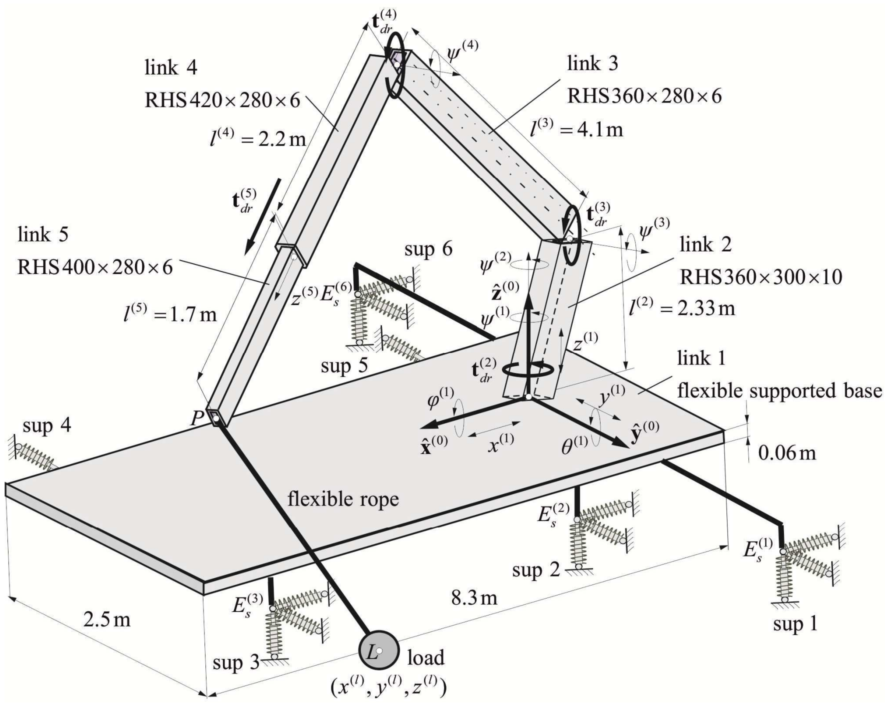

Figure 1. Model of the truck-mounted crane

A vector of the generalised (joint) coordinates of the developed model was determined in a form:

$$\mathbf{q} = \left[\mathbf{q}^{(c)^T} \quad \mathbf{q}^{(l)^T} \right]^T = \left[\tilde{\mathbf{q}}^{(1)^T} \quad \tilde{\mathbf{q}}^{(2)^T} \quad \tilde{\mathbf{q}}^{(3)^T} \quad \tilde{\mathbf{q}}^{(4)^T} \quad \tilde{\mathbf{q}}^{(5)^T} \quad \mathbf{q}^{(l)^T} \right]^T, \tag{1}$$

where: $\tilde{\mathbf{q}}^{(1)} = \left[x^{(1)} \quad y^{(1)} \quad z^{(1)} \quad \psi^{(1)} \quad \theta^{(1)} \quad \varphi^{(1)} \right]^T$, $\tilde{\mathbf{q}}^{(2)} = \left[\psi^{(2)} \right]$, $\tilde{\mathbf{q}}^{(3)} = \left[\psi^{(3)} \right]$,

$$\tilde{\mathbf{q}}^{(4)} = \left[\psi^{(4)} \right], \quad \tilde{\mathbf{q}}^{(5)} = \left[z^{(5)} \right], \; \mathbf{q}^{(l)} = \left[x^{(l)} \quad y^{(l)} \quad z^{(l)} \right]^T.$$

The matrices of the homogeneous transformations from the local coordinate systems of particular links of the model to the assumed reference system can be presented as:

$$\mathbf{T}^{(p)}\Big|_{p=1,\ldots,n_l} = \mathbf{T}^{(p-1)}\tilde{\mathbf{T}}^{(p)}, \tag{2}$$

where: $\mathbf{T}^{(0)} = \mathbf{I}$,

$$\tilde{\mathbf{T}}^{(1)} = \begin{bmatrix} 1 & -\psi^{(1)} & \theta^{(1)} & x^{(1)} \\ \psi^{(1)} & 1 & -\varphi^{(1)} & y^{(1)} \\ -\theta^{(1)} & \varphi^{(1)} & 1 & z^{(1)} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; \tilde{\mathbf{T}}^{(2)} = \begin{bmatrix} \mathrm{c}\psi^{(2)} & -\mathrm{s}\psi^{(2)} & 0 & 0 \\ \mathrm{s}\psi^{(2)} & \mathrm{c}\psi^{(2)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\tilde{\mathbf{T}}^{(3)} = \begin{bmatrix} \mathrm{c}\psi^{(3)} & -\mathrm{s}\psi^{(3)} & 0 & -l^{(2)}\,\mathrm{c}\alpha^{(2)} \\ 0 & 0 & 1 & 0 \\ -\mathrm{s}\psi^{(2)} & -\mathrm{c}\psi^{(2)} & 0 & l^{(2)}\,\mathrm{s}\alpha^{(2)} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tilde{\mathbf{T}}^{(4)} = \begin{bmatrix} \mathrm{c}\psi^{(4)} & -\mathrm{s}\psi^{(4)} & 0 & l^{(3)} \\ \mathrm{s}\psi^{(4)} & \mathrm{c}\psi^{(4)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\tilde{\mathbf{T}}^{(5)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & z^{(5)} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; \mathrm{s}\alpha^{(\beta)} = \sin\alpha^{(\beta)}, \; \mathrm{c}\alpha^{(\beta)} = \cos\alpha^{(\beta)}.$$

Models of a revolute joint and a prismatic joint with friction were developed for the needs of the analysis and they are presented in Figs. 2a and 2b, respectively.

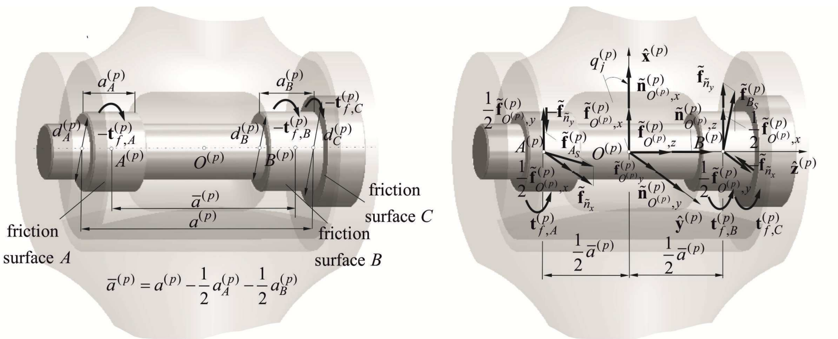

Figure 2a. Model of a revolute joint

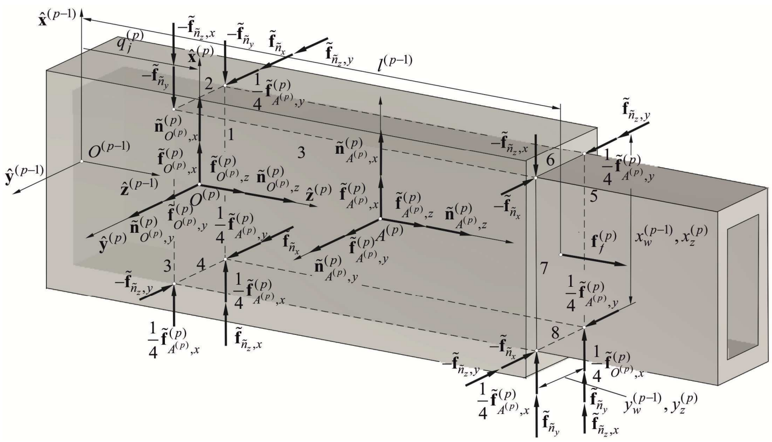

Figure 2b. Model of a prismatic joint

Values of friction torques $t_f^{(p)}$ in the revolute joints and friction force $f_f^{(p)}$ in the prismatic joint are calculated on basis of knowledge about joint forces and torques $\tilde{\mathbf{f}}_{O^{(p)}}^{(p)}$, $\tilde{\mathbf{n}}_{O^{(p)}}^{(p)}$ in those joints determined by the Newton-Euler recursive algorithm [7].

The equations of the crane model motion can be presented as:

$$\mathbf{A}\ddot{\mathbf{q}} = \mathbf{e} + \mathbf{f}_s + \mathbf{t}_{dr} - \mathbf{s}_f \ , \tag{3}$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{(c)} & 0 \\ 0 & \mathbf{A}^{(l)} \end{bmatrix}, \quad \mathbf{A}^{(c)} = \begin{bmatrix} \mathbf{A}_{1,1} & \cdots & \mathbf{A}_{1,j} & \cdots & \mathbf{A}_{1,n_l} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \mathbf{A}_{i,1} & \cdots & \mathbf{A}_{i,j} & \cdots & \mathbf{A}_{i,n_l} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ \mathbf{A}_{n_l,1} & \cdots & \mathbf{A}_{n_l,j} & \cdots & \mathbf{A}_{n_l,n_l} \end{bmatrix}, \quad \mathbf{A}^{(l)} = \begin{bmatrix} m^{(l)} & 0 & 0 \\ 0 & m^{(l)} & 0 \\ 0 & 0 & m^{(l)} \end{bmatrix},$$

$$\mathbf{A}_{i,j} = \sum_{p=\max\{i,j\}}^{n_l} \tilde{\mathbf{A}}_{i,j}^{(p)}, \ \tilde{\mathbf{A}}_{i,j}^{(p)}\Big|_{i,j=1,\dots,p} = \left(\tilde{a}^{(p)}_{n_{dof}^{(i-1)}+k,\, n_{dof}^{(j-1)}+l} \right)_{\substack{k=1,\dots,\tilde{n}_{dof}^{(i)} \\ l=1,\dots,\tilde{n}_{dof}^{(j)}}}, \ \tilde{a}_{i,j}^{(p)} = \mathrm{tr}\left\{ \mathbf{T}_i^{(p)} \mathbf{H}^{(p)} \mathbf{T}_j^{(p)^T} \right\},$$

$\mathbf{T}_i^{(p)} = \dfrac{\partial \mathbf{T}^{(p)}}{\partial q_i^{(p)}}$, $\mathbf{H}^{(p)}$ – pseudo-inertia matrix of link p,

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}^{(c)} \\ \mathbf{e}^{(l)} \end{bmatrix}, \ \mathbf{e}^{(c)} = \begin{bmatrix} \mathbf{e}_1 \\ \vdots \\ \mathbf{e}_i \\ \vdots \\ \mathbf{e}_{n_l} \end{bmatrix}, \ \mathbf{e}^{(l)} = \begin{bmatrix} 0 \\ 0 \\ m^{(l)} g \end{bmatrix}, \ \mathbf{e}_i = -\sum_{p=i}^{n_l} \left(\tilde{\mathbf{h}}_i^{(p)} + \tilde{\mathbf{g}}_i^{(p)} \right),$$

$m^{(l)}$ – mass of the load, g – acceleration of gravity,

$$\tilde{\mathbf{h}}_i^{(p)}\Big|_{i=1,\dots,p} = \left(\tilde{h}^{(p)}_{n_{dof}^{(i-1)}+k}\right)_{k=1,\dots,\tilde{n}_{dof}^{(p)}},\ \tilde{h}_i^{(p)} = \sum_{m=1}^{n_{dof}^{(p)}}\sum_{n=m}^{n_{dof}^{(p)}} \mathrm{tr}\left\{\mathbf{T}_i^{(p)}\mathbf{H}^{(p)}\mathbf{T}_{m,n}^{(p)^T}\right\}\dot{q}_m^{(p)}\dot{q}_n^{(p)},\ \mathbf{T}_{m,n}^{(p)} = \frac{\partial^2\mathbf{T}^{(p)}}{\partial q_m^{(p)}\partial q_n^{(p)}},$$

$$\tilde{\mathbf{g}}_i^{(p)}\Big|_{i=1,\dots,p} = \left(\tilde{g}^{(p)}_{n_{dof}^{(i-1)}+k}\right)_{k=1,\dots,n_{dof}^{(i)}},\quad \tilde{g}_i^{(p)} = m^{(p)}g\begin{bmatrix}\mathbf{j}_3^T & 0\end{bmatrix}\mathbf{T}_i^{(p)}\mathbf{r}_{C^{(p)}}^{(p)},$$

$m^{(p)}$ – mass of link p, $\mathbf{r}_{C^{(p)}}^{(p)}$ – vector of the position of mass centre $C^{(p)}$ of link p,

$$\mathbf{f}_s = \begin{bmatrix}\mathbf{f}_s^{(c)}\\ \mathbf{f}_s^{(l)}\end{bmatrix},\ \mathbf{f}_s^{(c)} = \mathbf{f}_{s,s}^{(c)} + \mathbf{f}_{s,r}^{(c)},\ \mathbf{f}_{s,s}^{(c)} = \begin{bmatrix}-\left(\frac{\partial E_{p,s}}{\partial\tilde{\mathbf{q}}^{(1)}} + \frac{\partial R_s}{\partial\dot{\tilde{\mathbf{q}}}^{(1)}}\right)^T & \mathbf{0}\end{bmatrix}^T,\ \mathbf{f}_{s,r}^{(c)} = \begin{bmatrix}-\left(\frac{\partial E_{p,r}}{\partial\mathbf{q}^{(c)}} + \frac{\partial R_r}{\partial\dot{\mathbf{q}}^{(c)}}\right)\end{bmatrix}^T,$$

$$\mathbf{f}_s^{(l)} = \begin{bmatrix}-\left(\frac{\partial E_{p,r}}{\partial\mathbf{q}^{(l)}} + \frac{\partial R_r}{\partial\dot{\mathbf{q}}^{(l)}}\right)\end{bmatrix}^T,$$

$$\frac{\partial E_{p,s}}{\partial\tilde{\mathbf{q}}^{(1)}} = \sum_{i=1}^{n_s}\sum_{\alpha\in\{x,y,z\}} s_{s,\alpha}^{(i)}\frac{e_{s.\alpha}^{(i)}}{l_{s,\alpha}^{(i)}}\mathbf{U}_{E_s}^{(i)^T}\mathbf{U}_{E_s}^{(i)}\tilde{\mathbf{q}}^{(1)},\ \frac{\partial R_s}{\partial\dot{\tilde{\mathbf{q}}}^{(1)}} = \sum_{i=1}^{n_s}\sum_{\alpha\in\{x,y,z\}} d_{s,\alpha}^{(i)}\mathbf{U}_{E_s}^{(i)^T}\mathbf{U}_{E_s}^{(i)}\dot{\tilde{\mathbf{q}}}^{(1)},$$

$$e_{s,\alpha}^{(i)} = l_{s,\alpha}^{(i)} - l_{s,\alpha,0}^{(i)},\ l_{s,\alpha}^{(i)} = \left|\mathbf{U}_{E_s}^{(i)}\tilde{\mathbf{q}}^{(i)}\right|,\ \mathbf{U}_{E_S}^{(i)} = \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 0 & z_{E_S}^{(i)} & -y_{E_S}^{(i)}\\ 0 & 1 & 0 & -z_{E_S}^{(i)} & 0 & x_{E_S}^{(i)}\\ 0 & 0 & 1 & y_{E_S}^{(i)} & -x_{E_S}^{(i)} & 0\end{array}\right],$$

$s_{s,\alpha}^{(i)}, d_{s,\alpha}^{(i)}$ – stiffness and damping coefficient of support i in direction α,

$l_{s,\alpha}^{(i)},\ l_{s,\alpha,0}^{(i)}$ – current and initial length of support i in direction α,

$$\frac{\partial E_{p,r}}{\partial q_j^{(c)}} = s_r\delta_r\frac{e_r}{l_r}\mathbf{r}_{PL}^T\mathbf{J}\mathbf{T}_j^{(5)}\mathbf{r}_P^{(5)},\ \frac{\partial R_r}{\partial\dot{q}_j^{(c)}} = d_r\delta_r\frac{\dot{\mathbf{r}}_{PL}}{l_r}\mathbf{r}_{PL}^T\mathbf{J}\mathbf{T}_j^{(5)}\mathbf{r}_P^{(5)},$$

$$\frac{\partial E_{p.r}}{\partial q_j^{(l)}} = -s_r\delta_r\frac{e_r}{l_r}\mathbf{r}_{PL}^T\mathbf{j}_j,\ \frac{\partial R_r}{\partial\dot{q}_j^{(l)}} = -d_r\delta_r\frac{\dot{\mathbf{r}}_{PL}}{l_r}\mathbf{r}_{PL}^T\mathbf{j}_j,$$

s_r, d_r – stiffness and damping coefficient of the hoist rope,

$\mathbf{r}_P^{(5)}$ – vector of the position of point P in the coordinate system of link 5,

$$\mathbf{r}_{PL} = \mathbf{J}\left(\mathbf{r}_P^{(0)} - \mathbf{r}_L^{(0)}\right),\ l_r = \mathbf{r}_{PL}^T\mathbf{r}_{PL},\ e_r = l_r - l_{r,0},\ \delta_r = \begin{cases}1, & e_r > 0\\ 0, & e_r \le 0\end{cases},$$

$\mathbf{r}_\alpha^{(0)}\Big|_{\alpha\in\{P,L\}}$ – vector of the position of points P and L in the reference system,

l_r, $l_{r,0}$ – current and initial length of the hoist rope,

$$\mathbf{J} = \left[\begin{array}{c|c|c|c}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{array}\right] = \left[\begin{array}{c|c|c|c}\mathbf{j}_1 & \mathbf{j}_2 & \mathbf{j}_3 & \mathbf{0}\end{array}\right],\ \mathbf{t}_{dr} = \begin{bmatrix}\mathbf{0} & t_{dr}^{(2)} & t_{dr}^{(3)} & t_{dr}^{(4)} & t_{dr}^{(5)} & \mathbf{0}\end{bmatrix}^T,$$

$$t_{dr}^{(p)} = -\left(\frac{\partial E_{p,dr}^{(p)}}{\partial q_j^{(p)}} + \frac{\partial R_{dr}^{(p)}}{\partial \dot{q}_j^{(p)}} \right),\ \frac{\partial E_{p,dr}^{(p)}}{\partial q_j^{(p)}} = -s_{dr}^{(p)} \left(q_{dr}^{(p)} - q_j^{(p)} \right),\ \frac{\partial R_{dr}^{(p)}}{\partial \dot{q}_j^{(p)}} = -d_{dr}^{(p)} \left(\dot{q}_{dr}^{(p)} - \dot{q}_j^{(p)} \right),$$

$s_{dr}^{(p)}, d_{dr}^{(p)}$ – stiffness and damping coefficient of the drive,

$q_{dr}^{(p)}$ – assumed driving functions,

$$\mathbf{s}_f = \begin{bmatrix} \mathbf{0} & t_f^{(2)} & t_f^{(3)} & t_f^{(4)} & f_f^{(5)} & \mathbf{0} \end{bmatrix}^T,\ t_f^{(p)} = t_{f,A}^{(p)} + t_{f,B}^{(p)} + t_{f,C}^{(p)},\ f_f^{(p)} = \mu^{(p)} \tilde{f}^{(p)},$$

$$t_{f,A}^{(p)} = \frac{1}{2} \mu_A^{(p)} \tilde{f}_{A^{(p)}}^{(p)} d_A^{(p)},\ t_{f,B}^{(p)} = \frac{1}{2} \mu_B^{(p)} \tilde{f}_{B^{(p)}}^{(p)} d_B^{(p)},\ t_{f,C}^{(p)} = \frac{1}{2} \mu_C^{(p)} \left| \tilde{f}_{O^{(p)},z}^{(p)} \right| \frac{d_C^{(p)^3} - d_B^{(p)^3}}{d_C^{(p)^2} - d_B^{(p)^2}},$$

$$\tilde{f}_{A^{(p)}}^{(p)} = \sqrt{ \left(\frac{1}{2} \tilde{f}_{O^{(p)},x}^{(p)} - \tilde{f}_{\tilde{n}_y} \right)^2 + \left(\frac{1}{2} \tilde{f}_{O^{(p)},y}^{(p)} + \tilde{f}_{\tilde{n}_x} \right)^2 },$$

$$\tilde{f}_{B^{(p)}}^{(p)} = \sqrt{ \left(\frac{1}{2} \tilde{f}_{O^{(p)},x}^{(p)} + \tilde{f}_{\tilde{n}_y} \right)^2 + \left(\frac{1}{2} \tilde{f}_{O^{(p)},y}^{(p)} - \tilde{f}_{\tilde{n}_x} \right)^2 },\ \tilde{f}_{\tilde{n}_x} = \frac{\tilde{n}_{O^{(p)},x}^{(p)}}{\overline{a}^{(p)}},\ \tilde{f}_{\tilde{n}_y} = \frac{\tilde{n}_{O^{(p)},y}^{(p)}}{\overline{a}^{(p)}},$$

$$\tilde{f}^{(p)} = \sum_{i=1}^{8} \tilde{f}_i^{(p)},\ \tilde{f}_1^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},x}^{(p)} - \tilde{f}_{\tilde{n}_y} + \tilde{f}_{\tilde{n}_z,x} \right|,\ \tilde{f}_2^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},y}^{(p)} + \tilde{f}_{\tilde{n}_x} + \tilde{f}_{\tilde{n}_z,y} \right|,$$

$$\tilde{f}_3^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},x}^{(c,p)} - \tilde{f}_{\tilde{n}_y} - \tilde{f}_{\tilde{n}_z,x} \right|,\ \tilde{f}_4^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},y}^{(p)} + \tilde{f}_{\tilde{n}_x} - \tilde{f}_{\tilde{n}_z,y} \right|,\ \tilde{f}_5^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},x}^{(p)} + \tilde{f}_{\tilde{n}_y} + \tilde{f}_{\tilde{n}_z,x} \right|,$$

$$\tilde{f}_6^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},y}^{(p)} - \tilde{f}_{\tilde{n}_x} + \tilde{f}_{\tilde{n}_z,y} \right|,\ \tilde{f}_7^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},x}^{(p)} + \tilde{f}_{\tilde{n}_y} - \tilde{f}_{\tilde{n}_z,x} \right|,\ \tilde{f}_8^{(p)} = \left| \frac{1}{4} \tilde{f}_{A^{(p)},y}^{(p)} - \tilde{f}_{\tilde{n}_x} - \tilde{f}_{\tilde{n}_z,y} \right|,$$

$$\tilde{f}_{\tilde{n}_x} = \frac{1}{2} \frac{\tilde{n}_{A^{(p)},x}^{(p)}}{l^{(p-1)} - \left| q_j^{(p)} \right|},\ \tilde{f}_{\tilde{n}_y} = \frac{1}{2} \frac{\tilde{n}_{A^{(p)},y}^{(p)}}{l^{(p-1)} - \left| q_j^{(p)} \right|},\ \tilde{f}_{\tilde{n}_z,x} = \frac{1}{2} \frac{y_z^{(p)}}{\left(x_z^{(p)^2} + y_z^{(p)^2} \right)} \tilde{n}_{A^{(p)},z}^{(p)},$$

$$\tilde{f}_{\tilde{n}_z,y} = \frac{1}{2} \frac{x_z^{(p)}}{\left(x_z^{(p)^2} + y_z^{(p)^2} \right)} \tilde{n}_{A^{(p)},z}^{(p)},\ \tilde{f}_{A^{(p)},\alpha}^{(p)} = \tilde{f}_{O^{(p)},\alpha}^{(p)} \Big|_{\alpha \in \{x,y,z\}},$$

$$\tilde{n}_{A^{(p)},x}^{(p)} = \tilde{n}_{O^{(p)},x}^{(p)} + \frac{1}{2} \tilde{f}_{O^{(p)},y}^{(p)} \left(l^{(p-1)} - \left| q_j^{(p)} \right| \right),\ \tilde{n}_{A^{(p)},y}^{(p)} = \tilde{n}_{O^{(p)},y}^{(p)} - \frac{1}{2} \tilde{f}_{O^{(p)},x}^{(p)} \left(l^{(p-1)} - \left| q_j^{(p)} \right| \right),$$

$$\tilde{n}_{A^{(p)},z}^{(p)} = \tilde{n}_{O^{(p)},z}^{(p)}.$$

The LuGre friction model adopted by the authors of this work, describing friction coefficients $\mu\Big|_{\substack{\mu=\mu_{\alpha}^{(p)}\big|_{\alpha\in\{A,B,C\}} \\ \mu=\mu^{(p)}}}$ in a form of two differential equations of a first order, has already been presented in details in other works by them, namely in the publication devoted to dynamics of spatial linkages [11], and also in the article dealing with the grab crane [6].

The formulated equations of motion were solved by the Runge-Kutta method of the fourth order with a fixed-step.

3. Results of numerical calculations

Some results of the numerical calculations presenting a trajectory of the load moved, determined in plane $\hat{\mathbf{x}}^{(0)}\hat{\mathbf{y}}^{(0)}$ of the reference system in the case of taking into account or omitting friction in the crane joints, are presented in Fig. 3.

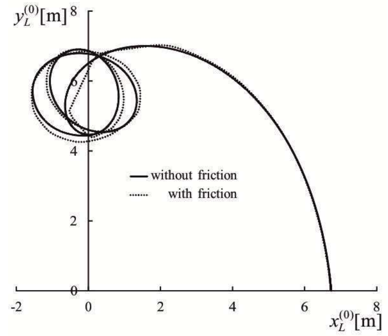

Figure 3. Trajectory of the load

An influence of friction on time courses of the values of the drive torques in the revolute joints and the drive force in the prismatic joint is presented in Fig. 4.

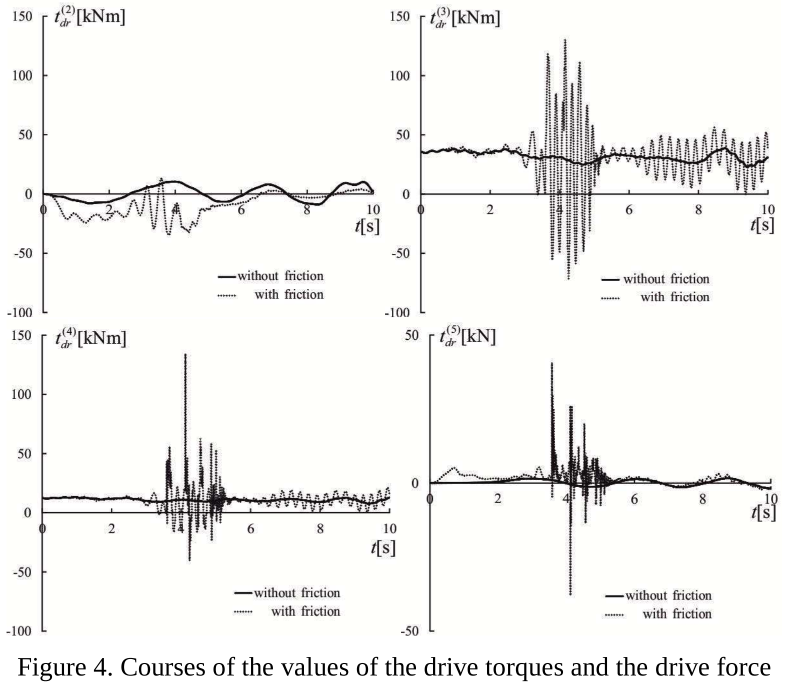

Figure 4. Courses of the values of the drive torques and the drive force

The performed calculations showed that friction in the joints of the modeled crane and also – as it was to be expected – flexibility of its support system and hoist rope had a significant influence on the crane dynamics, changing significantly courses of the determined parameters.

4. Conclusions

A dynamics analysis of the selected truck-mounted crane is presented in the work. The developed mathematical model can be treated as a virtual prototype of a real crane helpful while performing a process of its designing, and also while developing control algorithms. High degree of advancement of the prepared model provides – according to the authors – a possibility of a precise reflection of real system behavior in the dynamics conditions, what should make correctness of the calculation results reliable. However, the final verification of correctness of the prepared model can be made by experimental tests of a real system.

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