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SYMBOL DEFINITION OF STRUCTURAL CHARACTERISTICS OF MULTIDIMENSIONAL SURFACES

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Abstract. A hypersurface in four-dimensional space E_4, with generating line, satisfying the generalized condition of incidence $\left(e^{1,0}_{4,2}\right)^2 e^{1,0}_{4,1}$ is described.

Keywords: ruled surface, Radishchev diagram, symbol definition

1 Preamble

Conditions for the existence of n - dimensional space of two-parameter manifolds (congruence) k - surface, denoted $W(2, k)$ or simply W, are as follows [1,2]:

$$\dim\prod_i (e^{k,\dots,0}_{\dots}) = \sum_i \dim(e^{k,\dots,0}_{\dots})_i = (k+1)(n-k)-2. \tag{1}$$

For n=4 and k=1 we obtain

$$\dim\prod_i (e^{1,0}_{\dots}) = \sum_i \dim(e^{1,0}_{\dots})_i = (1+1)(4-1)-2 = 4. \tag{2}$$

Examples of four-dimensional hypersurfaces in symbolic form, dimension in the amount equal to four are:

$$\begin{aligned}
&1.\ \left(e^{1,0}_{4,2}\right)^2;\\
&2.\ \left(e^{1,0}_{4,2}\right)^2 \cdot e^{1,0}_{4,1};\\
&3.\ \left(e^{1,0}_{4,2}\right)^2 e^{1,0}_{3,2};\\
&4.\ \left(e^{1,0}_{4,1}\right)^2;\\
&5.\ \left(e^{1,0}_{3,2}\right)^2;\\
&6.\ e^{1,0}_{4,1} \cdot e^{1,0}_{3,2};\\
&7.\ e^{1,0}_{4,2} \cdot e^{1,0}_{4,0};\\
&8.\ e^{1,0}_{4,2} \cdot e^{1,0}_{3,1}.
\end{aligned} \tag{3}$$

2 Theorem

Investigate four-dimensional hypersurface $\left(e^{1,0}_{4,2}\right)^2 e^{1,0}_{4,1}$.

Hypersurface in four-dimensional space E_4, with generating line, satisfying the generalized condition of incidence $\left(e^{1,0}_{4,2}\right)^2 e^{1,0}_{4,1}$, is a hypersurface of second order of the first class [3].

According to the formula sharing criteria specified conditions (4) we find c_i [2]:

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$$\sum_{i=0}^{m} c_i = a_i - n + m + k. \tag{4}$$

Then we have

$$\sum_{i=0}^{m} c_i = 4 + 1 - 4 + 1 + 2 = 4 . \tag{5}$$

Decomposition conditions $e_{4,1}^{1,0}$

$$\begin{array}{ll} 0 \le c_0 \le 1 & 0,\ 1 \\ 1 < c_1 \le 4 & 2,\ 3,\ 4 \end{array} \tag{6}$$

will lead to the expression $\left(e_{4,0}^{1,0} + e_{3,1}^{1,0}\right) \cdot e_{4,2}^{1,0}$.

We expand the condition $e_{4,0}^{1,0}$ and check, whether there is a set of digits c_i, which would satisfy the system (4):

$$\begin{array}{llll} 0 \le c_0 \le 0 & 0 & 0 \le c_0 \le 1 & 0,\ 1 \\ 0 < c_1 \le 4 & 1,\ 2,\ 3,\ 4 & 1 < c_0 \le 3 & 2,\ 3 \end{array} \tag{7}$$

and obtain $2e_{3,0}^{1,0}$. Multiplication by $e_{4,1}^{1,0}$ will determine the order of the hypersurface

$$2e_{3,0}^{1,0} \cdot e_{4,1}^{1,0} = 2e_{1,0}^{1,0} . \tag{8}$$

We find the product geometric conditions in the following amounts of basic conditions

$$\sum_{i=0}^{m} c_i = 3 + 0 - 4 + 1 + 1 = 1 . \tag{9}$$

Decomposition conditions $e_{3,0}^{1,0}$

$$\begin{array}{llll} 0 \le c_0 \le 0 & 0 & 0 \le c_0 \le 1 & 0,\ 1 \\ 0 < c_1 \le 3 & 1,\ 2,\ 3 & 1 < c_0 \le 2 & 2 \end{array} \tag{10}$$

give us $2e_{3,0}^{1,0} \cdot e_{2,1}^{1,0}$.

Multiplication by $e_{3,2}^{1,0}$ will determine the class of the hypersurface:

$$\left(2e_{3,0}^{1,0} \cdot e_{2,1}^{1,0}\right) \cdot e_{3,2}^{1,0} = 0 + e_{1,0}^{1,0} . \tag{11}$$

So the order hypersurface is equal to two and class one.

Hypersurface $\left(e_{4,2}^{1,0}\right)^2 \cdot e_{4,1}^{1,0}$ is defined by two guide planes, which do not intersect in a straight line in E_4, and generating a direct parallel to the plane.

KN (K_1N_1, K_2N_2, K_3N_3) – generating a hypersurface (Fig. 1).

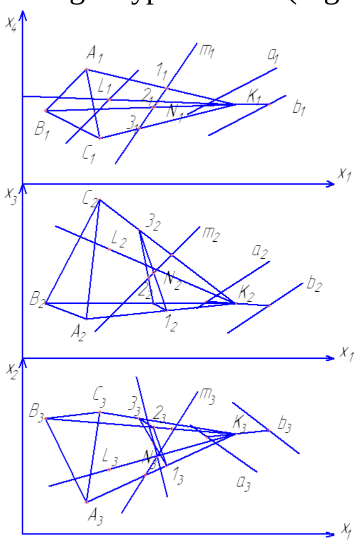

Figure 1: Construction of the generating of a hypersurface $\left(e_{4,2}^{1,0}\right)^2 \cdot e_{4,1}^{1,0}$

3 Issue

In this article the questions of the theory of enumerative geometry in the study and design of ruled varieties of four-dimensional space are considered. Methods of enumerative geometry can formalize the raw data of the problem, determine the order and class of optimal models constructed spaces of different dimensions and to determine the consistency of geometric conditions, explore options for setting the surface of the drawing Radishchev.

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SYMBOLICZNA DEFINICJA STRUKTURALNYCH CHARAKTERYSTYK POWIERZCHNI WIELOWYMIAROWYCH

W pracy opisuje się strukturę powierzchni prostokreślnych przestrzeni czterowymiarowej spełniających warunek incydencji $\left(e_{4,2}^{1,0}\right)^2 \cdot e_{4,1}^{1,0}$.