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Optimization of photonic crystal filter based on slack variables

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The improved model based on slack variables is designed to optimize a photonic crystal filter. An optimization model of the filter is established, with the transmission quantity in a stop-band as the objective function and the transmission quantity outside a stop-band as the constraint condition. The improved model is obtained by introducing slack variables in the infeasible region. With the treatment of objective and constraint functions, the optimization problem and the improved model are solved by sequential linear programming. Then the convergent solution satisfying the constraints can be found. The change of the objective function and design variables with iteration steps is presented by numerical examples, and the influence of by the constraints on the central frequency of the stop-band is discussed, indicating that the improved model is available and effective for optimization of a photonic crystal filter.

Keywords: photonic crystal filter, slack variables, optimization.

1. Introduction

Photonic crystals are a kind of artificial materials that exhibit periodic arrays of dielectric constant. It is well-known that photonic crystals have a special spectral structure, a so-called photonic band gap. This feature can be employed to design optical filters [1–4]. Compared with the traditional optical filter, the photonic crystal filter has advantages of high quality factor, flexible structure design, small size, low loss, large power capacity and other high performance filtering characteristics. Especially, with its micrometer structure size, it has a huge superiority in the miniaturization and integration of optical devices [5, 6]. Therefore, it is of great theoretical significance and application value to study the optimal design of photonic crystal filters.

The common methods used to optimize the photonic crystal filter are the following: finite-difference time-domain [7], particle swarm optimization [8], response surface methodology [9, 10], *etc*. In [9] and [10], the optimization model of the filter is established on the base of analysis and it is dealt with response surface methodology. The model is solved using sequential quadratic programming and the optimal parameters are obtained. However, for this optimization model, it must have a feasible solution that is satisfying the constraints in the optimization process, otherwise the optimization will be terminated. In practical applications, due to the filtering performance of the filter, it is often required to adjust the constraints, which may lead to no feasible solution. Hence this paper improves the above optimization model by introducing slack variables. First, the periodic length a, the dielectric thickness d, the relative permittivity ε_r of the dielectric are the three major factors in determining the stop-band characteristic of the photonic crystal structures. An optimization model of the filter is established, which takes the transmission quantity in the stop-band as the objective function and the transmission quantity outside the stop-band as the constraint condition. Then the inequality constraints in the optimization model are transformed into the equality constraints by using the method of adding slack variables, and an improved model with no feasible solution is obtained. With the treatment of objective and constraint functions, the optimization problem and the improved model are solved by sequential linear programming. A respective program has been written and numerical examples demonstrate its precision and efficiency.

2. Optimal design of photonic crystal filter

2.1. The establishment of optimization model

A photonic crystal filter structures in a waveguide is shown in Fig. 1. It is composed of a dielectric layer and an air layer arrayed alternately. In the design of a high quality band-stop filter, photonic crystals have been used primarily because of their photonic band gap, in which the propagation of light is prohibited. Furthermore, the smaller transmission coefficient in the stop-band is and the higher transmission coefficient in the pass-band is, the better the property of the filter is [11, 12]. When the width of the stop-band is fixed, we hope the area surrounded by the transmission coefficient curve and horizontal axis (frequency axis) should be the maximum. This area is defined as the negative of transmission quantity at a corresponding frequency band. Thus, this

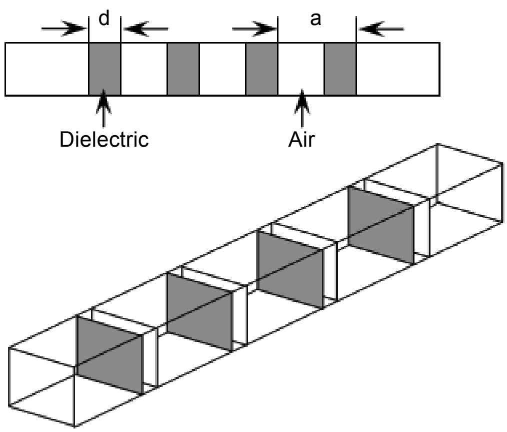

Fig. 1. The structure of a photonic crystal filter.

study establishes the optimization model with the transmission quantity in the stop-band as the objective function and the transmission quantity outside the stop-band as the constraint condition [9, 10]:

$$\begin{cases} \text{Find:} & a,\ d,\ \varepsilon_{\mathrm{r}} \\ \text{Minimize:} & G = -A_{\mathrm{sb}} = \int_{f_2}^{f_3} S_{21}\,\mathrm{d}f \\ \text{Subject to:} & \underline{a} \le a \le \overline{a} \\ & \underline{d} \le d \le \overline{d} \\ & d/a \le k \\ & \underline{\varepsilon}_{\mathrm{r}} \le \varepsilon_{\mathrm{r}} \le \overline{\varepsilon}_{\mathrm{r}} \\ & A_{\mathrm{L}} = \int_{f_1}^{f_2} (-S_{21})\,\mathrm{d}f \le T_{\mathrm{L}} \\ & A_{\mathrm{R}} = \int_{f_3}^{f_4} (-S_{21})\,\mathrm{d}f \le T_{\mathrm{R}} \end{cases} \tag{1}$$

where periodic length a, dielectric thickness d, relative permittivity ε_{r} of the dielectric are the design variables; $\mathbf{x} = (a, d, \varepsilon_{\mathrm{r}})^{\mathrm{T}}$ represents the design variable vector, f_2 and f_3 are the lower and upper bound of the stop-band, f_1 is the lower bound of the concerned band on the left of the stop-band, f_4 is the upper bound of the concerned band on the right of the stop-band; S_{21} is the transmission coefficient, A_{sb} is the area surrounded by the transmission coefficient curve and horizontal axis (frequency axis); G is the transmission quantity at the region between f_2 and f_3. The higher value of A_{sb} is, the lower is the value of G. Similarly, A_{L} and A_{R} are the negatives of transmission quantities at corresponding regions; T_{L} and T_{R} are the maximal values allowed by transmission quantity's absolute value at the corresponding region; $\underline{a}$, $\overline{a}$, $\underline{d}$, $\overline{d}$, $\underline{\varepsilon}_{\mathrm{r}}$, $\overline{\varepsilon}_{\mathrm{r}}$ are the lower and upper bound on the design variables (a, d, ε_{r}), respectively; k is a positive number less than 1 because d is less than a.

The initial estimate values of the design variables are obtained by the following design formula [7]:

$$\begin{cases} \varepsilon_{\mathrm{e}} = \dfrac{d}{a}\varepsilon_{\mathrm{r}} + \dfrac{1-d}{a} \\ \lambda_{\mathrm{g}} = \dfrac{\lambda/\sqrt{\varepsilon_{\mathrm{e}}}}{\sqrt{1-(\lambda/\lambda_{\mathrm{c}})^2}} \\ a = \lambda_{\mathrm{g}}/2 \end{cases} \tag{2}$$

where ε_{e} is the effective relative permittivity, λ and λ_{g} are the vacuum wavelength and waveguide wavelength corresponding to the center frequency of the stop-band; λ_{c} is the wavelength corresponding to the cut-off frequency of the rectangular waveguide TE_{10} mode.

2.2. The improved model with the method of introducing slack variables

When seeking solutions to the above optimization model by using sequence programming, there must be a feasible solution satisfying all the constraints, otherwise the optimization would come to a failure. Here the improved model is obtained by introducing slack variables. For the case of no feasible solution, the inequality (equality) constraints are left plus or minus a nonnegative new variable, which is called the slack variable. The introduction of a slack variable is to solve the problem in a larger feasible region, and to find a minimum solution that violates the constraint condition in the infeasible region. The result is assigned to the **x** as the initial value of the next cycle [13].

The general programming problem can be transformed into the following linear programming problem:

$$\begin{cases} \text{Find:} & \mathbf{x} \\ \text{Minimize:} & \mathbf{C}^{\mathrm{T}}\mathbf{x} \\ \text{Subject to:} & \mathbf{A}\mathbf{x} = \mathbf{B} \quad \text{or} \quad \mathbf{A}\mathbf{x} \le \mathbf{B} \quad \text{or} \quad \mathbf{A}\mathbf{x} \ge \mathbf{B} \\ & \underline{\mathbf{x}} \le \mathbf{x} \le \overline{\mathbf{x}} \end{cases} \tag{3}$$

The above equation is a standard linear programming problem. The sequence approximation method is used to solve the standard model, and if there is a feasible solution, the iteration is continued until the convergent solution is found. In the case of no feasible solution, the slack variables should be added.

1. If the constraint is **Ax** = **B**, one should add the slack variables and convert it into two inequality constraints

$$\sum_{j=1}^{n} a_{ij} x_j + \varepsilon_i \ge b_i \tag{4}$$

$$\sum_{j=1}^{n} a_{ij} x_j - \varepsilon_i \le b_i \tag{5}$$

2. If the constraint is **Ax** ≤ **B**, one should subtract a nonnegative variable, and convert it into an equality constraint

$$\sum_{j=1}^{n} a_{ij} x_j - \varepsilon_i = b_i \tag{6}$$

3. If the constraint is **Ax** ≥ **B**, one should add nonnegative variables, and convert it into an equality constraint

$$\sum_{j=1}^{n} a_{ij} x_j + \varepsilon_i = b_i \tag{7}$$

Then, based on the above discussion, we have:

$$\begin{cases} \text{Find:} & \mathbf{x}, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m \\ \text{Minimize:} & \varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_m \\ \text{Subject to:} & \mathbf{A}_1\mathbf{x}_1 \pm \{\boldsymbol{\varepsilon}\}_{m_1} = \mathbf{B}_1 \\ & \mathbf{A}_2\mathbf{x}_2 \pm \{\boldsymbol{\varepsilon}\}_{m_2} \le \mathbf{B}_2 \quad \text{or} \quad \mathbf{A}_2\mathbf{x}_2 \pm \{\boldsymbol{\varepsilon}\}_{m_2} \ge \mathbf{B}_2 \\ & \underline{\mathbf{x}} \le \mathbf{x} \le \overline{\mathbf{x}}, \qquad \varepsilon_i \ge 0 \end{cases} \tag{8}$$

where m is the number of constraints and $m_1 + m_2 = m$. In the optimization model (1), the constraint conditions

$$A_{\mathrm{L}} = \int_{f_1}^{f_2} (-S_{21})\mathrm{d}f \le T_{\mathrm{L}}$$

$$A_{\mathrm{R}} = \int_{f_3}^{f_4} (-S_{21})\mathrm{d}f \le T_{\mathrm{R}}$$

often lead to no feasible solution. For this case, the two constraints are improved by introducing slack variables and the inequality constraints are converted into the equality constraints:

$$A_{\mathrm{L}} - \varepsilon_{\mathrm{L}} = T_{\mathrm{L}} \tag{9a}$$

$$A_{\mathrm{R}} - \varepsilon_{\mathrm{R}} = T_{\mathrm{R}} \tag{9b}$$

where ε_{L} and ε_{R} are the slack variables introduced in the constraints of the left of the stop-band and the right of the stop-band, respectively. Thus the optimization model becomes:

$$\begin{cases} \text{Find:} & a, d, \varepsilon_{\mathrm{r}}, \varepsilon_{\mathrm{L}}, \varepsilon_{\mathrm{R}} \\ \text{Minimize:} & \varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{R}} \\ \text{Subject to:} & \underline{a} \le a \le \overline{a} \\ & \underline{d} \le d \le \overline{d} \\ & d/a \le k \\ & \underline{\varepsilon}_{\mathrm{r}} \le \varepsilon_{\mathrm{r}} \le \overline{\varepsilon}_{\mathrm{r}} \\ & A_{\mathrm{L}} - \varepsilon_{\mathrm{L}} = T_{\mathrm{L}} \\ & A_{\mathrm{R}} - \varepsilon_{\mathrm{R}} = T_{\mathrm{R}} \\ & \varepsilon_{\mathrm{L}} \ge 0 \\ & \varepsilon_{\mathrm{R}} \ge 0 \end{cases} \tag{10}$$

Once the optimization model (1) encounters an infeasible solution, iterations can be continued by using the improved model (10) until a convergent solution is found.

2.3. Objective and constraint function

In order to use the sequential linear programming, we need to process the objective function and the optimization function. The first derivative of the objective function G can be expressed as

$$\mathbf{g} = \left(\frac{\partial G}{\partial a}, \frac{\partial G}{\partial d}, \frac{\partial G}{\partial \varepsilon_{\mathrm{r}}}\right)^{\mathrm{T}} \tag{11}$$

For the constraint function, increase the move limits. In the optimization process, suppose $\underline{d}_i^{(v)}$ and $\bar{d}_i^{(v)}$ are the lower and upper bound of the move limit of the i-th component of the design variable in v-th iteration. Here we use the move limits instead of the variable interval constraint $\underline{x}_i \le x_i \le \bar{x}_i$ in the optimization model (1); it can be expressed in the form of components

$$\underline{d}_i^{(v)} \le x_i \le \bar{d}_i^{(v)} \tag{12}$$

For the constraint $A_{\mathrm{L}} = \int_{f_1}^{f_2}(-S_{21})\mathrm{d}f \le T_{\mathrm{L}}$, the first derivative of G is as follows:

$$\mathbf{g}_{\mathrm{L}} = \left(\frac{\partial A_{\mathrm{L}}}{\partial a}, \frac{\partial A_{\mathrm{L}}}{\partial d}, \frac{\partial A_{\mathrm{L}}}{\partial \varepsilon_{\mathrm{r}}}\right)^{\mathrm{T}} \tag{13}$$

In v-th iteration, suppose that the current value of the design variable is $\mathbf{x}^* = (a^*, d^*, \varepsilon_{\mathrm{r}}^*)^{\mathrm{T}}$, the transmission quantity on the left of the stop-band is A_{L}^*; $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^*$ is the design variable of the incremental vector, then the constraint condition can be expressed as

$$\mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x} \le T_{\mathrm{L}} - A_{\mathrm{L}}^* + \mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x}^* \tag{14}$$

Similarly, for the constraint $A_{\mathrm{R}} = \int_{f_3}^{f_4}(-S_{21})\mathrm{d}f \le T_{\mathrm{R}}$, we have

$$\mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x} \le T_{\mathrm{R}} - A_{\mathrm{R}}^* + \mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x}^* \tag{15}$$

where $\mathbf{g}_{\mathrm{R}}$ is the first derivative of A_{R}, the transmission quantity on the right of the stop -band is A_{R}^*.

The constraint conditions in the improved model (10) are processed by the same method. Therefore, the constraints $A_{\mathrm{L}} - \varepsilon_{\mathrm{L}} = T_{\mathrm{L}}$ and $A_{\mathrm{R}} - \varepsilon_{\mathrm{R}} = T_{\mathrm{R}}$ can be written as follows:

$$\mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x} = T_{\mathrm{L}} + \varepsilon_{\mathrm{L}}^* - A_{\mathrm{L}}^* + \mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x}^* \tag{16}$$

$$\mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x} = T_{\mathrm{R}} + \varepsilon_{\mathrm{R}}^{*} - A_{\mathrm{R}}^{*} + \mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x}^{*} \tag{17}$$

where both $\varepsilon_{\mathrm{L}}^{*}$ and $\varepsilon_{\mathrm{R}}^{*}$ are the current values in ν-th iteration.

2.4. The improved model used for solving

Based on the above discussion, the optimization model (1) and the improved model (10) can be converted into the following linear programming problem:

$$\begin{cases} \text{Find:} & a,\, d,\, \varepsilon_{\mathrm{r}} \\ \text{Minimize:} & G = \mathbf{g}^{\mathrm{T}}\mathbf{x} \\ \text{Subject to:} & \underline{d}_1 \le a \le \overline{d}_1 \\ & \underline{d}_2 \le d \le \overline{d}_2 \\ & d/a \le k \\ & \underline{d}_3 \le \varepsilon_{\mathrm{r}} \le \overline{d}_3 \\ & \mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x} \le T_{\mathrm{L}} - A_{\mathrm{L}}^{*} + \mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x}^{*} \\ & \mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x} \le T_{\mathrm{R}} - A_{\mathrm{R}}^{*} + \mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x}^{*} \end{cases} \tag{18}$$

$$\begin{cases} \text{Find:} & a,\, d,\, \varepsilon_{\mathrm{r}},\, \varepsilon_{\mathrm{L}}^{*},\, \varepsilon_{\mathrm{R}}^{*} \\ \text{Minimize:} & \varepsilon_{\mathrm{L}}^{*} + \varepsilon_{\mathrm{R}}^{*} \\ \text{Subject to:} & \underline{d}_1 \le a \le \overline{d}_1 \\ & \underline{d}_2 \le d \le \overline{d}_2 \\ & d/a \le k \\ & \underline{d}_3 \le \varepsilon_{\mathrm{r}} \le \overline{d}_3 \\ & \mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x} = T_{\mathrm{L}} + \varepsilon_{\mathrm{L}}^{*} - A_{\mathrm{L}}^{*} + \mathbf{g}_{\mathrm{L}}^{\mathrm{T}}\mathbf{x}^{*} \\ & \mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x} = T_{\mathrm{R}} + \varepsilon_{\mathrm{R}}^{*} - A_{\mathrm{R}}^{*} + \mathbf{g}_{\mathrm{R}}^{\mathrm{T}}\mathbf{x}^{*} \\ & \varepsilon_{\mathrm{L}}^{*} \ge 0 \\ & \varepsilon_{\mathrm{R}}^{*} \ge 0 \end{cases} \tag{19}$$

In the process of solving by the sequential linear programming method, the optimization model (18) is used for the iterative optimization of the feasible solution. For the case of no feasible solution, the iteration will be continued by using the improved model (19). The combination of the two is to find the convergent solution that is satisfying the constraint conditions.

3. Numerical results

For photonic crystal filter structures in the waveguide as shown in Fig. 1, the width and height of the waveguide are 57 and 23 mm, respectively. Let the number of the waveguide period be 9 in this paper. The center frequency of 5.5 GHz and bandwidth of 2 GHz is designed by using the improved model. Choosing $f_1 = 3$ GHz, $f_2 = 4.5$ GHz, $f_3 = 6.5$ GHz, $f_4 = 8$ GHz, $\underline{a} = 0.002$ mm, $\overline{a} = 100$ mm, $\underline{d} = 0.001$ mm, $\overline{d} = 100$ mm, $\underline{\varepsilon}_r = 1.1$, $\overline{\varepsilon}_r = 10$, and $k = 0.9$, the initial design variables are $a = 20$ mm, $d = 12$ mm, $\varepsilon_r = 3.35$. Three selections of constraint conditions are discussed as follows: $T_L = T_R = 2.0$ (1); $T_L = 2.5$, $T_R = 1.5$ (2); and $T_L = 2.7$, $T_R = 1.0$ (3).

Numerical example 1. In the case of $T_L = T_R = 2.0$, Table 1 gives the optimization process and results of 21 iterations. We see that the value of A_L does not satisfy the

T a b l e 1. Optimization process and result of $T_L = T_R = 2.0$.

Iterative step	G	A_L	A_R	Design variable		
				a [mm]	d [mm]	ε_r
0	−42.2288	4.84208	0.07740	20	12	3.35
1	−44.1091	3.38961	0.09461	20.1159	11.8	3.15
2	−43.3218	3.21069	0.27698	20.0682	11.62	2.97
3	−40.8282	2.24587	0.93305	19.6632	11.458	2.80800
4	−41.3677	2.03503	1.44938	20.0277	11.3122	2.72107
5	−42.9436	1.99682	1.47435	20.3557	11.1810	2.69475
6	−44.3407	1.99510	1.40714	20.6510	11.0629	2.67854
7	−45.4101	1.99697	1.33941	20.9167	10.9566	2.66275
8	−46.2761	1.99813	1.27739	21.1558	10.8609	2.64734
9	−46.9758	1.99881	1.22047	21.3711	10.7748	2.63287
10	−47.5170	1.99927	1.16795	21.5648	10.6974	2.61959
11	−48.0607	1.99582	1.48036	21.3904	10.6276	2.64584
12	−48.5157	1.99957	1.41350	21.5473	10.5649	2.63590
13	−48.9452	1.99660	1.71961	21.4061	10.5284	2.65740
14	−49.3098	1.99980	1.64924	21.5332	10.4575	2.64943
15	−49.6522	1.99741	1.93346	21.4188	10.4118	2.66704
16	−49.9571	1.99980	2.00254	21.4392	10.3706	2.66976
17	−50.2140	1.99960	1.99673	21.4992	10.3335	2.66736
18	−50.4334	1.99968	1.99740	21.5493	10.3002	2.66550
19	−50.6248	1.99975	1.99780	21.5939	10.2702	2.66389
20	−50.7922	1.99980	1.99816	21.6334	10.2432	2.66389
21	−50.9391	1.99984	1.99846	21.6685	10.2188	2.66132

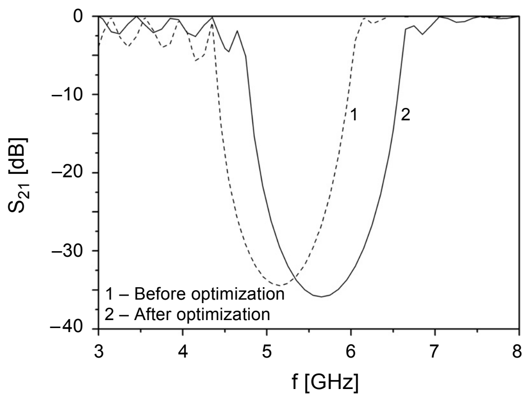

Fig. 2. Comparison of stop-band characteristics before and after optimization.

constraints of the optimization model at the beginning, with the increase of the number of iterations, the constraints A_L and A_R are gradually approaching 2.0. The objective function value exhibit fluctuations in the first four optimizations, which is the result of using the improved model under the more restrictive constraints. After 4 iterations, the objective function values decrease gradually, and finally keep constant at about –50.9. When the function value G converges, the design variable a have a trend of a slow increase and variables d and ε_r still show a little decrease. After 21 iterations, we obtain the optimized function values, where a = 21.6685 mm, d = 10.2188 mm, ε_r = 2.66132, respectively.

Figure 2 shows the stop-band characteristics before optimization and after optimization. It is observed that after optimization the stop-band of the filter is deeper and wider than that before optimization. Thus, the filtering performance of the photonic crystal filter is improved. Furthermore, the stop-band center frequency of this filter is 5.15 GHz before optimization. Under the action of the constraint conditions, the center frequency of the stop-band has been moved, the optimized stop-band center frequency is 5.65 GHz and the bandwidth is 2 GHz, whereas it has a certain deviation from the center frequency of the expected 5.5 GHz.

Numerical example 2. When the constraint conditions are T_L = 2.5 and T_R = 1.5, the optimization process is given in Table 2. Clearly, with the increase of the number of iterations, the constraint A_L is convergent at about 2.5 and A_R is convergent at about 1.5. The objective function values have a trend of a slow decrease and finally the function value converges to –53.8. Design variable a is gradually increasing, while d and ε_r are slowly decreasing. Finally, the optimum point of design variables corresponds to a = 21.4525 mm, d = 10.2702 mm, ε_r = 2.7814.

Figure 3 gives the comparison of stop-band characteristics before and after optimization. We can see that the stop-band center frequency is 5.15 GHz before optimization, the optimized stop-band center frequency is 5.55 GHz and the bandwidth is 2 GHz, indicating that the optimization process is effective. There is still a 0.05 GHz

Table 2. Optimization process and result of $T_L = 2.5$, $T_R = 1.5$.

Iterative step	G	A_L	A_R	Design variable		
				a [mm]	d [mm]	ε_r
0	−42.2288	4.84208	0.0774	20	12	3.35
1	−44.6931	3.48970	0.08039	20.2782	11.8	3.15
2	−46.0365	3.27765	0.19891	20.7282	11.62	2.9867
3	−45.3924	3.01578	0.39018	20.9773	11.4580	2.82470
4	−42.8240	2.07711	0.83193	20.6128	11.3122	2.67890
5	−46.1779	2.50882	0.48661	20.9409	11.1810	2.74577
6	−47.2421	2.50807	0.71367	20.6456	11.0629	2.79466
7	−48.0158	2.50353	1.04550	20.3799	10.9566	2.83924
8	−48.9887	2.49497	1.07993	20.6190	10.8609	2.81712
9	−49.7924	2.49674	1.08598	20.8343	10.7748	2.79949
10	−50.4482	2.50084	1.40284	20.6406	10.6974	2.83387
11	−51.0591	2.49798	1.40090	20.8149	10.6276	2.81883
12	−51.5958	2.49981	1.50345	20.8563	10.5649	2.82114
13	−52.0431	2.49899	1.48861	20.9948	10.5084	2.80990
14	−52.4385	2.49953	1.49175	21.0977	10.4575	2.80296
15	−52.7844	2.49970	1.49293	21.1888	10.4118	2.79697
16	−53.0909	2.49982	1.49416	21.2681	10.3706	2.79202
17	−53.3633	2.49990	1.49521	21.3375	10.3335	2.78787
18	−53.6055	2.49995	1.49609	21.3986	10.3002	2.78437
19	−53.8208	2.49998	1.49683	21.4525	10.2702	2.78140

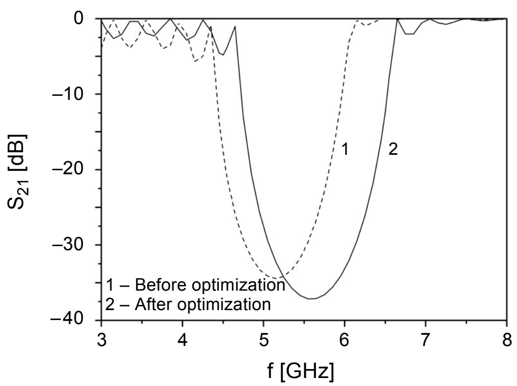

Fig. 3. Comparison of stop-band characteristics before and after optimization.

deviation, but the stop-band of the filter is deeper and wider than that before optimization and the filter performance is improved.

Numerical example 3. Here we let $T_L = 2.7$ and $T_R = 1.0$. Table 3 gives the optimization process and results of 20 iterations. A_L is eventually convergent at about 2.7 and A_R is convergent at about 1.0. The objective function value is reduced in the whole

T a b l e 3. Optimization process and result of $T_L = 2.7$, $T_R = 1.0$.

Iterative step	G	A_L	A_R	Design variable		
				a [mm]	d [mm]	ε_r
0	−42.2288	4.84208	0.0774	20	12	3.35
1	−44.8993	3.59998	0.07556	20.3432	11.8	3.15
2	−46.6362	3.21206	0.16622	20.7932	11.62	3.00894
3	−46.9114	3.15445	0.32142	21.1982	11.458	2.87256
4	−47.0464	2.98762	0.43884	21.5627	11.3122	2.77836
5	−45.8861	2.56373	0.55192	21.8907	11.1810	2.64714
6	−48.3119	2.81356	0.54587	21.5955	11.0629	2.74541
7	−48.6781	2.63235	0.53434	21.3298	10.9566	2.74623
8	−50.0825	2.72421	0.60628	21.0906	10.8609	2.80370
9	−50.7571	2.70535	0.84286	20.8754	10.7748	2.83587
10	−51.4466	2.69687	0.85380	21.0691	10.6974	2.81848
11	−52.0713	2.69808	0.84588	21.2434	10.6276	2.80520
12	−52.6716	2.70326	1.02247	21.1321	10.5649	2.82690
13	−53.0241	2.69843	0.99597	21.2733	10.5200	2.81383
14	−53.4444	2.69969	0.99629	21.3787	10.4691	2.80733
15	−53.7981	2.69984	0.99662	21.4704	10.4234	2.80185
16	−54.1026	2.70000	0.99694	21.5497	10.3822	2.79747
17	−54.3662	2.70010	0.99727	21.6187	10.3451	2.79393
18	−54.5957	2.70016	0.99759	21.6790	10.3118	2.79107
19	−54.7967	2.70019	0.99789	21.7318	10.2818	2.78874
20	−54.9736	2.70020	0.99818	21.7783	10.2548	2.78684

iterative process, and finally keeps constant at about –54.9, where a, d and ε_r are 21.7783 mm, 10.2548 mm and 2.78684, respectively.

It can be seen that the optimized center frequency stop-band is 5.5 GHz and the bandwidth is 2 GHz, as shown in Fig. 4, which achieves the expected design purpose.

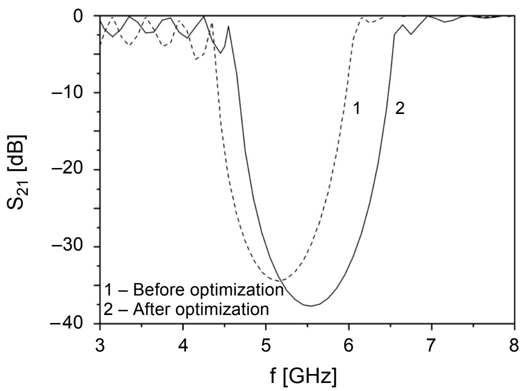

Fig. 4. Comparison of stop-band characteristics before and after optimization.

After optimization, the filtering performance of the photonic crystal filter is obviously improved by the increased width and depth of the stop-band.

4. Conclusion

This paper designs an improved model based on slack variables to optimize a photonic crystal filter. Firstly, the optimization model of the photonic crystal filter is constructed with the objective function of minimizing the transmission quantity in the stop-band and the constraint that the absolute value of transmission quantity is less than the maximum allowable value. Then the optimization model is improved by introducing slack variables to find a minimum solution which violates the constraint condition in the infeasible region. In the case of no feasible solution, by the change of the objective function and the design variables with iteration steps, the stop-band characteristics of the filter can be evaluated and the results before and after optimization are presented in numerical examples. Furthermore, the influence of constraints on the central frequency of the stop-band is discussed. The conclusions are drawn as follows:

1. By changing the constraint conditions on two sides of the stop-band, we can adjust the center frequency of the photonic crystal filter.
2. Depth and width of the stop-band is improved obviously after optimization, indicating that the filtering performance of the photonic crystal filter has been improved.
3. The improved model proposed in this paper is practicable and effective to solve the optimization problem of no feasible solution for a photonic crystal filter.

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