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Estimation of the Crushed Ore Particles Density in the Pulp Flow Based on the Dynamic Effects of High-Energy Ultrasound

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The method of automatic measurement of the ore particles density in the pulp flow by measuring the amount of high-frequency volume ultrasonic oscillations attenuation, which have passed a fixed distance in the test medium under the influence of high-energy ultrasound dynamic effects is considered.

The results of ultrasonic field parameters calculation and spatial simulation of high-energy ultrasound radiation pressure effect on the pulp flow, as well as the results of modeling the trajectory of ore particles displacement of three fractions in the pulp flow under the influence of high-energy ultrasound radiation pressure are presented.

Keywords: concentration; high-energy ultrasound; bulk-mode ultrasonic waves; particle size distribution; pulp characteristics.

1. Introduction

In order to control ore concentration effectively we need three types of controlled parameters characterizing the required quality and quantity of the processed ore materials as well as the production situations and the equipment state (Kosharskiy, Sitkovskiy, 1977; Sihul, 1989; Protsuto, 1987).

In (Kosharskiy, Sitkovskiy, 1977; Sihul, 1989; Protsuto, 1987; Rzhevskiy, Yamshchikov, 1968; Brazhnikov *et al.*, 1975; Humaniuk, 1970; Yamshchikov, Korobeinikov, 1967; Brajnikov, 1975; *Ultrasound*, 1979; Bergman, 1957) various ultrasound control methods and devices used in technological process automation are described. The authors indicate that such advantages of these methods as accuracy and reliability in measuring aggressive medium parameters make them some of the most prospective approaches in developing measuring complexes for process automated systems.

The known methods of ultrasound control over the pulp parameters make it possible to distinguish its two basic characteristics – density and grain-size composition (Morkun *et al.*, 2014a; 2014b; 2014c; 2014d; 2015a; 2015b; 2015c). To measure these parameters volume ultrasonic waves are usually used.

In (Morkun *et al.*, 2015a; 2015b) revealed that control of the useful component content and the minerals disclosure degree at a known particle size of the analyzed particle, which is ground in the process of ore dressing can be reduced to measuring the density of this particle.

2. Materials and methods

Let's consider a method of ore particle density automated control in the pulp flow based on measuring the intensity of high-frequency ultrasonic waves passing through the analyzed medium and the dynamic effects of the high-energy ultrasound.

As a result of the radiation pressure of the high-energy ultrasound, the size redistribution of crushed ore particles occurs in the measuring zone. In case of the pulp flow constant speed these redistribution characteristics are determined by the ultrasound field intensity, the solid pulp concentration and properties.

To solve the above-mentioned task one should define the analytical dependencies of the ultrasound field parameters, the crushed ore concentration and size in the pulp for each point of the simulated space.

Figure 1 reveals the results of the space simulation of the ultrasound field parameters and radiation pressure impact of the high-energy ultrasound source on the pulp flow in Matlab (Holzbecher, 2012; Hultiaev, 1999). To make our analysis simple we present the flow section in different planes and perform a step digitization of the ultrasound intensity in the space coordinates.

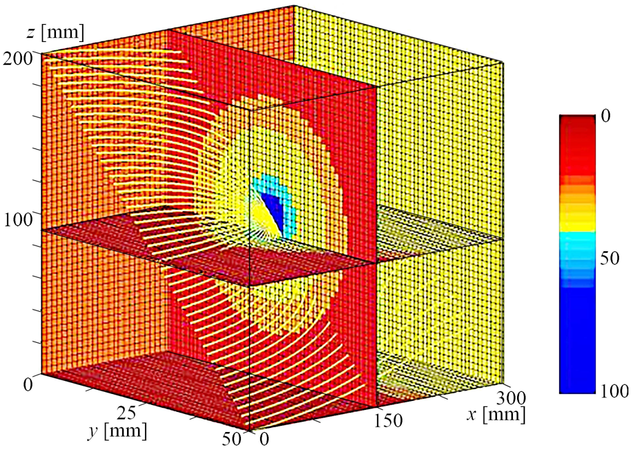

Fig. 1. The results of calculating the parameters of the ultrasound field and the space simulation of the high-energy ultrasound radiation pressure on the pulp flow.

We assess the influence of the ultrasound pressure on the changes in particles concentration of r radius. Let the pulp with velocity of V flows in the positive direction of the axis OX (Fig. 2). Let's denote by $n_r(Z,t)$ the concentration of the particles of radius r at the depth Z at the moment of t.

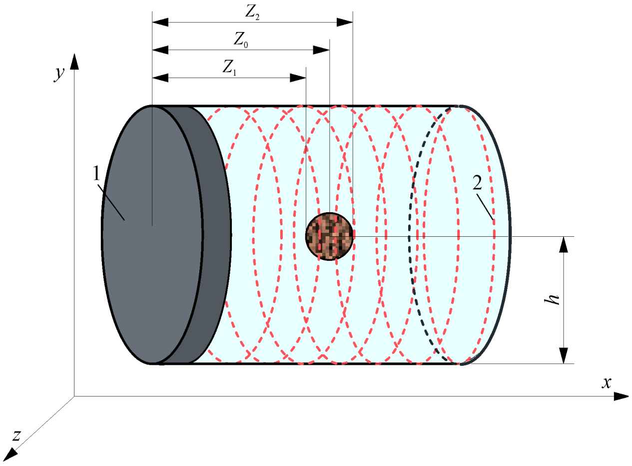

Fig. 2. The particle motion in the intensive ultrasound field.

The timing point is from the moment of the ultrasound exposure. In order to describe time variation of the particle concentration, we should write down a balance equation used for deriving transfer equations such as a diffusion equation, a heat equation, etc. The equation will look like

$$\frac{\partial n_r(Z,t)}{\partial t} = -\frac{\partial}{\partial Z}\left[V_r(Z,t)n_r(Z,t)\right], \tag{1}$$

where $V_r(Z,t)$ is the velocity of particle displacement of r radius and the coordinate Z in the ultrasound field.

The velocity is directed along the axis OZ, that is, it is perpendicular to the pulp flow. In a general case, it depends on the time t as the ultrasound exposure changes the particle concentration resulting in the changes of the ultrasound intensity and the particle displacement velocity. This fact considerably complicates the equation (1), so let's consider that the velocity depends on the coordinate Z only.

If we turn to a new variable in (1)

$$\varphi = \int\limits_0^Z \frac{\mathrm{d}Z'}{V_r(Z')}, \tag{2}$$

its solution under the given initial and boundary conditions

$$n_r(Z,0) = n_0; \quad n_r(0,t) = 0$$

will be as follows

$$n_r(Z,t) = \frac{n_0 V_r\left(\varphi^{-1}\left(\int\limits_0^z \frac{\mathrm{d}Z'}{V_r(Z')} - t\right)\right)}{V_r(Z)} \cdot St\left(\int\limits_0^z \frac{\mathrm{d}Z'}{V_r(Z')} - t\right), \tag{3}$$

where $St(X)$ is a step function with the property:

$$St(X) = \begin{cases} 0, & X < 0, \\ 1, & X \geq 0, \end{cases}$$

$\varphi^{-1}(X)$ is an inverse function (2).

We find the particle displacement velocity of r radius in the ultrasound field. The force of the radiation pressure on the particle in the plane wave is determined by the formula

$$F_r = \overline{E}(\sigma_s + \sigma_p) - \overline{E}\oint I_v \cos v \,\mathrm{d}S, \tag{4}$$

where $\overline{E}$ is the time-average energy density in the incident wave; σ_s and σ_p are effective scattering and absorption cross-sections; v is the angle between the directions of the incident and scattered waves; I_v is the value of the scattered wave intensity at an angle v.

The formula (4) was obtained by Westervelt (Rosenberg, 1967) and is a general expression for the radiation pressure force.

We obtain a similar expression for the radiation pressure force, presented by means of full and differential cross-sections of ultrasound scattering and absorption on the particles

$$F_r = \frac{I}{c}(\sigma_p + \sigma_s \mu), \tag{5}$$

where I is the incident wave intensity; c is its propagation velocity;

$$\mu = \frac{2\pi}{\sigma_s} \int_{-1}^{1} \mathrm{d}\cos v \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\cos v)(1 - \cos v).$$

For the spherical particles of r radius, the differential effective cross-section of scattering looks like

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\cos v) = \frac{r^2}{9}(kr)^4 \left(a_1 - \frac{3}{2}a_2 \cos v\right)^2, \tag{6}$$

where $a_1 = 1 - \frac{rc^2}{\rho_s c_s^2}$, $a_2 = 2\frac{\rho_s - \rho}{2\rho_s + \rho}$; ρ_s, c_s is the particle density and the ultrasound velocity in the particle material; ρ is the medium density.

So, at high frequencies $\sigma_p \ll \sigma_s$, substituting Eq. (6) into (5) we obtain

$$F_r = \frac{4}{9}\pi r^2 (kr)^4 \left(a_1^2 + a_1 a_2 + \frac{3}{4}a_2^2\right)\frac{I}{c}. \tag{7}$$

In the steady-state conditions, when the particle displacement velocity in the ultrasound field is constant, the condition takes place

$$F_r - F_c = 0, \tag{8}$$

where F_c is the resistance force determined by the Stokes formula

$$F_c = 6\pi r \eta V_r. \tag{9}$$

Substituting Eqs. (7) and (9) into (8), we find the velocity of the steady particle motion

$$\begin{aligned} V_r(Z) &= \frac{F_r(Z)}{6\pi r \eta} \\ &= \frac{2r(kr)^4}{27\eta c}\left(a_1^2 + a_1 a_2 + \frac{3}{4}a_2^2\right) I_0 e^{-\alpha z}. \end{aligned} \tag{10}$$

We suppose here that the ultrasonic wave intensity changes exponentially and the coefficient α depends on the sound frequency ν_o.

Thus, the dependency of the particle displacement velocity on its coordinate Z can be presented as follows

$$V_r(Z) = \beta e^{-\alpha z}, \tag{11}$$

where

$$\beta = \frac{2r(kr)^4}{27\eta c} I_0 \left(a_1^2 + a_1 a_2 + \frac{3}{4}a_2^2\right).$$

In order to find the function in an explicit form (3), we need to know the function φ and its inverse function. Substituting Eq. (11) into (2), we obtain

$$\varphi = \int_0^z \frac{\mathrm{d}Z'}{\beta e^{-\alpha z'}} = \frac{1}{\alpha\beta}(e^{\alpha z} - 1), \tag{12}$$

and the inverse function looks like

$$Z = \frac{1}{\alpha}\ln(1 + \alpha\beta\varphi). \tag{13}$$

Considering Eqs. (11) and (13) the particle concentration $n_r(Z,t)$ is determined by the formula

$$n_r(Z,t) = n_0 \frac{e^{\alpha z}}{e^{\alpha z} - \alpha\beta t} St(e^{\alpha z} - 1 - \alpha\beta t). \tag{14}$$

In the real pulp, solid particles are of various sizes. Let's describe the size distribution of particles by the function $f_\eta(r)$. Besides, particles have different density depending mostly on the particle size. Let's assume that $\widetilde{\varphi}(r,\rho)$ is the distribution function of particles as to their density of r radius.

To assess experimentally the impact of the particle displacement under the action of the intensive ultrasound one should determine the changes in the ultrasound signal with the frequency ν in the direction perpendicular to the pulp motion. Let's call this ultrasound signal sounding. The controlled zone is a cylinder of the radius R and the height l (the distance between the radiator and the receiver of the sounding signal). The cylinder axis coordinates are in Fig. 2.

Under these conditions the sounding signal attenuation at the moment t is determined by the formula

$$\begin{aligned} I_\nu &= I_0 \int_{z_1}^{z_2} \mathrm{d}Z\, 2\sqrt{R^2 - (R + Z_1 - Z)^2} \\ &\cdot \exp\left\{-\frac{Wl}{\aleph}\int_0^\infty \mathrm{d}r\, f_\eta(r)\,\sigma(r,v)\right. \\ &\left.\cdot\left[\int_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho\, \widetilde{\phi}(r,\rho) a^* St(e^{\alpha z_0} - 1 - \alpha\beta t)\right]\right\}, \end{aligned} \tag{15}$$

where

$$a^* = \frac{e^{\alpha Z}}{e^{\alpha Z} - \alpha\beta t}.$$

If we assume that the cross-section sizes of the sounding zone are small, by means of the mean-value theorem the expression (15) can be written as

$$\begin{aligned} I_\nu &= I_0 \pi R^2 \exp\left\{-\frac{Wl}{\aleph}\int_0^\infty \mathrm{d}r\, f_\eta(r)\,\sigma(r,v)\right. \\ &\left.\cdot\left[\int_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho\, \widetilde{\phi}(r,\rho) b^* St(e^{\alpha z_0} - 1 - \alpha\beta t)\right]\right\}, \end{aligned} \tag{16}$$

where

$$b^* = \frac{e^{\alpha z_0}}{e^{\alpha z_0} - \alpha\beta t}.$$

In the formulae (15) and (16), $\sigma(r,v)$ determines the full cross-section of the ultrasound attenuation with the frequency ν on the particle of r radius

$$\aleph = \int\limits_0^\infty \mathrm{d}r\, f_\eta(r)\frac{4\pi}{3}r^3.$$

We create two signals according to the results of measuring the intensity or the amplitude of the sounding ultrasonic wave. One signal is determined by the ultrasound attenuation in the pulp without the intensive ultrasound field impact

$$S_0 = \ln\left(\frac{\widetilde{I}_0\pi R^2}{\widetilde{I}_v}\right) = \frac{Wl}{\aleph}\int\limits_0^\infty \mathrm{d}r\, f_\eta(r)\sigma(r,v), \qquad (17)$$

and the other is determined under the radiation pressure on the pulp particles

$$S_1 = \ln\left(\frac{I_0\pi R^2}{I_v}\right) = \frac{Wl}{\aleph}\int\limits_0^\infty \mathrm{d}r\, f_\eta(r)\sigma(r,v) \cdot \left[\int\limits_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho\, \widetilde{\phi}(r,\rho)c^* St(e^{\alpha z_0} - 1 - \alpha\beta t)\right], \qquad (18)$$

where

$$c^* = \frac{e^{\alpha z_0}}{e^{\alpha z_0} - \alpha\beta t}.$$

Let's find the ratio of these signals

$$\frac{S_1}{S_0} = \frac{d^*\left[\int\limits_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho\, \widetilde{\phi}(r,\rho)e^* St(e^{\alpha z_0} - 1 - \alpha\beta t)\right]}{d^*}, \qquad (19)$$

where

$$d^* = \int\limits_0^\infty \mathrm{d}r\, f_\eta(r)\sigma(r,v),$$

$$e^* = \frac{e^{\alpha z_0}}{e^{\alpha z_0} - \alpha\beta t}.$$

As the expression (19) reveals, this value depends on the ultrasound field intensity, exposure time and particle distribution as to density and size. Let's analyze this expression.

If the value βt is such that the step function argument takes positive values, then

$$\frac{S_1}{S_0} \approx \frac{e^{\alpha z_0}}{e^{\alpha z_0} - \alpha\langle\beta t\rangle}, \qquad (20)$$

where $\langle\beta t\rangle$ is a value average in density and particle size, that is

$$\frac{e^{\alpha z_0}}{e^{\alpha z_0} - \alpha\langle\beta t\rangle} = \frac{\int\limits_0^\infty \mathrm{d}r\, f_\eta(r)\sigma(r,v)\int\limits_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\rho\, \widetilde{\phi}(r,\rho)f^*}{\int\limits_0^\infty \mathrm{d}r\, f_\eta(r)\sigma(r,v)},$$

where

$$f^* = \frac{e^{\alpha z_0}}{e^{\alpha z_0} - \alpha\beta t}.$$

As the intensive ultrasound exposure time depends on the pulp motion velocity along the axis X, then $t \approx h/V$. Thus, changing the relative position of the sounding channel and the ultrasound field, we can change the exposure time t while the ultrasound field intensity generally influences the value β.

The choice of the ultrasound intensity and frequency and the position of the sounding channel is based on the maximum sensitivity of the value S_1/S_0 to the changed density of particles.

The dependency of the signal S_1/S_0 on the value aZ_0 under different values $\langle\beta t\rangle$ shown in Fig. 3. As we can see from the figure, the function maximum (20) falls within $\alpha Z_0 = 1$. Under the given position Z_0, it allows us to determine the frequency ν_0 because $\alpha(v_0) = 1/Z_0$.

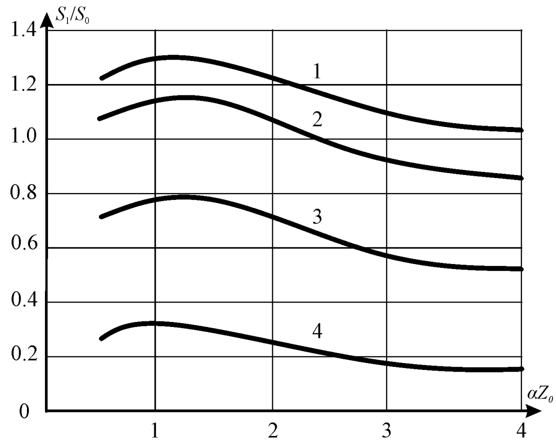

Fig. 3. The dependency of the signal S_1/S_0 on the value aZ_0 under the fixed values $\langle\beta t\rangle$: 1 – $\langle\beta t\rangle \cdot Z_0^{-1} = 0.6$; 2 – $\langle\beta t\rangle \cdot Z_0^{-1} = 0.75$; 3 – $\langle\beta t\rangle \cdot Z_0^{-1} = 1.0$; 4 – $\langle\beta t\rangle \cdot Z_0^{-1} = 1.25$.

The variation of the particles density as to the average value $\overline{\rho}_s$ by 30% causes the change of β by 6%. The dependency of the value S_1/S_0 on $\langle\beta t\rangle$ under the fixed values αZ_0 shown in Fig. 4. This dependency allows us to find the area of the maximum sensitivity to the changes of β making it possible to choose the ultrasound intensity.

Thus, the signal value S_1/S_0 allows us to determine the density of the solid pulp particles.

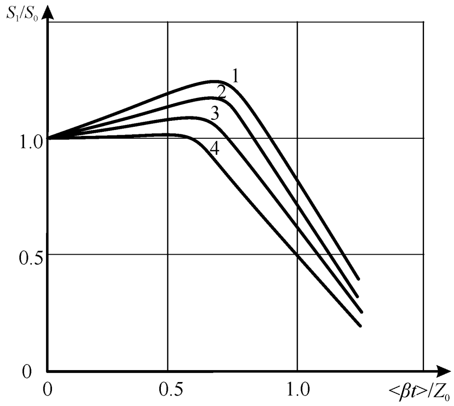

Fig. 4. The dependency of the signal S_1/S_0 on the value $\langle \beta t \rangle$ under the fixed values aZ_0: 1 – $aZ_0 = 1.0$; 2 – $aZ_0 = 2.0$; 3 – $aZ_0 = 3.0$; 4 – $aZ_0 = 4.0$.

3. Results

The given method of ultrasound control of the solid pulp particles density was realized by means of the developed hardware-in-the-loop complex. The calculation of the high-intensity ultrasound power for the designed displacement of the crushed ore particles of some quantity in the pulp flow is based on the above-obtained results of investigating the ultrasound impulse front propagation by means of the package HIFU Simulator v1.2 (Soneson, 2011).

Figure 10 reveals the results of the trajectory simulation for ore particle displacement of three size divisions in the pulp flow under the high-energy ultrasound radiation pressure. The positions of particles of each size division are joined by solid lines at the tenth step.

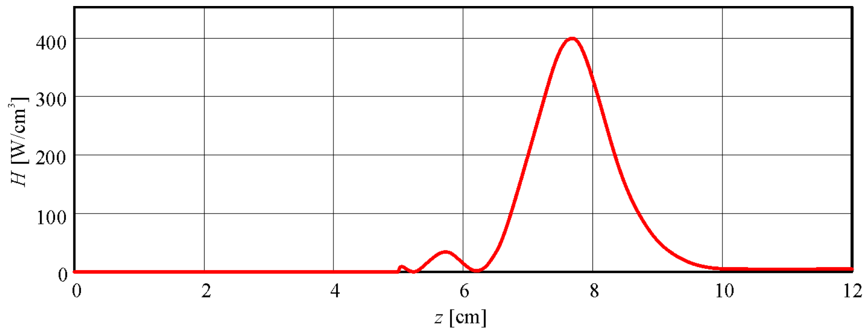

Fig. 5. The axis power of ultrasonic radiation.

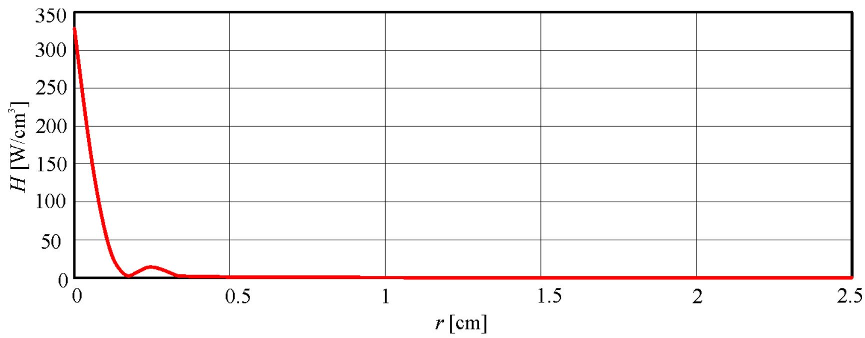

Fig. 6. Radial power in the ultrasonic radiation focus.

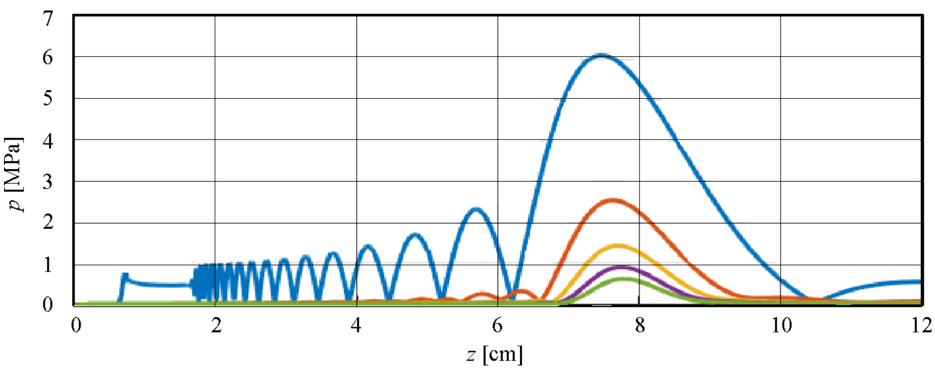

Fig. 7. Axial pressure distribution of the first five harmonics of ultrasonic radiation.

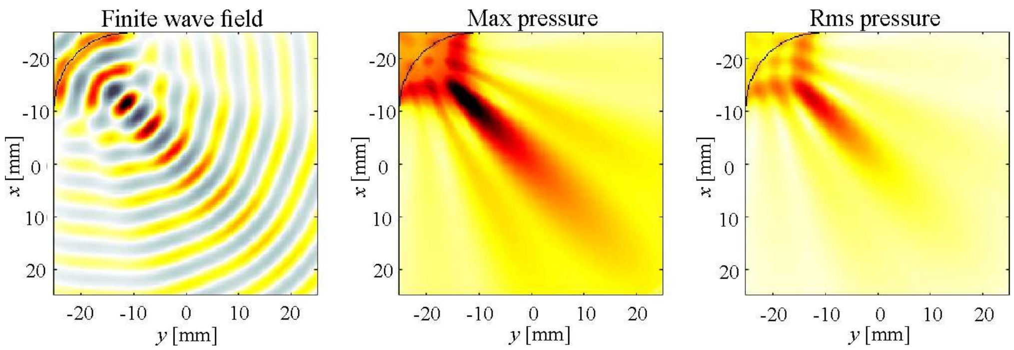

Fig. 8. Modeling results of the high-energy ultrasound focusing in iron-ore pulp.

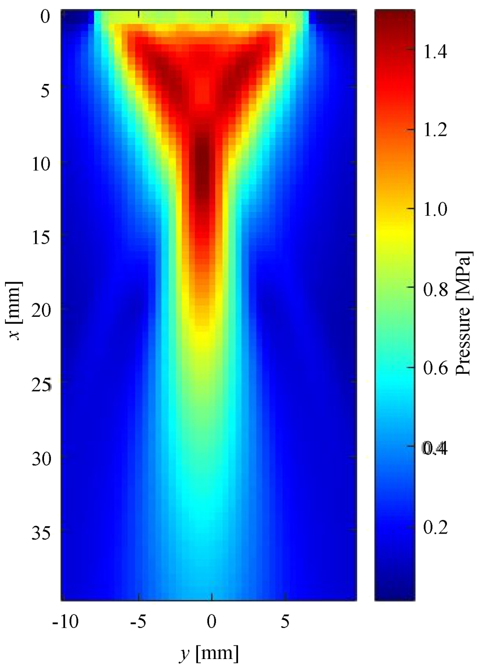

Fig. 9. Modeling results of the high-energy ultrasound pressure in iron ore pulp (density 1350 kg/m^3).

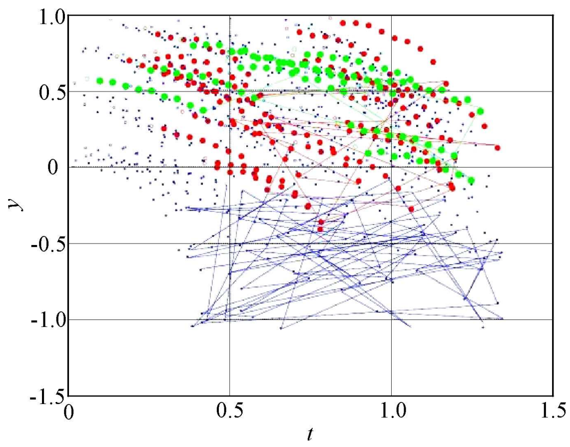

Fig. 10. Simulation results of three-radius ore particles displacement under the radiation pressure of the high-energy ultrasound.

The developed programme calculates the high-energy ultrasound intensity at some point of the measuring zone for performing the designed displacement of the crushed ore particles of certain quantity and the solid pulp fraction change under the controlled radiation pressure of the high-energy ultrasound. The root-mean-square deviation between the model and the experiment at the control points of the grain-size characteristics made 0.87%. The bulk-mode ultrasonic waves of 5–10 MHz were used to measure the solid particle density of the iron ore pulp of less than 150 μm in size. If the solid pulp concentration in the controlled medium was stable, the calculation error of the crushed ore particle density did not exceed 1%.

4. Conclusions

The density of the crushed material particles in the pulp flow is determined by measuring the changed attenuation value of the high-frequency bulk-mode ultrasound waves covering the fixed distance in the analyzed medium under of the high-energy ultrasound dynamic effects.

It is advisable to combine the suggested method with the measuring channel to determine the solid pulp concentration. For this purpose, one can apply the methods and means described in (Morkun *et al.*, 2014d; 2015a; 2015b; 2015c).

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