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Exponential stability of impulsive delayed nonlinear hybrid differential systems

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Abstract: In recent years, with the rapid development of digital components, digital electronic computers, especially microprocessors, digital controllers have replaced analog controllers on many occasions. The application of digital controller makes the performance analysis of impulsive system more and more important. This paper considers global exponential stability (GES) of impulsive delayed nonlinear hybrid differential systems (IDNHDS).Through the application of the Lyapunov method and the Razumikhin technique, a series of uncomplicated and useful guiding principles have been obtained. The results of a numerical simulation are presented to demonstrate that the method is correct and effective.

Key words: impulsive delayed nonlinear hybrid systems, global exponential Lyapunov function, Razumikhin technique

1. Introduction

Time-delay impulsive systems are widely used in engineering, control technology, biology, economics, communication networks and other fields [1–6]. Therefore, it is very important to study the characteristics of these systems. Among them, stability analysis is an important aspect of studying such systems [7–9]. It is particularly important, especially for the study of exponential stability. Some achievements have been made in the exponential stability analysis of such systems. In [10–12], the authors have studied the exponential stability of such systems by using the Lyapunov and Razumikhin technique.

The research on a hybrid dynamical system (HDS) has become a hotspot in recent years [13, 14]. The hybrid systems provide convenient applications such as robotics, integrated circuit design, automated highway systems, complex physical phenomena. V. Lakshikantham and X.Z. Liu were the first to introduce the concept of the impulsive hybrid systems [15]. In their paper, they established a comparison principle and some stability criteria for such systems. Recently, hybrid impulsive dynamical systems are studied extensively [16-21]. However, further research on the HDS is needed. Results of studying the HDS with delay are rare [22–24]. In [23] the author investigated globally asymptotically and exponentially the stability issue of non-linear impulsive and switching time-delay systems. However, the conclusion of the paper needs a set of conditions, and the time delay is less than the impulsive interval length. In [24], by using a Lyapunov function and Lyapunov functional, the local uniform stability of a time-delay linear hybrid system is studied for two cases with delay independent or delay dependent.

In this paper, by using the Lyapunov method and the Razumikhin technique, we consider the global exponential stability (GES) problems for impulsive delayed nonlinear hybrid differential systems (IDNHDs) for any time delay. Furthermore, the conclusions of this paper can be used to study the GES of the linear impulsive HDS with time delays. Therefore, the results of this paper generalize the results of [24] and study the GES under the same conditions. In addition, the results show that, for a delayed nonlinear or linear differential hybrid system. Even if the system may be unstable, the impulses can help the GES.

The sections of this paper are as follows: In the second section, we will introduce some symbols and definitions. In the third section, we obtain some criteria for GES of impulsive delay nonlinear or linear mixed differential systems with arbitrary delays. In the fourth section, we discuss two examples validate the conclusions in this paper. In the fifth section, the work is briefly discussed.

2. Preliminaries

The symbols and definitions used in this paper are described below. $R = (-\infty, +\infty)$ is the set of real numbers. $R^+ = [0, +\infty)$ is the set of nonnegative real numbers and $N = \{0, 1, 2, \ldots\}$ is the set of natural numbers. For the vector $u \in R^n$, its transpose is denoted as u^T. $\|u\|$ is defined as the norm of vector u. $R^{n\times n}$ is the real matrix with the order $n \times n$.

Consider the following IDNHDS:

$$\begin{cases} \dot{x}(t) = Ax(t) + f(t, x(t), x(t-\tau(t))) + M_k x_k\,, & t \neq t_k\,,\ \ t \geq t_0\,, \\ x(t) = C_k x(t^-), & t = t_k\,,\ \ k \in N, \\ x_{t_0} = \varphi, & \end{cases} \tag{1}$$

where $t \geq t_0$, $\varphi \in PC([-\tau, 0], R^n)$, $x(t) \in R^n$, A, B and C_K, $M_K \in R^{n\times n}$, the time sequence $\{t_k\}_{k=1}^{+\infty}$ satisfy $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim\limits_{k\to\infty} t_k = +\infty$. The time delay $0 \leq \tau(t) \leq \tau \leq +\infty$, $x(t^+) = \lim\limits_{s\to 0^+} x(t+s)$ and $x(t^-) = \lim\limits_{s\to 0^-} x(t+s)$.

Let

$$PC([-\tau, 0],\ R^n) = \{\phi : [-\tau,\ 0] \to R^n\}\,.$$

$\phi(t)$ is continuous except for a limited number of points $\hat{t}$. At the points, the condition that $\phi(\hat{t}^{+})$ and $\phi(\hat{t}^{-})$ exist and $\phi(\hat{t}^{+}) = \phi(\hat{t}^{-})$ is satisfied.

For

$$\Psi \in PC([-\tau, 0],\ R^n),$$

the norm Ψ is defined as:

$$\|\Psi\|_\tau = \sup_{-\tau \le s \le 0} \|\Psi\|\,.$$

$$x_t, x_{t^-} \in PC\big([-\tau,\ 0],\ R^n\big)$$

are defined by $x_t(s) = x(t+s)$ and $x_{t^-}(s) = x(t^- + s)$ for $s \in [-\tau, 0]$, respectively.

Assuming that the initial conditions satisfy, system (1) has unique solutions. Denote by $x(t) = x(t, t_0, \varphi)$ the solution of (1) such that $x_{t_0} = \varphi$.

Further we assume that all the solutions x_t of system (1) are continuous except at some breakpoints t_k, $k \in N$, at which $x(t)$ is right-continuous.

Let

$$x(t_0) = (x_1(t_0),\ x_2(t_0)) = \varphi = (-1, 1), \qquad \Delta t_k = t_{k+1} - t_k\,, \quad k \in N.$$

Obviously, $x(t) = 0$ is the zero solution of (1).

The following definitions are available.

Definition 1. If there are some constants $\lambda > 0$ and $K > 0$ for any initial condition $x_{t_0} = \varphi$, the following inequality is established:

$$\|x(t, t_0, \varphi)\| \le K\|\varphi\|_\tau e^{-\lambda(t-t_0)}, \quad t \ge t_0\,,$$

where

$$(t_0, \varphi) \in R^+ \times PC\,([-\tau, 0],\ R^n)\,.$$

Then the zero solution of (1) is globally exponentially stable. That is, (1) is a global stability system [4].

Definition 2. Function $V : R^+ \times R^N \to R^N$ is said to belong to the class υ_0, if (i) V is continuous in each of the sets $[t_k,\ t_{k+1}) \times R^n$, and it exists for each $x \in R^n$, Ψ, $\lim\limits_{(t,y)\to(t_k^-,x)} V(t, y) = V(t_k^-, x)$; as well (ii) $V(t, x)$ is locally Lipschitzian in all $x \in R^n$, and for all υ_0, $V(t, 0) = t_0$ [4].

Definition 3. Given a function $V : R^+ \times R^n \to R^+$, and an upper right-hand derivative of V with respect to system (1), it is defined by [4].

$$D^+V(t, x(t)) = \lim_{\delta\to 0^+} \sup \frac{1}{\delta}\big[V(t+\delta,\ x(t+\delta)) - V(t, x(t))\big].$$

3. Main results

In this section, some simple and practical criteria will be drawn. These criteria in GES are directed against the system (1). We will combine the Lyapunov-Razumikhin technology with some analysis methods.

Theorem 1. Suppose that P is a square matrix with the order of n, and P is symmetric and positive definite. Suppose that λ_3 and λ_4 are the maximum eigenvalues of

$$P^{-1}\left(A^T P + PA + 2PP + l_1\right) \quad \text{and} \quad l_2 P^{-1},$$

respectively, and suppose that λ_{M_k} and λ_{C_k} are the maximum eigenvalues of

$$P^{-1}C_k^T M_k^T M_k C_k \quad \text{and} \quad P^{-1}C_k^T P C_k,$$

respectively.

Defining $\lambda_5 = \sup\limits_{k\in N} \lambda_{Mk}$, $\lambda_6 = \sup\limits_{k\in N} \lambda_{Ck}$ and $0 \le \lambda_6 \le 1$.

Assume that the constants satisfying $-\ln\lambda_6 \ge 1$, $\lambda > 0$ and $\sigma > 0$, for all $k \in N$. The above constant satisfies the following conditions:

(i) $$F(\lambda) = \sigma - \lambda - \left(\lambda_3 + \frac{\lambda_4}{\lambda_6}e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\right) \ge 0,$$

(ii) $$\ln\lambda_6 < -(\sigma+\lambda)(t_{k+1} - t_k),$$

(iii) Existing two numbers $l_1 > 0$ as well as $l_2 > 0$,

and satisfying

$$\|f(t, x(t), x(t-\tau(t)))\|^2 \le l_1\|x(t)\|^2 + l_2\|x(t-\tau(t))\|^2.$$

Then, for any fixed delays $0 \le \tau(t) \le \tau < +\infty$, the zero solution of (1) is globally exponentially stable, and the convergence rate is $\dfrac{\lambda}{2}$.

Proof

Let $x(t) = x(t, t_0, x_0)$ be any solution of systems (1), which initial state it is.

Constructing a Lyapunov function:

$$V(t, x(t))\, x^T(t)Px(t). \tag{2}$$

Therein, $\lambda_1 > 0$ is the smallest eigenvalue of P, and $\lambda_2 > 0$ is the largest eigenvalue of P, λ_1 and λ_2 meet the following condition:

$$\lambda_1\|x(t)\|^2 \le V(t, x(t)) \le \lambda_2\|x(t)\|^2. \tag{3}$$

The following inequalities will be proved:

$$V(t, x(t)) \le \lambda_2 K\|x_0\|_\tau^2 e^{-\lambda(t-t_0)}, \qquad t \in [t_k, t_{k+1}), \quad k \in N. \tag{4}$$

Calculating the right upper derivative of $V(t, x(t))$,

$$\begin{gathered} D^+V(t,x(t)) = \\ = x^T(t)\left(A^T P + PA\right)x(t) + 2x^T(t)Pf(t, x(t), x(t-\tau(t)) + 2M_k x_k x(t)) \le \\ \le x^T(t)\left(A^T P + PA + PP\right)x(t) + f(t, x(t), x(t-\tau))\, f(t, x(t), x(t-\tau(t))) + \\ + x_k^T M_k^T M_k x_k + x^T(t)PPx(t) \le \\ \le x^T(t)\left(A^T P + PA + PP + l_1 p^{-1} P\right)x(t) + l_2 x^T x(t-\tau(t))P^{-1}Px(t-\tau(t)) + \\ + x_k^T M_k^T M_k x_k \le \lambda_3 V(t, x(t)) + \lambda_4 V(t-\tau(t), x))) + x_k^T M_k^T M_k x_k\,. \end{gathered} \tag{5}$$

Because of condition (ii) is satisfying, we can get:

$$-\ln\lambda_6+\lambda\tau-(\sigma+\lambda)(t_{k+1}-t_k)>0. \tag{6}$$

From (6), we choose $K\geq 1$, which satisfies:

$$1<e^{(\sigma+\lambda)(t_1-t_0)}\leq K\leq -\ln\lambda_6 e^{\lambda\tau-(\sigma+\lambda)(t_1-t_0)}e^{(\sigma+\lambda)(t_1-t_0)}. \tag{7}$$

Then it follows that:

$$\|x_0\|_\tau^2<\|x_0\|_\tau^2 e^{\sigma(t_1-t_0)}\leq K\|x_0\|_\tau^2 e^{-\lambda(t_1-t_0)}. \tag{8}$$

First, we need to prove following condition:

$$V\Big(t,x(t)\leq\lambda_2K\|x_0\|_\tau^2e^{-\lambda(t-t_0)}\Big),\quad t\in[t_0,\,t_1). \tag{9}$$

For the proof of the upper form, we need to prove that

$$V\Big(t,x(t)\leq\lambda_2K\|x_0\|_\tau^2e^{-\lambda(t_1-t_0)}\Big),\quad t\in[t_0,\,t_1). \tag{10}$$

If the above model is not set up by (3) and (9), there is some $\bar{t}\in(t_0,t_1)$ satisfying the following formula:

$$\begin{aligned}V\Big(\bar{t},x\left(\bar{t}\right)\Big)>\lambda_2K\|x_0\|_\tau^2e^{-\lambda(t_1-t_0)}\geq\lambda_2\|x_0\|_\tau^2e^{-\lambda(t_1-t_0)}\\ >\lambda_2\|x_0\|_\tau^2\geq V(t_0+s,x(t_0+s)),\quad s\in[-\tau,0].\end{aligned}$$

The above formula implies that some $t^*\in\Big(t_0,\bar{t}\Big)$ exists, such as:

$$V(t^*,x(t^*))=\lambda_2M\|x_0\|_\tau^2e^{-\lambda(t_1-t_0)} \tag{11}$$

and

$$V(t,x(t))\leq V(t^*,x(t^*)),\quad t\in[t_0-\tau,t^*]. \tag{12}$$

$t^{**}\in[t_0,\,t^*)$ also exists, such as:

$$V(t^{**},x(t^{**}))=\lambda_2\|x_0\|_\tau^2, \tag{13}$$

as well as

$$V(t^{**},x(t^{**}))\leq V(t,x(t)),\quad t\in[t^{**},t^*]. \tag{14}$$

So, for any parameter $s\in[-\tau,0]$, there are the following inequalities:

$$\begin{aligned}V(t+s,x(t+s))\leq\lambda_2K\|x_0\|_\tau^2e^{-(t_1-t_0)}\leq\frac{\lambda_2}{\lambda_6}e^{\lambda\tau-(\sigma+\lambda)(t_1-t_0)}e^{(\sigma+\lambda)(t_1-t_0)}\|x_0\|_\tau^2\leq\\ \leq\frac{\lambda_2}{\lambda_6}e^{\lambda\tau-(\sigma+\lambda)(t_1-t_0)}e^{(\sigma+\lambda)(t_1-t_0)}\|x_0\|_\tau^2\leq\frac{e^{\lambda\tau}}{\lambda_6}\lambda_2\|x_0\|_\tau^2=\frac{e^{\lambda\tau}}{\lambda_6}V(t^{**},x(t^{**}))\leq\\ \leq\frac{e^{\lambda\tau}}{\lambda_6}V(t,x(t)),\quad t\in[t^{**},t^*]\,.\end{aligned} \tag{15}$$

From (4), (13) and (14), we can get:

$$x_0^T M_0^T M_0 x_0 = x_0^T M_0^T M_0 P^{-1} P x_0 \le \lambda_5 V(t_0, x(t_0)) \\ \le \lambda_5 \lambda_2 \|x_0\|_\tau^2 \le \lambda_5 V(t, x(t)), \quad t \in [t^{**}, t^*] . \tag{16}$$

Because the conditions (i), (5), (15) and (16) are satisfying, we can get:

$$D^+ V(t, x(t)) \le \left(\lambda_3 + \frac{\lambda_4}{\lambda_6} e^{\lambda \tau} + \frac{\lambda_5}{\lambda_6} \right) V(t, x(t)) \le (\sigma - \lambda) V(t, x(t)), \quad t \in [t^{**}, t^*] . \tag{17}$$

From (8), (11), (12), (13) and (14), we can get:

$$V(t^*, x(t^*)) \le V(t^{**}, x(t^{**})) e^{(\sigma-\lambda)(t^*-t^{**})} = \lambda_2 \|x_0\|_\tau^2 e^{(\sigma-\lambda)(t^*-t^{**})} < \\ < \lambda_2 \|x_0\|_\tau^2 e^{\sigma(t_1-t_0)} \le \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(t_1-t_0)} = V(t^*, x(t^*)) . \tag{18}$$

There is a contradiction in the above conclusion. Therefore, Formula (9) is established, and Formula (4) is true for $k = 0$.

We assume that (4) holds for $k = 0, 1, 2, \ldots, m$ ($m \in N$, $m \ge 0$), i.e.

$$V(t, x(t)) \le \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(t-t_0)}, \qquad t \in [t_k, t_{k+1}), \quad k = 0, \ldots, m. \tag{19}$$

Next, we will prove the establishment of (4) for $k = m + 1$, i.e.:

$$V(t, x(t)) \le \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_{m+1}, t_{m+2}) . \tag{20}$$

If (20) does not hold, then we define

$$\bar{t} = \inf \left\{ t \in [t_{m+1}, t_{m+2}) \mid V(t, x(t)) > \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(t-t_0)} \right\} .$$

By condition (ii) and (19), the following formula is obtained:

$$V(t_{m+1}, x(t_{m+1})) = x^T(t_{m+1}^-) C_{m+1}^T P C_{m+1} x(t_{m+1}^-) \le \lambda_6 V\left(t_{m+1}^-, x(t_{m+1}^-)\right) \le \\ \le \lambda_6 \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(t_{m+1}-t_0)} = \lambda_6 \lambda_2 K \|x_0\|_\tau^2 e^{\lambda(\bar{t}-t_{m+1})} e^{-\lambda(\bar{t}-t_0)} < \\ < \lambda_6 \lambda_2 e^{\lambda(t_{m+2}-t_{m+1})} K \|x_0\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} < \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \tag{21}$$

and so $\bar{t} \ne t_{m+1}$.

From the continuity of $V(t, x(t))$ in $[t_{m+1}, t_{m+2})$, we have:

$$v\left(\bar{t}, x\left(\bar{t}\right)\right) = \lambda_2 K \|x_0\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}, \quad t \in \left[t_{m+1}, \bar{t}\right] . \tag{22}$$

From (21), we know there is some $t^* \in \left(t_m, \bar{t}\right)$ such as:

$$V(t^* x(t^*)) = \lambda_6 \lambda_2 e^{\lambda(t_{m+2}-t_{m+1})} K \|x_0\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \tag{23}$$

and

$$V(t^* x(t^*)) \le V(t, x(t)) \le V\left(\bar{t}, x\left(\bar{t}\right)\right), \quad t \in \left[t^*, \bar{t}\right] . \tag{24}$$

According to (19), (21), (23) and (24), we have:

$$\begin{aligned}x^T_{m+1}M^T_{m+1}M_{m+1}x_{m+1} &= x^{-T}_{m+1}C^T_{m+1}M^T_{m+1}M_{m+1}C_{m+1}x^-_{m+1} \le \lambda_5 V\left(t^-_{m+1}, x\left(t^-_{m+1}\right)\right) = \\ &= \lambda_2 e^{\lambda(t_{m+2}-t_{m+1})}K\|x_0\|^2_\tau e^{-\lambda(\bar{t}-t_0)} = \frac{\lambda_5}{\lambda_6}\lambda_6\lambda_2 e^{\lambda(t_{m+2}-t_{m+1})}\|x_0\|^2_\tau\, e^{-\lambda(\bar{t}-t_0)} = \\ &= \frac{\lambda_5}{\lambda_6}V(t^* x\,(t^*)) \le \frac{\lambda_5}{\lambda_6}V(t, x(t)), \qquad t \in \left[t^*, \bar{t}\right].\end{aligned} \tag{25}$$

Meanwhile, for any $t \in \left[t^*, \bar{t}\right]$, $s \in [-\tau, 0]$. Then either $t + s \in [t_0 - t, t_{m+1})$ or $t + s \in \left[t_{m+1}, \bar{t}\right)$. Two cases will be discussed as follows:

If $t + s \in [t_0 - t, t_{m+1})$, than, based on (19), we obtain:

$$\begin{aligned}V(t + s, x(t + s)) &\le \lambda_2 K\|x_0\|^2_\tau e^{-\lambda(t_1-t_0)}e^{-\lambda s} \le \lambda_2 K\|x_0\|^2_\tau e^{-\lambda(\bar{t}-t_0)}e^{\lambda(\bar{t}-t)}e^{\lambda\tau} \\ &\le \lambda_2 e^{\lambda\tau}e^{\lambda(t_{m+2}-t_{m+1})}K\|x_0\|^2_\tau e^{-\lambda(\bar{t}-t)}.\end{aligned} \tag{26}$$

If $t + s \in \left[t_{m+1}, \bar{t}\right)$ then using (22) we get:

$$V(t + s, x(t + s)) \le \lambda_2 K\|x_0\|^2_\tau e^{-\lambda(\bar{t}-t_0)} \le \lambda_2 e^{\lambda\tau}e^{\lambda(t_{m+2}-t_{m+1})}K\|x_0\|^2_\tau e^{-\lambda(\bar{t}-t_0)}. \tag{27}$$

From (26) and (27), in any case, we have for any $s \in [-\tau, 0]$

$$V(t + s, x(t + s)) \le \frac{e^{\lambda\tau}}{\lambda_6}V\left(t^*, x\left(t^*\right)\right) \le \frac{e^{\lambda\tau}}{\lambda_6}V(t, x(t)), \qquad t \in \left[t^*, \bar{t}\right]. \tag{28}$$

Finally, from (i), (5) and (28), the following can be obtained:

$$D^+V(t, x(t)) \le \left(\lambda_3 + \frac{\lambda_4}{\lambda_6}e^{\lambda\tau}\right)V(t, x(t)) \le (\sigma - \lambda)V(t, x(t)).$$

Therefore, by condition (ii), the following results can be obtained:

$$\begin{aligned}V\left(\bar{t}, x\left(\bar{t}\right)\right) &\le V(t^*, x\,(t^*))\, e^{(\sigma-\lambda)\left(\bar{t}-t^*\right)} = \lambda_6\lambda_2 e^{\lambda(t_{m+1}-t_m)}K\|x_0\|^2_\tau e^{-\lambda\left(\bar{t}-t_0\right)}e^{(\sigma-\lambda)\left(\bar{t}-t^*\right)} \\ &< \lambda_2 e^{-(\sigma+\lambda)(t_{m+3}-t_{m+1})}e^{\lambda(t_{m+2}-t_{m+1})}K\|x_0\|^2_\tau e^{-\lambda\left(\bar{t}-t_0\right)}e^{(\sigma-\lambda)\left(\bar{t}-t^*\right)} = \\ &= \lambda_2 K\|x_0\|^2_\tau e^{-\sigma(t_{m+2}-t_{m+1})}e^{(\sigma-\lambda)\left(\bar{t}-t^*\right)}e^{-\lambda\left(\bar{t}-t_0\right)} \\ &< \lambda_2 K\|x_0\|^2_\tau e^{-\lambda\left(\bar{t}-t_0\right)} = V\left(\bar{t}, x\left(\bar{t}\right)\right).\end{aligned} \tag{29}$$

The above results are contradictory. This means that the hypothesis is not valid. Therefore, (4) is established for $k = m + 1$ and by mathematical induction, we can set up (4) for any $k \in N$.

From (3), we can obtain:

$$\|x(t)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}K\|x_0\|_\tau e^{\frac{-\lambda}{2}(t-t_0)}}t \ge t_0, \tag{30}$$

which implies that for any fixed delays $0 \le \tau(t) \le \tau \le +\infty$, the zero solution of system (1) is globally exponentially stable with the convergence rate $\frac{\lambda}{2}$. Then, we complete the proof of Theorem 1.

Corollary 1. Let λ_3, λ_4, λ_5, λ_6 and condition (iii) be precisely the same as that of Theorem 1, and assume that for all $k \in N$

$$\text{(iv)} \qquad \left(\lambda_3 + \frac{\lambda_4}{\lambda_5} e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\right)(t_{k+1} - t_k) < -\ln \lambda_6\,.$$

Then for any fixed delays $0 \le \tau(t) \le \tau \le +\infty$, the zero solution of system (1) is globally exponentially stable.

Proof

The condition (i) of Theorem 1 is equivalent to the following conditions:

$$(1)' \ \sigma \ge \lambda + \left(\lambda_3 + \frac{\lambda_4}{\lambda_5} e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\right).$$

Moreover, both conditions $(1)'$ and (ii) are equivalent to the following conditions:

$$\left(2\lambda + \lambda_3 + \frac{\lambda_4}{\lambda_5} e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\right)(t_{k+1} - t_k) < -\ln \lambda_6\,, \quad k \in N. \tag{31}$$

If the real number $\lambda > 0$ is small enough, the above results can be obtained by conditional (iv). Actually, we introduce below a function for all $k \in N$:

$$H(\lambda) = \left(2\lambda + \lambda_3 + \frac{\lambda_4}{\lambda_5} e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\right)(t_{k+1} - t_k) + \ln \lambda_6\,, \tag{32}$$

which can be generated by condition (iv) that $H(0) < 0$. Because of the continuity of $H(\lambda)$, we can deduce that there exists a small enough real number $\lambda > 0$ which makes $H(\lambda) < 0$. At the same time, inequality (31) is established by Theorem 1. That is to say, Corollary 1 is set up.

The results of Theorem 1 in this paper can be used to handle the GES problem for the Theorem 1 of [24] impulsive delayed linear dynamical hybrid systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau) + M_k x_k\,, & t \ne t_k\,, \quad t \ge t_0\,, \\ x(t) = C_k x(t^-), & t = t_k\,, \quad k \in N, \end{cases} \tag{33}$$

in which $f\colon R^+ \times R^n \times R^n \to R^n$ is a continuous vector-valued function and satisfying $f(t; 0, 0) = 0$. Obviously, system (33) is a generalization of the systems discussed in [24].

4. Numerical simulations

Example 1. Consider the IDNHDS as follows:

$$\begin{cases} \dot{x}(t) = -0.73x(t) + \dfrac{2x(t-\tau(t))}{1 + 3.8x^2(t-\tau(t))} + \dfrac{3}{k+2} x_k, & t \ne t_k\,, \quad t \ge t_0\,, \\ x(t) = \dfrac{k+1}{2k^2+1} x(t^-), & t = t_k\,, \quad k \in N, \end{cases} \tag{34}$$

where

$$0 \le \tau(t) \le \tau < +\infty, \quad x(t) \in R, \quad f(t, x(t), x(t-\tau(t))) = \frac{2x(t-\tau(t))}{1+3.8x^2(t-\tau(t))}$$

by Theorem 1.

Let $l_1 = 0$, $l_2 = 2$ satisfy condition (iii).

Let $P = 13$, then $\lambda_3 = 24.54$, $\lambda_4 = 0.0385$, $\lambda_5 = 0.1731$, $\lambda_6 = 0.0769$, which imply conditions (i) and (ii) of Theorem 1 hold. By condition (iv) of Corollary 1, if $\Delta t_k < 0.094$, the zero solution occurs if system (34) is globally exponentially stable with any time delay. The numerical simulation with $\Delta t_k = 0.09$, $\tau = 0$ and initial $x(t_0) = \varphi = 1$ is given in Fig. 1. When looking at the bifurcation diagram of Fig. 2, we find that the zero solution of system (34) is globally exponentially stable for $0 < \Delta t_k \le \Delta t_k^* \approx 2.39$. Fig. 3 shows that the zero solution of system (34) is globally exponentially stable with $\Delta t_k = 2.1$.

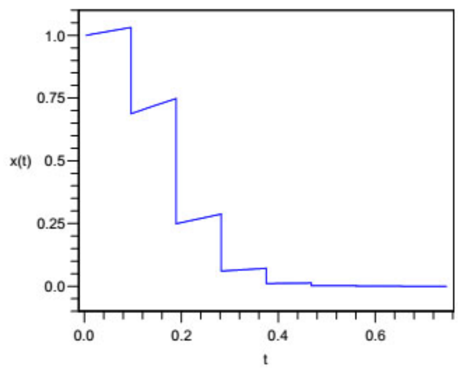

Fig. 1. Global exponential stability of system (34) with $\Delta t_k = 0.09$

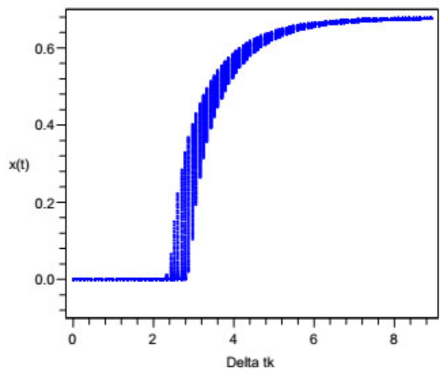

Fig. 2. Bifurcation diagrams of system (34) for $\Delta t_k = 0.013$ over (0.9)

Let

$$f(\lambda) = \left(-\ln \lambda_6 \big/ 2\lambda + \lambda_3 + \frac{\lambda_4}{\lambda_6}e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\right)$$

by the inequality of (31), if $\Delta t_k < f(\lambda)$, the zero solution of system (1) or (33) is globally exponentially stable.

So, if we require the global exponentially convergence rate of system (1) greater than or equal to any given rate $\frac{\lambda^*}{2}$, we can choose suitable Δt_k, such that systems (1) or (33) are globally exponentially stable with exponential convergence rate $\frac{\lambda}{2} \ge \frac{\lambda^*}{2} > 0$ (see Fig. 4, where D is a globally exponentially stable region).

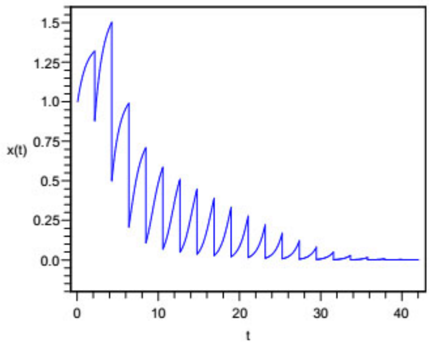

Fig. 3. Global exponential stability of system (34) with $\Delta t_k = 2.1$

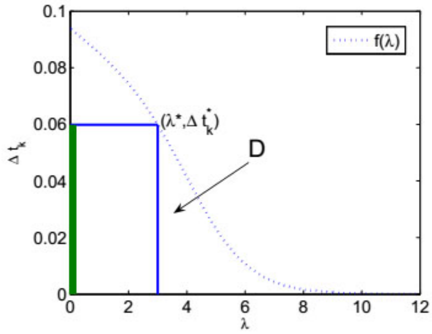

Fig. 4. Global exponential stable region of system (34) for $(\lambda, \Delta t_k)$

Example 2. Consider the IDNHDS as follows:

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{4} & 5 \end{pmatrix} x(t) + \begin{pmatrix} 2 & 2 \\ \frac{1}{3} & 6 \end{pmatrix} x(t-\tau(t)) + \begin{pmatrix} \frac{1}{k+1} & 0 \\ 0 & \frac{1}{k+1} \end{pmatrix} x_k, & t \neq t_k, \ t \geq t_0, \\ x(t) = \begin{pmatrix} \frac{3k+1}{k^2+1} & 0 \\ 0 & \frac{4k+1}{k^2+1} \end{pmatrix} x(t^-), & t = t_k, \ k \in N, \end{cases} \tag{35}$$

where $x(t) = (x_1(t), x_1(t)) \in R^2$.

Let

$$P = \begin{bmatrix} 9 & 0 \\ 0 & 12 \end{bmatrix},$$

then $\lambda_3 = 30.0371$, $\lambda_4 = 3.44449\lambda_5 = 0.1302$, $\lambda_6 = 0.5208$, which imply Theorem 1 holds. By the condition of Corollary 1, if $\Delta t_k < 0.0159$, the zero solution of (34) is globally exponentially stable for any time delay. The numerical simulation with $\Delta t_k = 0.013$, $\tau = 0$ and initial $x(t_0) = (x_1(t_0), x_2(t)) = \varphi = (-1.1)$ is given in Fig. 5.

Furthermore, we also assume:

$$P = \begin{bmatrix} 9 & 0 \\ 0 & 12 \end{bmatrix}, \qquad \Delta t_k = 0.013, \qquad \tau = 0$$

and initial $x(t_0) = (x_1(t_0), x_2(t)) = \varphi = (-1.1)$ system (34) is unstable without impulse (see Fig. 6).

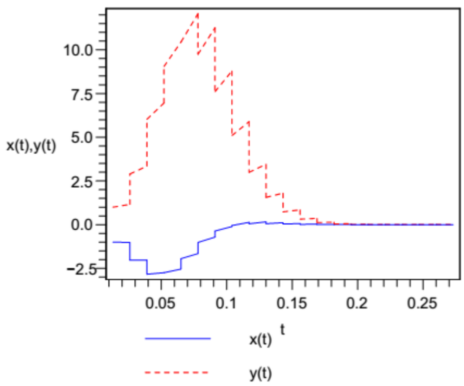

Fig. 5. Global exponential stability of system (34) with $\Delta t_k = 0.013$

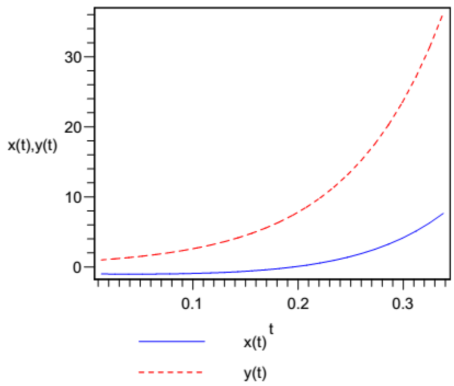

Fig. 6. Instability of system (34) without impulses

5. Conclusions

In this paper, the GES of impulsive delayed nonlinear and linear hybrid differential systems has been investigated. The combination of the Razumikhin technology and Lyapunov method is adopted. Some GES criteria have been established. In this paper, we have extended some known results existing in the literature. Examples are given to verify our results. Furthermore, by the simulation, impulse can stabilize an unstable dynamical system, which has more practical application value.

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