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Calculating surface current distribution in antenna array in the presence of mutual coupling by analytical solving of Pocklington's integral equation

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Abstract: In this article, the current distribution of an antenna array in the presence of mutual coupling is calculated analytically by solving Pocklington's integral equation. Block-pulse and Galerkin's functions are used for numerical solving of Pocklington's integral equation. In this work, the surface current distribution can be achieved for an antenna array in receiving mode, with any arbitrary structure and various numbers of elements. In all previous works, the authors have been tried to solve Pocklington's integral equation for a single half dipole antenna in transmitting mode. Pocklington's equation is somehow difficult to work with because of the singularity and existence of a sharp peak for a small value of wire's radius. In order to calculate surface current distribution, for thin wires, singularity part is extracted from the kernel in aforementioned integral. Hence, the kernel is decomposed into singular and nonsingular parts. An inter-element mutual coupling effect between array elements and self-coupling for each element are assumed in this case. The validity of the proposed methodology is tested by numerical simulation results. The accuracy of the proposed method is evaluated by the multiple signal classification (MUSIC) algorithm for different scenarios to direction of arrival (DOA) estimation.

Key words: antenna array, current distribution, DOA estimation, MUSIC algorithm, mutual coupling effect, Pocklington's integral equation

1. Introduction

Recently in the telecommunication industry, researchers have focused on using antenna arrays for applications in areas such as mobile communications, radar and sonar systems, medical imaging and multiple-input multiple-output systems (MIMO). Antenna arrays in receiving mode are capable of scanning the space and providing spatial information about signal sources. Direction of arrival (DOA) estimation of an arrival signal is an important application in the antenna array in the receiving mode and is the base of most of adaptive array processing algorithms.

Practically, these algorithms may often be seriously affected by mutual coupling between array elements. The mutual coupling among the array elements can significantly degrade the performance of DOA estimation techniques and signal processing algorithms [1–6]. Hereupon, this undesired effect has been attractive for the researchers who study in the context of adaptive array processing and DOA estimation. Several papers have also been published presenting different algorithms such as a full-wave method, calibration method, conventional-mutual-impedance method (CMIM), and receiving-mutual-impedance method (RMIM) for compensating and decoupling the mutual coupling effect [1–8]. The method which uses the mutual-impedance method (MIM) is a new and interesting method that is worth investigating. The calculation of the mutual impedance is based on an estimated current distribution on the antenna array excited by a plane wave [5]. To the best of our knowledge previous researchers have mostly tried to solve Pocklington's integral equation for an antenna array in transmitting mode. In the present work, the surface current distribution was achieved for an antenna array in receiving mode by solving Pocklington's integral equation.

Pocklington's integral equation sets a relation between the current distribution on a wire antenna and the affected by an electric field on its surface. One of the first digital computer solutions of the Pocklington's equation was reported in 1965 [9]. Pocklington's integral equation has been applied to specify the current along a straight thin-wire segment embedded in a homogeneous lossless dielectric [10]. A few researchers have solved the thin-wire current by using the analytical method of Pocklington's integral equation. Pocklington's equation is somehow difficult to work because of the singularity and existence of a sharp peak in an integral kernel for a small value of wire's radius. Richmond has presented a solution for this problem by a numerical method, while the analytical method for solving this problem has been presented in [11] that are based on the decomposition of the kernel in the singular and non-singular parts. Recently, the attention of researchers has been focused to a wavelet-based moment method for solving Pocklington's integral equation [12–16]. In [12], Pocklington's integral equation is simply solved by an effective method proposed based on Haar wavelets. The aim of utilizing these wavelets is to reduce Pocklington's integral equation computational complexity to algebraic equations. Moreover, the authors in [12] have investigated the performance of their method for convergence and the sparseness of the resulted matrix equation by using numerical examples. In all previous works, the authors have been trying to solve Pocklington's integral equation for a single half dipole antenna in transmitting mode. In [15, 16] a fast efficient algorithm based on the method of moments (MOM) and Haar wavelets in order to obtain the current distribution on the antenna is proposed. In order to generate a system of the matrix equations for a transmitting dipole antenna from the thin-wire electric field integral equation (EFIE), applied a wavelet expansion from the Haar orthogonal wavelet. The result shows a good convergence at the feeding point by using Pocklington's integral equation. Acharjee A. et al. [17] obtained current distributions for a half-wave and full-wave dipole antenna and a three-element Yugi–Uda antenna array by solving Pocklington's integral equation. By using current distribution, they obtained and analyzed a radiation pattern and input impedance of a wire antenna.

A new formulation of Pocklington's equation for thin wires has been presented in [18]. This formulation for improper integrals was performed using the exact kernel. The results of applying exact kernel in solving Pocklington's integral equation leads to numerically stable and good convergence with a pulse function point matching solution. In [19], the current distribution of a dipole antenna is considered as the sinusoidal structure for finite diameter wires. To find a more accurate current distribution on a barrel shaped wire, Pocklington's integral equation is suggested. Therefore, by knowing the voltage at the feed terminals of a dipole antenna and find the current distribution by solving Pocklington's integral equation, the input impedance and radiation pattern can be acquired. In [20] Hallen and Pocklington's integral equations were solved by using a method of moments which is a numerical technique. So that, calculated current distribution of the antenna, MOM has been used on Pocklington's and Hallen integral equations for a transmitting antenna, by point matching and Galerkin functions. Due to the calculation of the surface current distribution in thin wires, different basis functions are used to solve Pocklington's and Hallen integral equations. Orthogonal piecewise constant basis functions, such as a block-pulse function [21] and Haar-wavelet function [12] were used to solve the integral equations.

In the majority of papers on the calculation of current distribution, the single antenna was considered in transmitting mode, whereas in this research, the antenna array in receiving mode in the presence of mutual coupling is evaluated. The novelty of this paper is to calculate the surface current distribution of antenna array elements by analytical solving of Pocklington's integral equation. Due to the analytical solution of Pocklington's integral equation, the kernel is decomposed into non-singular and singular parts. The non-singular part due to slow changes could be calculated numerically and the singular part must be calculated analytically. In this way, all mutual coupling effects between the array elements are taken into account. The array is used in receiving mode with arbitrary array element arrangement and an optional number.

As far as the excitations of the transmitting and receiving arrays are different, they have different mechanisms that cause mutual coupling. Consequently, the mutual-coupling effect for an antenna array in the transmitting and receiving modes are two different problems, which should be treated differently [19]. For analyzing a mutual coupling effect on antenna arrays in both transmitting and receiving mode, current distribution on the antenna elements needs to be calculated. The calculation of current distribution can be done both by solving integral equations or by utilizing available software such as FEKO [8] or NEC [22]. Since different studies are available in transmitting mode on analytical solving of Pocklington's integral equation in spite of availability of different software applications, it was decided to show its practicality for calculating current distribution by solving integral equations in an antenna array in the presence of mutual coupling in receiving mode. On the other hand, the compensation of mutual coupling effects of antenna arrays, using signal processing algorithms, is highly dependent on the accurate computation of current distribution on the body of antenna elements. Therefore, in order to make sure of the accuracy, we carried out the computation independently, by solving Pocklington's integral equations.

The rest of the paper is organized as follows: in section 2, Pocklington's integral equation and a mutual coupling effect are introduced. Kernel decomposition and calculation of current distribution on the antenna array in the presence of the mutual coupling effect are presented in section 3. In section 4, modeling the load impedance Z_L at the terminals of the dipole elements are introduced. Simulation results of the surface current distribution for a four-element uniform linear array (ULA) are presented in section 5. Finally, the conclusion is discussed in section 6.

2. Pocklington's integral equation and mutual coupling effect

Pocklington's equation is a principle of thin-wire antenna analysis and appears in lots of researches and books [23–26]. The geometry from which Pocklington's integral equation can be achieved is shown in Fig. 1. As it can be seen in Fig. 1, the wire antenna is located along the x-axis. The current distribution is limited to the central line of the wire antenna and directed along the z-axis [27].

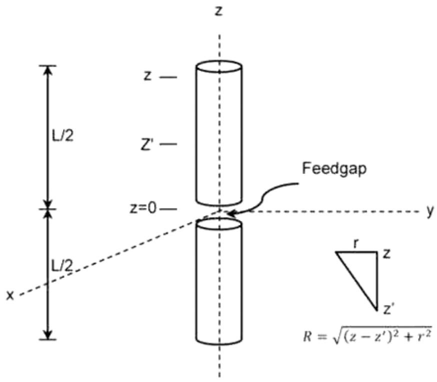

Fig. 1. Integral equation formulation [27]

A brief explanation of the integral usage can be offered by supposing a traditional model that assumes an antenna made of a perfect thin conductor compared with the wavelength of the electromagnetic field. The Pocklington approach assumes that the surface current distribution can be simulated by a current string parallel to the antenna axis. Pocklington's equation for a wire of length L centered and directed along the z-axis is given as follows:

$$\left(\frac{\partial^2}{\partial z^2} + k^2\right) A(z) = -j\omega\mu\varepsilon E_z^i(z). \tag{1}$$

In Equation (1), $k = 2\pi/\lambda$ (λ wavelength) is the wave number, ω (rad/s) is the angular frequency, ε is the permittivity of the medium, and $E_z^i(z)$ is an external plane wave excitation. According to the current distribution in the surface conductor of antenna, the vector potential, $A(z)$, for a dipole can be written as:

$$A(z) = \frac{\mu}{4\pi} \int\limits_{-L/2}^{L/2} I_Z(z')G(z,z')\,\mathrm{d}z'. \tag{2}$$

In Equation (2), L is the length of dipole and $G(z,z')$ is the thin-wire kernel:

$$G(z,z') = \frac{e^{-jkR}}{R}. \tag{3}$$

In order to investigate coupling effects of an array, the parameter R can be calculated differently for each state of self and mutual effects. To calculate self-coupling between two segments of a dipole, R, is [28].

$$R = \sqrt{(z-z')^2 + r^2}\,. \tag{4}$$

To calculate mutual coupling, the distance between two segments of two antennas, R, is:

$$R = \sqrt{(z-z')^2 + ((p-1)d)^2}\,, \qquad p = 1, 2, \ldots, P. \tag{5}$$

In Equation (4), z and z' belong to a dipole, while in Equation (5) they belong to two different dipoles. In the above equation, r is the radius of the dipole and P is a total number of array elements and d is the distance between each element of the array. Substituting Equation (2) into Equation (1), one obtains.

$$\frac{\mu}{4\pi}\int\limits_{-L/2}^{L/2} I_Z(z')\left(\frac{\partial^2}{\partial z^2}G(z,z') + kG(z,z')\right)\mathrm{d}z' = -j\omega\mu\varepsilon E_z^i(z). \tag{6}$$

The achieved term from the second order derivative of the kernel is denoted by $K(z,z')$.

$$K(z,z') = \left(\frac{\partial^2}{\partial z^2} + k^2\right)G(z,z'). \tag{7}$$

Therefore, Equation (1) could be rewritten as:

$$\frac{\mu}{4\pi}\int\limits_{-L/2}^{L/2} I_Z(z')K(z,z')\,\mathrm{d}z' = -j\omega\mu\varepsilon E_z^i(z). \tag{8}$$

A solution of Equation (8) in the MOM formulation requires a block-pulse function and Galerkin function as a basis function and weight function, respectively. $I_z(z')$ is an unknown function in the left-hand side of Equation (8) which can be extended in term of a basis functions $f_n(z')$.

$$I_Z(z') = \sum_{n=1}^{PM} a_n f_n(z'). \tag{9}$$

M is the total segment number of each array element, PM is the total number of array segments, a_n is the current envelope coefficient and $f_n(z')$ is the basis function that is defined in section 3. By using the basis function, $f_n(z')$, and weight functions, $f_m(z')$, in Equation (8) and using Equation (9), one obtains.

$$\frac{\mu}{4\pi}\sum_{n=1}^{PM} a_n \int\limits_{f_m} f_m(z) \int\limits_{f_n} f_n(z')K(z,z')\,\mathrm{d}z'\,\mathrm{d}z = -\int\limits_{f_m} f_m(z)\,j\omega\mu\varepsilon E_z^i(z)\,\mathrm{d}z. \tag{10}$$

Both operators $\int_{f_m}$ and $\int_{f_n}$ indicate integration over a range of real numbers, for $f_m(z) \neq 0$ and $f_n(z) \neq 0$, respectively. The matrix form of Equation (10) is as follows:

$$\boldsymbol{Z}\boldsymbol{a} = \boldsymbol{b}. \tag{11}$$

In Equation (11), vector $\boldsymbol{a}$ is the unknown columned vector of size $PM * 1$ which contains the samples related to the current distribution over the all antenna elements and vector $\boldsymbol{b}$ is the known columned vector of size $PM * 1$ and matrix $\mathbf{Z}$ with size $PM * PM$ indicates the coefficient matrix, which z_{mn} represents the coupling between two differential segments of the antenna array. Equation (11) may be solved numerically for $\boldsymbol{a}$, and hence yields an approximate solution to Equation (9). Matrix $\mathbf{Z}$ in Equation (11) can be obtained for $m = 1, 2, \ldots, PM$, $n = 1, 2, \ldots, PM$ [6]:

$$z_{mn} = \int\limits_{f_m} f_m(z) \int\limits_{f_n} f_n(z')K(z,z')\,\mathrm{d}z'\,\mathrm{d}z. \tag{12}$$

For $m \neq n$ the entry z_{mn} in Equation (12) of the coefficient matrix $\mathbf{Z}$ can be evaluated by using the numerical quadrature rule [29]. When $m = n$ the integrand of Equation (12) is very sharply peaked, particularly for a small value of r. Therefore, applying the kernel decomposition is advantageous to extract and isolate the singularity.

In the next section, a procedure is presented to calculate surface current distribution for a thin wires antenna array and a singularity part is extracted from the kernel in Pocklington's integral equation. The results of the modified kernel may be applied to considerable improvement of the computational efficiency of certain moment method formulation.

3. Kernel decomposition and calculation of current distribution on the antenna array in the presence of mutual coupling effect

In this section kernel decomposition, a block pulse basis function and Galerkin weight functions are introduced. Richmond in [9] presents a specific form of Pocklington's integral equation which is evaluated by MOM [28, 30]. Since the field point is located in neighborhood of a source point, the kernel has a sharp peak, calculations require significant care. Due to avoiding this matter, Richmond has proposed a method in [9] which suggested changing of variables. Another approach eliminates singularity part of the kernel by decomposing the kernel into two parts. One part that can be integrated numerically and another part is the singular part which can be integrated analytically [11]. From the aspect of computational, using the kernel is advantageous in extracting and isolation the singularity. So $K(z,z')$ is as follows:

$$K(z,z') = \frac{e^{-jkR}}{R^5}\left[(1+jkR)(2R^2-3a^2)+(kRa)^2\right]. \tag{13}$$

This can be done by decomposition Equation (13) of the form:

$$K(z,z') = K^{(n)}(z,z') + K^{(s)}(z,z'). \tag{14}$$

In this equation, $K^{(s)}(z,z')$ and $K^{(n)}(z,z')$ represent the singular and non-singular parts of kernel $K(z,z')$, respectively.

$$K(z,z') = \frac{\left[e^{-jkR}+jkR-1\right]\left[(1+jkR)(2R^2-3a^2)+(kRa)^2\right]}{R^5} + \\ + \frac{R^2/2\left[\left(2+\left({}^{ka}\!/_{2}\right)^2(kR)^2\right)-(3+j2kR)(kR)^2\right]}{R^5}, \tag{15}$$

$$K^{(s)}(z,z') = \frac{k^2}{R}\left[1-\frac{1}{8}(ka)^2\right] + \frac{2}{R^3}\left[1-\frac{1}{4}(ka)^2\right] - \frac{3a^2}{R^5}. \tag{16}$$

By using Equation (14), we can express Equation (12) as:

$$z_{mn} = z_{mn}^{(n)} + z_{mn}^{(s)}. \tag{17}$$

In Equation (17), $z_{mn}^{(n)}$ and $z_{mn}^{(s)}$ are non-singular and singular parts give as:

$$z_{mn}^{(n)} = \int\limits_{f_m} f_m(z) \int\limits_{f_n} f_n(z') K^{(n)}(z,z')\,\mathrm{d}z\,\mathrm{d}z', \tag{18}$$

$$z_{mn}^{(n)} = \int\limits_{f_m} f_m(z) \int\limits_{f_n} f_n(z') K^{(s)}(z,z')\,\mathrm{d}z\,\mathrm{d}z'. \tag{19}$$

The block-pulses are the orthogonal functions that are defined as:

$$B(z) = \begin{cases} 1 & \frac{n-1}{N}L \le z \le \frac{n}{N}L \\ 0 & \text{otherwise} \end{cases}. \tag{20}$$

Galerkin and point matching or collocation choices are two interesting choices of weight functions. Weighting functions in the Galerkin method are assumed to be the same as a block-pulse function [28]. By assuming a block pulse as a basis function and Galerkin as a weight function, Equations (18) and (19) can be written as:

$$z_{mn}^{(n)} = \int\limits_{T_m}^{T_m+1} \int\limits_{T_n}^{T_n+1} K^{(n)}(z,z')\,\mathrm{d}z\,\mathrm{d}z', \tag{21}$$

$$z_{mn}^{(s)} = \int\limits_{T_m}^{T_m+1} \int\limits_{T_n}^{T_n+1} K^{(s)}(z,z')\,\mathrm{d}z\,\mathrm{d}z'. \tag{22}$$

The integrand of the integral in Equation (21) is well behaved and, as a consequence may be efficiently and accurately evaluated numerically. The integrand of the integral in Equation (20) contains a singularity and can be evaluated analytically as fol1ows [12]:

$$z_{mn}^{(s)} = h(\tau_{n+1},\tau_{m+1}) - h(\tau_{n+1},\tau_m) - h(\tau_n,\tau_{m+1}) + h(\tau_n,\tau_m), \tag{23}$$

$$\frac{\partial^2 h(z,z')}{\partial z \partial z'} = K^{(s)}(z,z'). \tag{24}$$

By using Equations (16) and (22), $h(z,z')$ could be achieved as:

$$\begin{aligned} h(z,z') = {} & \left(-\frac{1}{8}k^4a^2 + \frac{3}{2}k^2\right)R - \frac{1}{R} + \\ & + \left(-\frac{1}{8}k^4a^2 + k^2\right)\left(z\ln(-z+z'+R) + z'\ln(z-z'+R)\right). \end{aligned} \tag{25}$$

Consider the problem, as the arbitrary array, consisting of P dipole elements, with an arbitrary separation, d. When the array is under external plane wave excitation, the surface current distribution on each dipole element must be compatible with Equation (1) for an antenna array in receiving mode. The dipoles are considered as thin wires with perfect conductivity of length L, having a circular cross section of radius r and parallel to the z-axis, so that $L \gg r$, $\lambda \gg r$. Due to the MOM manner and using the approximation of the thin wire antenna, each dipole element is subdivided into M equal segments, hence, the length of each dipole will be equal to $dz = L/M$. The incident wave over the length of the wire $E^i_{zp}(z')$, is the incident wave over the length of p^{th} wire element, so z' varies in $[-L/2,\ L/2]$ interval. For a plane wave excitation arriving from (θ, φ), where φ and θ are the azimuth and elevation angles, respectively, $E^i_{zp}(z)$ is written as [6]:

$$E^i_{zp}(z') = E_0 \sin\theta \, e^{jk(x_p \sin\theta \cos\varphi + y_p \sin\theta \sin\varphi + z' \cos\varphi)}, \qquad p = 1, 2, \ldots, P. \tag{26}$$

In Equation (26), E_0 represents the electric field intensity associated with an incident plane wave, and x_p and y_p are the x and y coordinates of the p^{th} wire element. In Equation (9), Equations 1 to M correspond to Pocklington's integral equation for the first element, and equations $(p-1)M$ to pM correspond to Pocklington's integral equation for the p^{th} element, where $p = 1, 2, \ldots, P$. The n^{th} element of the right-hand side of Equation (11), $\boldsymbol{b}$, is:

$$b_n = -j4\pi\omega\varepsilon \int\limits_{f_m} f_m(z) E^i_{zp}(z)\,\mathrm{d}z, \qquad (p-1)M + 1 \le n \le pM. \tag{27}$$

So by applying the Galerkin weight function, Equation (27) could be written as follows:

$$b_n = -j4\pi\omega\varepsilon \int\limits_{\tau_m}^{\tau_m+1} E^i_{zp}(z)\,\mathrm{d}z, \qquad (p-1)M + 1 \le n \le pM. \tag{28}$$

Thus, for the unknown vector $\boldsymbol{a}$, i.e., the current distribution on all of the wire elements of the antenna is obtained as follows:

$$\boldsymbol{a} = \boldsymbol{Z}^{-1}\boldsymbol{b}. \tag{29}$$

4. Modeling the load impedance Z_L at the terminals of the dipole elements

So far all wire elements in the array are assumed to be continuous, that is short circuited at their central segments. In practice, the elements are connected to receiving stages. Assume the equivalent input impedance of the line and receiver stage, connected to the p^{th} element, is Z^p_L, $p = 1, 2, \ldots, P$ (here $P = 4$). In this case, the boundary condition over the middle differential segment of each wire element, which is connected to Z^p_L, as in Fig. 2, is written as:

$$E^i_z(z) + E^s_z(z) = \frac{v_t}{\Delta z} = \frac{Z^p_L i_t}{\Delta z}. \tag{30}$$

Here, the terminal voltage, v_t, i.e. the voltage drop on Z^p_L, is taken into account. $E^i_z(z)$ and $E^s_z(z)$ are the incident and scattered fields, Δz is the length of the middle differential segment of the wire element, and i_t is the current at the input terminals of the element.

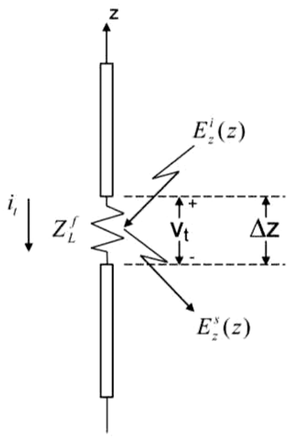

Fig. 2. Modeling the input impedance of the receiver at the terminals of the p^{th} antenna element, Z_L^p, $p = 1, 2, \ldots, P$ [6]

By adding $Z_L^p i_t/\Delta z$ terms to Hallen's integral equation and following Galerkin's testing procedure, with testing functions $f_m(z)$, a double integral term is added to some elements of the coefficient matrix $\mathbf{Z}$, as:

$$Z_{m,(2p-1)(M-1)/2} \to Z_{m,(2p-1)(M-1)/2} + \frac{Z_L^p}{2\eta\Delta z} \iint f_m(z') f_{(M-1)/2}(z') e^{-jk|z-z'|} \,\mathrm{d}z' \,\mathrm{d}z, \tag{31}$$

$M = (p-1)M + 1, (p-1)M + 2, \ldots, pM$, where, $p = 1, 2, \ldots, P$. The double-integral term represents the load impedance Z_L^p, connected to the terminals of the p^{th} wire element. Obviously, if the element is short-circuited, the term vanishes [6].

5. Simulation

In this section, the proposed methodology, outlined in the previous section, is implemented using the MATLAB software. In this method, all mutual coupling effects are considered to calculate surface current distribution in an antenna array. To evaluate the surface current distribution in an antenna array of the suggested method, two scenarios are presented.

In the first scenario, Pocklington's integral equation for the single element is considered for two wire lengths with $L = \lambda/2$ and $L = \lambda$. The parameters of the single dipole are defined as following: frequency $f = 3\mathrm{e}^8$ Hz, applied thin wire radius $a = 0.005\lambda$, the wavelength of the incident signal is $\lambda = c/f$, $c = 299792458$, $\beta = 2\pi/\lambda$, $\omega = (6\pi/\lambda) \times 10^8$ and $\varepsilon_0 = 8.854 \times 10^{-12}$. In both wire lengths, we considered rectangular pulse for $E_z^i(z)$ as follow:

$$E_z^i(z) = \begin{cases} \dfrac{1}{2\Delta} & \left| z - \dfrac{L}{2} \right| < \Delta \\ 0 & \text{otherwise} \end{cases} . \tag{32}$$

In this equation $\Delta = 0.001\lambda$. Current distribution $I_z(z')$ for $L = \lambda/2$ and $L = \lambda$ with $N = 100$, 500 was calculated by using Equation (9). The absolute value of normalized current distribution $I_z(z')/\|I_z(z')\|_2$ are plotted in Figs. 3 and 4 for $L = \lambda/2$ and $L = \lambda$ respectively. As can be seen in

Figs. 3 and 4, the solution of Pocklington's integral equation converges rapidly with increasing the value of N. The Haar wavelets method presented in [12, 15, 16] has limitations in selecting the number of segments, whereas, the proposed approach doesn't have any limitation in choosing the number of segments. Therefore, the value of N can be chosen as arbitrary. It can be concluded that using such techniques, leads to reduction of process time and computation rather than previous methods [12, 15, 16].

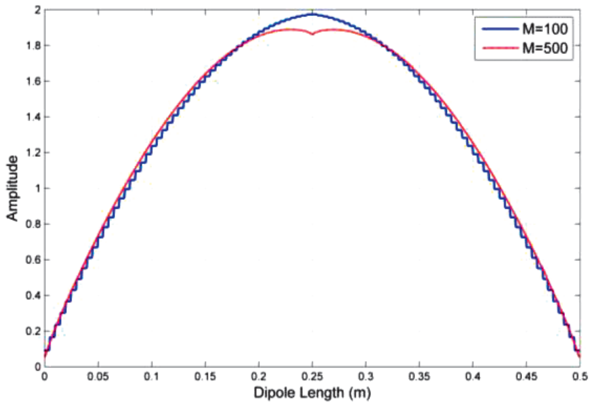

Fig. 3. Absolute value of normalized current distribution on single element for case $L = \lambda/2$

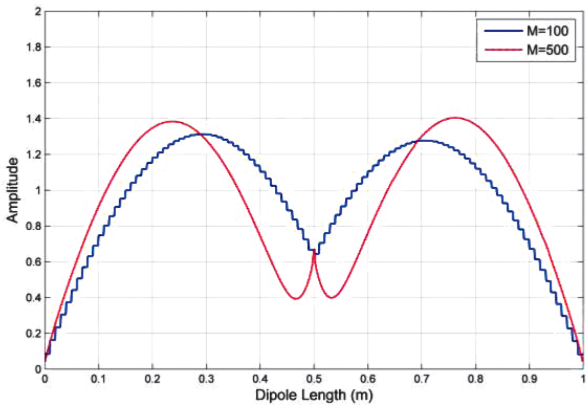

Fig. 4. Absolute value of normalized current distribution on single element for case $L = \lambda$

In the second scenario, a uniform linear antenna array consisting of four dipoles was considered as located in $x - z$ coordinate as shown in Fig. 5. The dipoles are considered as thin wires with perfect conductivity and the length of dipole is equal to $L = \lambda/2$, from $z = -L/2$ to $z = L/2$, having a circular cross section of radius $r = 0.005\lambda$, so that $L \gg r$, $\lambda \gg r$, and they are parallel to the z-axis. Each dipole is divided into $M = 500$ equal segments of length $dz = L/M$. The array excited by two incident plane waves from azimuth angles $\varphi = 45^\circ$, $\varphi = 60^\circ$ and an elevation angle $\theta = 90^\circ$. The separation between the dipoles is $d = \lambda/2$. The incoming signal parameters are presented in Table 1.

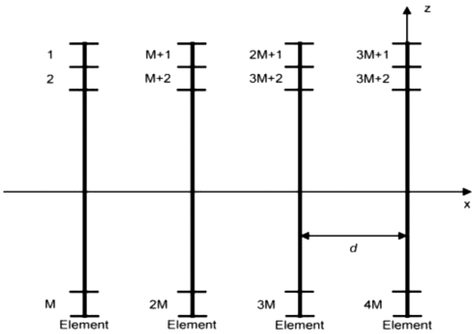

Fig. 5. Illustration of four dipole antennas array

Table 1. Incoming signal parameters

Parameters	First signal	Second signal
frequency	300 MHz	300 MHz
polarization	vertical	vertical
signal to noise ratio (SNR)	10 dB	10 dB
electric field intensity	1 V/m	1 V/m
direction of arrival (DOA)	$\varphi = 45^\circ$	$\varphi = 60^\circ$

The absolute value of each array's segment current distribution are plotted in Figs. 6 and 7 for two azimuth angles $\varphi = 45^\circ$, $\varphi = 60^\circ$, respectively. As can be seen in Figs. 6 and 7 the calculated current distribution depends on the direction of arrival of an incident signal to the array and by

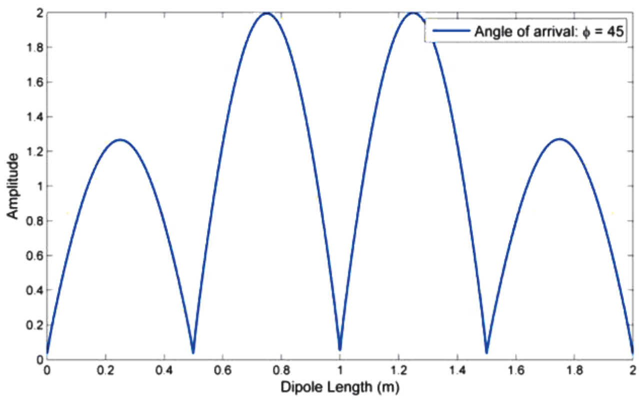

Fig. 6. Absolute value of normalized current distribution in an antenna array for signal DOA $\varphi = 45^\circ$

changing the direction of arrival, the peak of each curve will be changed. The results of the surface current distribution calculated from four dipoles with azimuth angles $\varphi = 45°$, $\varphi = 60°$ by the suggested method are listed in Table 2. To prove the correctness and accuracy of the calculated current distribution, the gained data has been evaluated by the MUSIC algorithm for DOA estimation of incident signals. Spatial spectrum of the MUSIC algorithm in Figs. 8 and 9 has a maximum value of about $\varphi = 45°$ and $\varphi = 60°$, respectively. As can be seen in Figs. 8 and 9, the measured DOA from the MUSIC algorithm has little different values in comparison to the real DOA. This proves that the current distribution obtained from the proposed method is correct. This difference is derived from the mutual coupling effects in antenna arrays, which can be reduced or eliminated by using compensation methods, such as the open circuit voltage method, receiving mutual impedance method and minimum norm method.

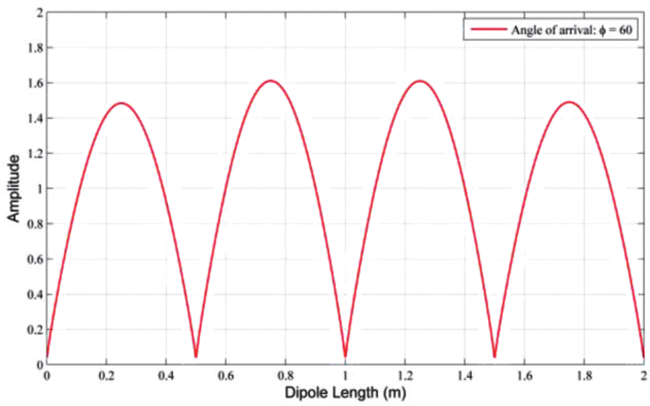

Fig. 7. Absolute value of normalized current distribution in an antenna array for signal DOA $\varphi = 60°$

Table 2. Comparisons of current distribution obtained from four dipoles with azimuth angles $\varphi = 45°$, $\varphi = 60°$

Element #	DOA	Dipole length in λ	Radius in λ	Current distribution
Element #1	$\varphi = 45°$	0.5λ	0.005λ	$1.24239732921767 + j0.246572680556934$
Element #2	$\varphi = 45°$	0.5λ	0.005λ	$-1.43967834114108 - j1.38180684460279$
Element #3	$\varphi = 45°$	0.5λ	0.005λ	$-0.893385348994451 + j1.78763209824354$
Element #4	$\varphi = 45°$	0.5λ	0.005λ	$1.09066636091737 - j0.652397934197564$
Element #1	$\varphi = 60°$	0.5λ	0.005λ	$1.46907882964834 + j0.209584324991737$
Element #2	$\varphi = 60°$	0.5λ	0.005λ	$-0.0222146309132732 - j1.6109282355556$
Element #3	$\varphi = 60°$	0.5λ	0.005λ	$-1.60799532140858 - j0.079833803622s893$
Element #4	$\varphi = 60°$	0.5λ	0.005λ	$0.161131122673406 + j1.48117771418671$

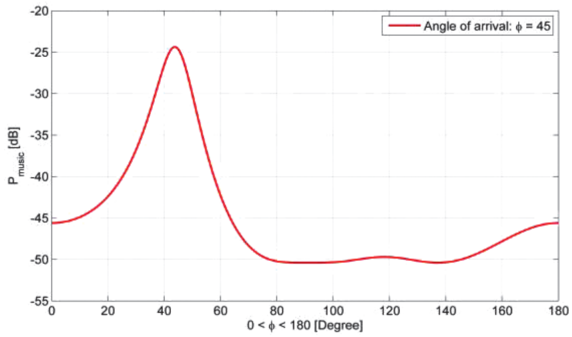

Fig. 8. Spatial spectrum of the MUSIC algorithm for direction of arrival detection of an incident signal from azimuth angles $\varphi = 45^\circ$ and elevation angle $\theta = 90^\circ$

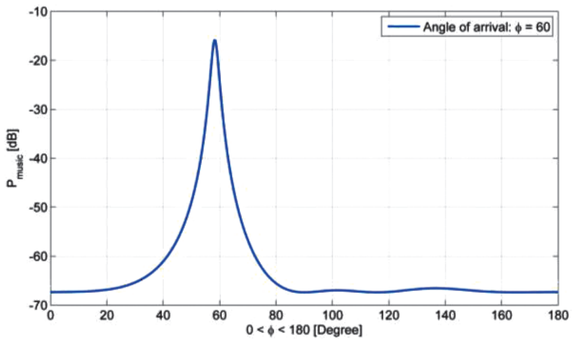

Fig. 9. Spatial spectrum of the MUSIC algorithm for direction of arrival detection of an incident signal from azimuth angles $\varphi = 60^\circ$ and elevation angle $\theta = 90^\circ$

6. Conclusions

The novelty of this paper is the calculation of the surface current distribution on antenna array elements in receiving mode by analytical solving of Pocklington's integral equation. Inter-element mutual coupling effects are taken into account to calculate surface current distribution. It is shown that the utilization of the MUSIC algorithm to the achieved current distribution by the proposed method leads to the proper performance of direction of arrival estimation, in terms of resolution and accuracy. By using a block-pulse function, the processing time and computational complexity have been decreased. The analytical solving of Pocklington's integral equation by the proposed method does not need consideration of boundary conditions, so the proposed method has less computation than Hallen's integral equation method to get surface current distribution. Therefore, the proposed method can be preferred over other methods to achieve the surface current distribution of an antenna array as a receiver in the presence of a mutual coupling effect.

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