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PROCESSING CHARACTERISTICS CORRECTION OF MEASURING SYSTEMS BY MEANS A DIFFERINTEGRAL OF VARIABLE ORDER

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Abstract – The paper presents new methods for correcting the processing characteristics of measurement systems based on a modified Grünwald-Letnikov fractional calculus definition. The presented methods are based on the determination of the fractional order as an estimation factor. Two methods are presented: a fractional order array and a fractional order function. Both methods can be used in DSP systems as methods to correct the processing characteristics of systems with measuring transducers and measurement systems in general.

Key words – fractional calculus, Grünwald-Letnikov, processing correction

INTRODUCTION

The theory of fractional integration and differentiation comes in many solutions, the best known of which are Grünwald-Letnikov differintegals, Riemann-Liouville and Caputo's fractional calculus [1-4]. The fractional calculus have found a range of applications, in particular, modelling of process dynamics and physical effects whose modelling with classic mathematical apparatus has not always been faithful to reality, e.g. modelling of such effects as memory process, PID controllers, robust control, heat transfer process, electrical drive, voltage regulator, charging and discharging of supercapacitors, robot manipulators, cell growth dynamics, biomedical engineering, image processing, chemical reaction processes, dynamics of automatic or electronic systems, photovoltaic systems, hybrid power systems or such non-technical issues as analysis of financial processes [5-19]. The fractional calculus seems an ideal tool for modelling of nonlinear and highly complex effects and processes. General dynamics models using fractional calculus and their particular solutions are now well described. The problem is to define a clear relationship between parameters of the real object model and the fractional order [20-23]. This paper attempts to delve into this topic from the point of view of applying the fractional calculus to the correction of measurement signal processing characteristics.

1 ASSUMPTIONS

1.1 ASSUMPTION 1

The measured signal $\hat{x}$ is assumed to be subject to measurement error and the reference signal x is assumed to be without measurement error (or subject to less error). The measured signal is usually time-shifted relative to the reference signal and has a different amplitude. These differences may be due to the characteristics of the measuring circuit, in which each circuit element (e.g. measuring transducer, A/D transducer, conditioner) introduces a measurement error. Examples of reference and measurement signals are shown in Fig. 1.

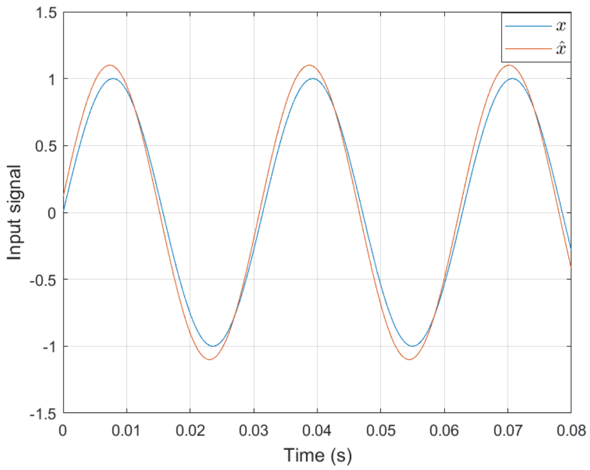

Fig. 1. Reference input signal $x(t)$ and measuring input signal $\hat{x}(t)$

1.2 ASSUMPTION 2

The definition of the fractional calculus is to follow the Newton-Leibniz theorem:

$$F'(a) = \lim_{(b-a)\to 0} \frac{F(b) - F(a)}{b - a} = \lim_{(b-a)\to 0} \frac{\int_a^b f(t)dt}{b - a}, \tag{1}$$

where F is a primitive function of f function. This assumption is satisfied by the modified Grunwald-Letnikov differintegral [23]:

$$\frac{d^n}{dt^\eta} g(t_0) = \lim_{dt_\eta \to 0} \frac{\sum_{m=0}^{n} (-1)^m \binom{n}{m} f(t_{n-m})}{dt^\eta} \equiv {}^{n}D^{\eta} g(t_0) dt^\eta, \tag{2}$$

where $d^n/dt^\eta\, g(t_0)$ and ${}^{n}D^{\eta} g(t_0)$ are notations of the fractional derivative connected to n-order derivative and η fractional order and $dt_\eta = dt + \Delta t = dt^\eta$, where Δt is an interval error.

2 VARIABLE ORDER MODEL

According to the Newton-Leibniz theorem, there is a following relation:

$$y(t) \to \frac{d}{dt}y(t) = x(t) \to \int x(t)\,dt = \int \left(\frac{d}{dt}y(t)\right)dt = y(t) \tag{3}$$

and similarly for the modified Grünwald-Letnikov differintegral (2) (it is not right for the classical Grünwald-Letnikov differintegral [1-2], [4]):

$$\hat{y}(t) \to {}^{1}D^{\eta}\hat{y}(t)dt^{\eta} = \hat{x}(t) \to {}^{1}D^{-\eta}\hat{x}(t)dt^{\eta} = {}^{1}D^{-\eta}\left({}^{1}D^{\eta}\hat{y}(t)dt^{\eta}\right)dt^{\eta} = \hat{y}(t) \tag{4}$$

Taking assumption that the η order is a correction factor for the measurement signal $\hat{x}(t)$, where in the ideal case $\hat{x}(t) = x(t)$, the differintegral of $\hat{x}(t)$ (for minus η) and the integral of $x(t)$ are equal to each other for a given η:

$${}^{1}D^{-\eta}\hat{x}(t)dt^{\eta} = \int x(t)\,dt \tag{5}$$

For a sequence of reference signal values:

$$x(t) = [x_1, x_2, \dots, x_n] \tag{6}$$

and a sequence of measurement signal values:

$$\hat{x}(t) = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n] \tag{7}$$

where: $x_1 = x(t_1)$, …, $x_i = x(t_i)$, $\hat{x}_1 = \hat{x}(t_1)$, …, $\hat{x}_i = \hat{x}(t_i)$, $i \in [1, \dots, n]$ and $dt = t_2 - t_1 = \cdots = dt = t_n - t_{n-1} = const.$, the integral within t_1 and t_2 has the form:

$$\int_{t_1}^{t_2} x(t)\,dt = \lim_{dt \to 0}(x_2 - x_1)dt = \Delta x_1 dt \tag{8}$$

and the differintegral is written as:

$${}^{1}D^{-\eta}\hat{x}(t)dt^{\eta_1} = \lim_{dt^{\eta} \to 0}(\hat{x}_2 - \hat{x}_1)dt^{\eta} = \Delta\hat{x}_1 dt^{\eta_1}. \tag{9}$$

Substituting equations (8) and (9) into (5):

$$\Delta\hat{x}_1 dt^{\eta_1} = \Delta x_1 dt. \tag{10}$$

Determining η order from (10):

$$\eta_1 = 1 + \log_{dt}\frac{\Delta x_1}{\Delta\hat{x}_1}. \tag{11}$$

For any of i the order (11) has form:

$$\eta_i = 1 + \log_{dt}\frac{\Delta x_i}{\Delta\hat{x}_i}. \tag{12}$$

where $\eta = [\eta_1, \eta_2, \dots, \eta_n]$, $\Delta x_i = x_i - x_{i-1}$ and $\Delta\hat{x}_i = \hat{x}_i - \hat{x}_{i-1}$.

Formula (12) yields a sequence of values of η order depending on the quotient of the increments $\Delta x(t)/\Delta\hat{x}(t)$. Since the determination of η is based on increments rather than function values, the characteristic $\eta_i(\Delta\hat{x})$ can be taken as a general case for the correction of any signal (for small values of dt). Assuming that the values of η_i are known, the estimated increment values of signal x is of the form (from equation (10)):

$$\Delta x_i = \Delta\hat{x}_i dt^{\eta_i - 1}. \tag{13}$$

The correction procedure consists of two stages:
1. Determination of η from a reference signal – equation (12).
2. Correction of the measurement signal on the basis of known values of η - equation (13).

3 PROCEDURE FOR DETERMINING THE η ORDER

The value of η_i can be determined from array data (chapter 2.2.1) or from the equation of function (chapter 3.1 and 3.2). All calculations and characteristics were performed in Matlab.

3.1 ARRAY η_i DATA

A block diagram of the system determining the order values η_i is shown in Fig. 1. The y signal is the input signal to the measuring system (e.g. measuring transducer), the Ideal System block is the reference measuring system and the Real System block is the measuring system with an error in relation to the Ideal System. The Factor η block determines an array of η order values based on equation (12).

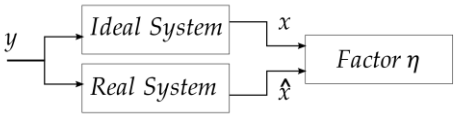

Fig. 2. Block diagram of the system determining the order values

A typical calibration signal for e.g. measuring transducers is a sine signal. For this signal, the increment of successive Δx values is not constant for a constant sampling value dt. It was therefore assumed that the sinusoidal signal would be the reference signal for determining η.

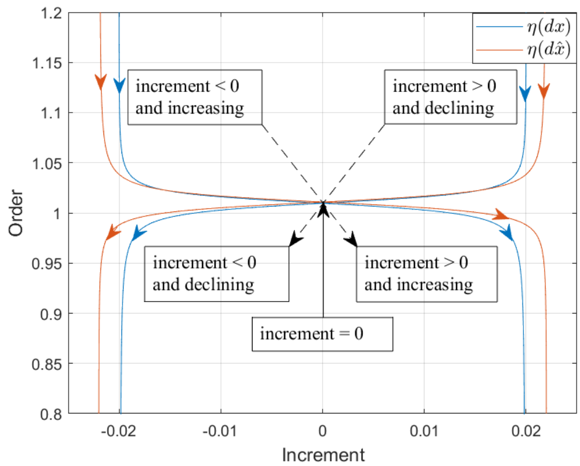

Fig. 3. The η order relative to the increments of the reference and measurement signals

The operation of the correction system is shown for example reference $x(t) = sin\, t$ and measurement $\hat{x}(t) = 1.1sin(t + \pi/30)$ signals and parameters dt=0.1 ms, f=200 rad/s. The characteristics $x(t)$ and $\hat{x}(t)$ are shown in Fig. 1. Fig. 3 shows the change in η relative to the increments of the reference and measurement signals. The direction of the characteristics depends on the sign of the signals and their increments. It can be seen from Fig. 3 that η for the reference and measurement signals (for small increments) are equal or approximately equal to each other. Discrepancies at the extremes are due to zero or near-zero values of the signal $x(t)$ or $\hat{x}(t)$, resulting in $(\Delta x_i)/(\Delta \hat{x}_i) \to 0$ or $(\Delta x_i)/(\Delta \hat{x}_i) = 0$ which, from the point of view of the definition of logarithm, is not permissible. This situation is due to the phase shift of the waveforms and its impact can be reduced by reducing the value of dt (in this case, the impermissible value from the definition of the logarithm $(\Delta x_i)/(\Delta \hat{x}_i) = 0$ should be disregarded) - in the further description this case is considered as a general one. Another solution is to scale the signals so that they do not reach 0 or to limit the η order.

3.2 Function of η_i

Using the data from the η_i array (chapter 3.1), the function can be determined that describes η_i (by Lagrange polynomials, for example). As the value of η_i depends on the direction of change of the increment, these must be two functions, where the choice of function is implemented by the logic circuit. For the signals in Fig. 1, the function η_i is described by example Lagrange polynomials:

$$\eta_i = \begin{cases} A_1 d\hat{x}^3 + A_2 d\hat{x}^2 + A_3 d\hat{x}^2 + A_1 d\hat{x} + A_0 \; for \\ (d\hat{x} < 0 \; AND \; d\hat{x} \uparrow < 0) \; OR \; (d\hat{x} > 0 \; AND \; d\hat{x} \uparrow < 0) \\ B_1 d\hat{x}^3 + B_2 d\hat{x}^2 + B_3 d\hat{x}^2 + B_1 d\hat{x} + B_0 \; for \\ (d\hat{x} > 0 \; AND \; d\hat{x} \downarrow > 0) \; OR \; (d\hat{x} < 0 \; AND \; d\hat{x} \downarrow > 0) \end{cases} \tag{14}$$

where the arrow corresponds to a negative or positive increment.

For $x(t) = sin\,t$ and $\hat{x}(t) = 1.1 sin(t + \pi/30)$:

$$A = [-1.108e + 04, -7.226, 1.793, 1.012] \tag{15}$$

$$B = [1.284e + 04, -27.280, -2.219, 1.014] \tag{16}$$

Fig. 4 compares η_i obtained from the η_i array and from the η_i function.

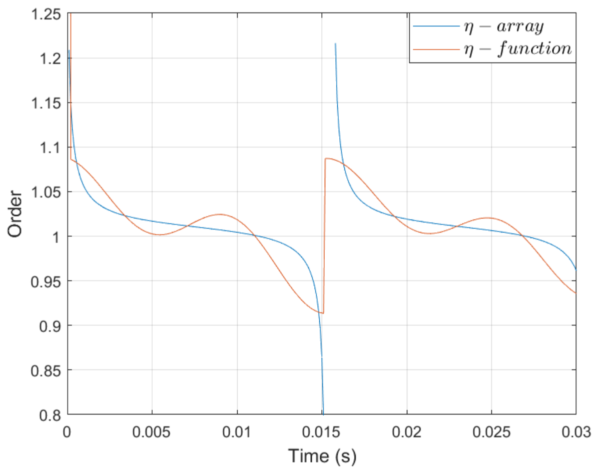

Fig. 4. The η-array and η-function

4 RESULTS

Fig. 5 shows the signals: $x(t)$ (reference), $\hat{x}(t)$ (measured), $\hat{x}\ (t) - array - estim$ (estimated from the array) and $\hat{x}\ (t) - func - estim$ (estimated from the function). The characteristics of the $x(t)$ and $\hat{x}\ (t) - array - estim$ signals coincide.

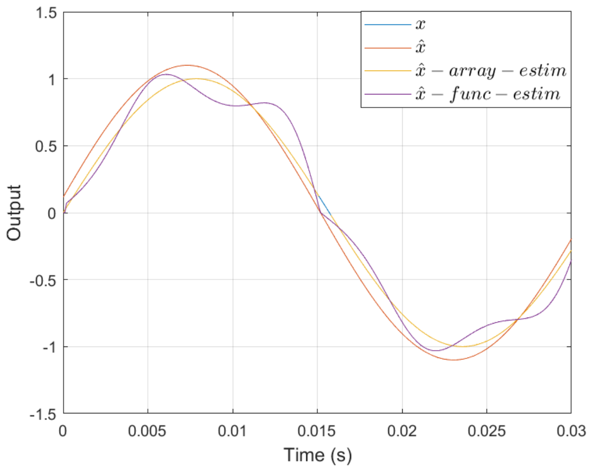

Fig. 5. Output signals

Fig. 6 shows the relative errors of the measured signal (Input Relative error), estimated from the array (Input Array Fractional Relative error) and estimated from the function (Input Function Fractional Relative error). The assessment of the estimation is shown in Table 1, where the assessment factor is the median.

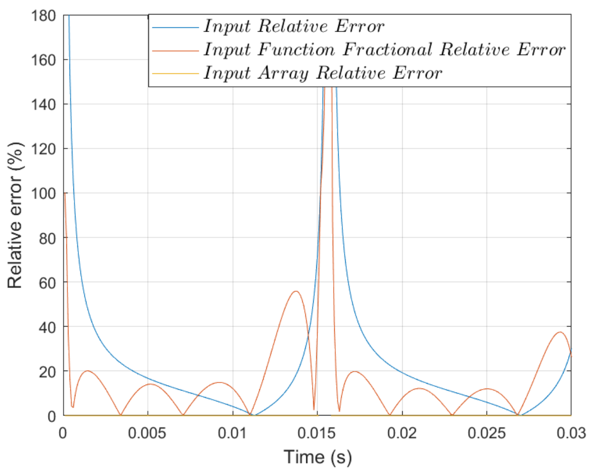

Fig. 6. Absolute values of the relative errors of the measured signal (Input Relative Error - IRE), the estimated from the array (Input Array Fractional Relative Error - IRFFE) and the estimated from the function (Input Function Fractional Relative Error - IARE

Table 1. Median of signals error absolute values

IRE	$IRFFE$	$IARE$
14.4880 %	11.8931 %	close to 10^{-13} %

5 DISCUSSION AND CONCLUSIONS

The estimation of the measured signal using the estimation array gives very good results with an error close to 0%. In this method, it is necessary to know the values of the signal increments and the corresponding η_i order values stored in the data array. It can be used in DSP systems with high computation speed and large memory capacity (e.g. in systems for the correction of non-linear processing characteristics of measuring transducers). Simpler to implement is the method where the order η_i is described by a function that has been previously determined from the data of the estimation array method. The disadvantage of this method is the inaccuracy of the estimation compared to the array method. However, the signal error after estimation is lower than without estimation by approximately 2.6%. This method uses a simple determination of the estimation equation using Lagrange method. It seems that better results can be achieved by determining a more precise equation. For better results, other estimation methods or data interpolation can be tried.

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