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Fusion Kalman filtration for distributed multisensor systems

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In the paper, fusion state hierarchical filtration for a multisensor system is considered. An optimal global Kalman filter is realized by a central node in the information form. The state estimate depends on local information that should be sent by local nodes. Two information structures are considered in the paper.

In the first case local estimates are based on the local measurement information. It leads to distributed Kalman filter fusion that is well known in a literature. In the second case local node has additionally global information of the system with one step delay. A synthesis of local filters is presented. An advantage of this structure is discussed.

Key words: fusion Kalman filtration, multisensor system, one step delay information

1. Introduction

Multisensor systems find applications in many areas such as aerospace, robotics, image processing, military surveillance, medical diagnosis. The advantage of using these systems over a systems with a single sensor results from e.g. improved reliability, robustness, extended coverage, improved resolution e.t.c. In these systems a state estimation problem is one of the critical concerns.

Theoretically, state estimate can be determined by using Kalman filter in a centralized structure. Conventional Kalman filtration requires that all process measurements are sent to a central station which determines an estimate of the state system. The centralized architecture produces an optimal estimate in a minimum mean square error (MMSE) sense, but it may imply low survivability and requires high processing and communication loads.

In order to integrate data from distributed sensors estimation fusion algorithms and appropriate architectures are proposed.

Fusion approaches have been researched for years and some results are known. In [5, 7, 12, 13] a centralized optimal state estimate is calculated from estimates determined by

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local nodes. The global estimate is equivalent to the optimal centralized one. In [2, 3, 4] are presented fusion algorithms guaranteed local optimality, only.

In [11] are presented different methodologies to obtain non-centralized state estimation algorithms and their implementations. A comprehensive review of the data state fusion state domain is given in [9].

In [1] was suggested a Kalman fusion filtering with feedback. Proposed hierarchical estimation equations can be implemented in a hierarchical structure where the central node calculates a global state prediction which is sent to local nodes. When a new local measurement is made, each node computes a local state estimate and communicates to the fusion center where a fusion is proposed. In [13] a performance analysis of those equations is given.

In this paper it is presented a hierarchical system with an information structure that leads to the equations suggested in [1]. Those equations are educed by directly derivation of Kalman filter. It is shown that for proposed architecture Kalman fusion is optimal and is equivalent to the corresponding centralized Kalman filtering formula. An advantage of this system is discussed.

2. Preleminaries

It is well known [10] that a minimum mean square error (MMSE) estimate $\hat{x}$ of a random signal x given information $\vec{i}$ is a conditional expectation $\hat{x} = E(x|\vec{i})$. For dynamical systems a state estimate $\hat{x}_{n|k}$ at time n given measurement information $\vec{i}_k = [i_0^T, i_1^T, ..., i_k^T]^T$ at time k has the form

$$\hat{x}_{n|k} = E(x_n|\vec{i}_k). \tag{1}$$

Thus, for $k = n$ we have

$$\hat{x}_{n|n} = E(x_n|\vec{i}_n) = E(x_n|\vec{i}_{n-1}, i_n) \tag{2}$$

where $\vec{i}_n = [\vec{i}_{n-1}^T, i_n^T]^T$.

If the random vector $[x_n^T, \vec{i}_{n-1}^T, i_n^T]^T$ is gaussian, then

$$\hat{x}_{n|n} = E(x_n|\vec{i}_n) = E(x_n|\vec{i}_{n-1}, i_n) = E(x_n|\vec{i}_{n-1}) + E(x_n|\widetilde{i}_{n|n-1}) - Ex_n \tag{3}$$

where

$$\widetilde{i}_{n|n-1} = i_n - E(i_n|\vec{i}_{n-1}). \tag{4}$$

If the random vector $[x_n^T, \widetilde{i}_{n|n-1}^T]^T$ is gaussian then

$$E(x_n|\widetilde{i}_{n|n-1}) = Ex_n + P_{x_n \widetilde{i}_{n|n-1}} P^{-1}_{\widetilde{i}_{n|n-1} \widetilde{i}_{n|n-1}} (\widetilde{i}_{n|n-1} - E\widetilde{i}_{n|n-1}) \tag{5}$$

where $P_{\alpha\beta}$ denotes a covariance matrix of the random vectors α and β.

Under above assumptions, the eqn. (3) can be written in the form

$$\hat{x}_{n|n} = \hat{x}_{n|n-1} + K_n[i_n - E(i_n|\vec{i}_{n-1})] \tag{6}$$

where

$$\hat{x}_{n|n-1} = E(x_n|\vec{i}_{n-1}) \tag{7}$$

and

$$K_n = P_{x_n\widetilde{i}_{n|n-1}} P^{-1}_{\widetilde{i}_{n|n-1}\widetilde{i}_{n|n-1}}. \tag{8}$$

The equations (6)-(8) are used for determination of a state estimate $\hat{x}_{n|n}$ given available information $\vec{i}_n$.

Let us consider a system described by a state equation

$$x_{n+1} = A_n x_n + w_n \tag{9}$$

and measurement equation

$$y_n = C_n x_n + r_n, \tag{10}$$

where x_n, y_n are the state and the measurement, respectively; A_n, C_n are the system and observation models, w_n, r_n are the state and measurement noises, respectively. It is assumed that $x_0 \sim N(\bar{x}_0, X_0)$, $w_n \sim N(\bar{w}_n, W_n)$, $r_n \sim N(0, R_n)$ and $x_n \in R^k$, $w_n \in R^k$, $y_n \in R^p$, $r_n \in R^p$; $A_n \in R^{k\times k}$, $C_n \in R^{p\times k}$. Additionally, w_n, r_m are gaussian white noise processes independent of each other and of the gaussian initial state x_0.

It is well known [10] that an optimal state estimate $\hat{x}_{n+1|n+1} = E(x_{n+1}|y_0, y_1, ..., y_{n+1})$ results from the eqn. (6)-(8) and has the form

$$\begin{aligned}\hat{x}_{n+1|n+1} &= \hat{x}_{n+1|n} + K_{n+1}\widetilde{y}_{n+1|n} = \hat{x}_{n+1|n} + K_{n+1}(y_{n+1} - \hat{y}_{n+1|n}) = \\ &= \hat{x}_{n+1|n} + K_{n+1}(y_{n+1} - C_{n+1}\hat{x}_{n+1|n})\end{aligned} \tag{11}$$

where

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{y}_n) = A_n\hat{x}_{n|n} + \bar{w}_n. \tag{12}$$

The matrix gain K_{n+1} is described as

$$K_{n+1} = P_{n+1|n}C^T_{n+1}(C_{n+1}P_{n+1|n}C^T_{n+1} + R_{n+1})^{-1} \tag{13}$$

where

$$P_{n+1|n} = E\widetilde{x}_{n+1|n}\widetilde{x}^T_{n+1|n} = E(x_{n+1} - \hat{x}_{n+1|n})(x_{n+1} - \hat{x}_{n+1|n})^T = A_nP_{n|n}A^T_n + W_n \tag{14}$$

and

$$P_{n|n} = E\widetilde{x}_{n|n}\widetilde{x}^T_{n|n} = E(x_n - \hat{x}_{n|n})(x_n - \hat{x}_{n|n})^T = (\mathbf{1} - K_nC_n)P_{n|n-1}. \tag{15}$$

An initial condition $\hat{x}_{0|0}$ results from (11)

$$\hat{x}_{0|0} = \hat{x}_{0|-1} + K_0(y_0 - C_0\hat{x}_{0|-1}) = \bar{x}_0 + K_0(y_0 - C_0\bar{x}_0). \tag{16}$$

The covariance matrix $P_{0|-1}$ can be determined as

$$P_{0|-1} = E\widetilde{x}_{0|-1}\widetilde{x}_{0|-1}^{T} = E(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T = X_0. \tag{17}$$

Classical covariance filter presented above can be described in an information form [6] as

$$\begin{aligned} P_{n+1|n+1}^{-1}\hat{x}_{n+1|n+1} &= P_{n+1|n}^{-1}\hat{x}_{n+1|n} + C_{n+1}^T(R_{n+1})^{-1}y_{n+1} \\ \hat{x}_{n+1|n} &= A_n\hat{x}_{n|n} + \bar{w}_n \\ P_{n+1|n+1}^{-1} &= P_{n+1|n}^{-1} + C_{n+1}^T(R_{n+1})^{-1}C_{n+1}. \end{aligned} \tag{18}$$

Information filter has some computational advantages in multisensor systems where the matrix $C_n^T(R_n)^{-1}C_n$ is usually of high dimension and nondiagonal.

A multisensor system is defined by the state model (9) and measurement equations

$$y_n^j = C_n^j x_n + r_n^j, \quad j = 1,...,M \tag{19}$$

where y_n^j is the measurement from the ith sensor.

It is assumed that $r_n^j \sim N(0, R_n^j)$, $y_n^j \in R^{p^j}$, $r_n^j \in R^{p^j}$; $C_n^j \in R^{p^j \times k}$. Additionally, w_n, r_m^j are gaussian white noise processes independent of each other and of the gaussian initial state x_0.

The classical covariance Kalman filter for the multisensor system defined as $\hat{x}_{n+1|n+1} = E(x_{n+1}|y_0, y_1, ..., y_{n+1})$ is described by the eqn. (11)-(15) where $y_n = [y_n^{1T}, ..., y_n^{MT}]^T$, $C_n = [C_n^{1T}, ..., C_n^{MT}]^T$, $r_n = [r_n^{1T}, ..., r_n^{MT}]^T$, $R_n = Er_n r_n^T = blockdiag\{R_n^1, ..., R_n^M\}$.

Consequently its information form (18) may be written as

$$\begin{aligned} P_{n+1|n+1}^{-1}\hat{x}_{n+1|n+1} &= P_{n+1|n}^{-1}\hat{x}_{n+1|n} + \sum_{j=1}^{M} C_{n+1}^{jT}(R_{n+1}^j)^{-1}y_{n+1}^j \\ \hat{x}_{n+1|n} &= A_n\hat{x}_{n|n} + \bar{w}_n \\ P_{n+1|n+1}^{-1} &= P_{n+1|n}^{-1} + \sum_{j=1}^{M} C_{n+1}^{jT}(R_{n+1}^j)^{-1}C_{n+1}^j. \end{aligned} \tag{20}$$

The global estimate performed by the central processor depends on information state vectors $C_{n+1}^{jT}(R_{n+1}^j)^{-1}y_{n+1}^j$ and information matrices $C_{n+1}^{jT}(R_{n+1}^j)^{-1}C_{n+1}^j$, $j = 1, ..., M$, that can be performed and transmitted by local nodes to the central node. It may increase processing speed.

Sometimes it is better to perform Kalman filtration by every local node upon its own available information and then transmit local state estimates to the fusion center, where a fusion is carried out. It will be discussed in the sequel.

3. Problem statement

Let us consider the multisensor system described by the state equation (9) with measurement equations (19).

The optimal Kalman filter realized by the central node is described by the information form (20). It is seen that the state estimate $\hat{x}_{n|n}$ depends on local information $C_n^{jT}(R_n^j)^{-1}y_n^j$ and $C_n^{jT}(R_n^j)^{-1}C_n^j$. This information should be sent by local nodes to the central node.

Let us assume that local nodes perform local state estimates of the system (9) basing on available local information.

Two information structures are considered in the paper.

In the first case local estimates $\hat{x}_{n|n}^j, \quad j = 1,...,M$, are based on the information $\vec{i}_n^j = [y_0^{jT},...,y_n^{jT}]^T$. It leads to distributed Kalman filter fusion without feedback.

In the second case local estimates $\hat{x}_{n|n}^j, \quad j = 1,...,M$, are based on the information $\vec{i}_n^j = [y_0^T,...,y_{n-1}^T, y_n^j]^T$. It leads to distributed Kalman filter fusion with feedback. Let us notice that local node has global measurement information of the system with one step delay.

Firstly, local estimation for assumed information structures are considered. Next, the possibility of fusion algorithms using the local estimates will be presented. In the feedback case an advantage of this structure will be discussed.

4. Kalman filtering without feedback

Let us consider local estimation problem for the multisensor system described by the state equation (9) and measurement equations (19) assuming that information available for the jth node at time n is defined as $\vec{i}_n^j = [y_0^{jT},...,y_n^{jT}]^T$.

Denote by
$\vec{y}_{n+1}^j = [y_0^{jT},...,y_n^{jT},y_{n+1}^{jT}]^T = [\vec{y}_n^{jT},y_{n+1}^{jT}]^T$, $\tilde{y}_{n+1|n}^j = y_{n+1}^j - E(y_{n+1}^j|\vec{y}_n^j)$ and $\vec{i}_n^j = \vec{y}_n^{jT}$, $i_{n+1}^j = y_{n+1}^{jT}$.

The local filtration problem for the jth mode is to find

$$\hat{x}_{n+1|n+1}^j = E(x_{n+1}|\vec{i}_{n+1}^j) = E(x_{n+1}|\vec{y}_{n+1}^j). \tag{21}$$

For the system described by the eqn. (9) and (19) the random vectors $[x_{n+1}^T, \vec{y}_n^{jT}, y_{n+1}^{jT}]^T$, $[y_{n+1}^{jT}, \vec{y}_n^{jT}]^T$ and $[x_{n+1}^T, \tilde{y}_{n+1|n}^{jT}]^T$ are gaussian.

Thus the covariance Kalman filter results directly from the section 2 and is described by the eqn. (11)-(15) with superscript "j" for all variables except of the state model parameters. Thus

$$\hat{x}_{n+1|n+1}^j = \hat{x}_{n+1|n}^j + K_{n+1}^j(y_{n+1}^j - C_{n+1}^j\hat{x}_{n+1|n}^j) \tag{22}$$

$$\hat{x}^j_{n+1|n} = E(x_{n+1}|\vec{y}^j_n) = A_n\hat{x}^j_{n|n} + \bar{w}_n \tag{23}$$

$$K^j_{n+1} = P^j_{n+1|n}C^{jT}_{n+1}(C^j_{n+1}P^j_{n+1|n}C^{jT}_{n+1} + R^j_{n+1})^{-1} \tag{24}$$

$$P^j_{n+1|n} = A_nP^j_{n|n}A^T_n + W_n \tag{25}$$

$$P^j_{n+1|n+1} = E(x_{n+1} - \hat{x}^j_{n+1|n+1})(x_{n+1} - \hat{x}^j_{n+1|n+1})^T = (\mathbf{1} - K^j_{n+1}C^j_{n+1})P^j_{n+1|n}. \tag{26}$$

An information local Kalman filter results from the eqn. (18) and is described as

$$\begin{aligned}
&(P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1} = (P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n} + C^{jT}_{n+1}(R^j_{n+1})^{-1}y^j_{n+1}\\
&\hat{x}^j_{n+1|n} = A_nP^j_{n|n}\hat{x}^j_{n|n} + \bar{w}_n\\
&(P^j_{n+1|n+1})^{-1} = (P^j_{n+1|n})^{-1} + C^{jT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1}.
\end{aligned} \tag{27}$$

From (27) we have

$$C^j_{n+1}(R^j_{n+1})^{-1}y^j_{n+1} = (P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1} - (P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n} \tag{28}$$

and

$$C^{jT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1} = (P^j_{n+1|n+1})^{-1} - (P^j_{n+1|n})^{-1}. \tag{29}$$

Global optimal state estimate results from (20). Using (28) and (29) in (20) gives

$$\begin{aligned}
&P^{-1}_{n+1|n+1}\hat{x}_{n+1|n+1} = P^{-1}_{n+1|n}\hat{x}_{n+1|n} + \sum_{j=1}^{M}(P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1} - (P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n}\\
&\hat{x}_{n+1|n} = A_nP_{n|n}\hat{x}_{n|n} + \bar{w}_n\\
&P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + \sum_{j=1}^{M}(P^j_{n+1|n+1})^{-1} - (P^j_{n+1|n})^{-1}.
\end{aligned} \tag{30}$$

Equations (30) describe fusion Kalman filter that generates optimal global state estimate according to (20). Local node needs its own local measurement to generate local state estimate. Communication from central node to the local nodes is no need. That is why this algorithm is known as the fusion algorithm without feedback.

5. Kalman filtering with feedback

Let us assume that the local estimate of the state x_{n+1} is based on the local information $\vec{i}^j_{n+1,f} = [y^T_0, \ldots, y^T_n, y^{jT}_{n+1}]^T$. Because of nonconventional information structure an algorithm of Kalman filtration will be presented in more details.

Denote by $\vec{y}^j_{n+1,f} = [y^T_0, \ldots, y^T_n, y^{jT}_{n+1}]^T = [\vec{y}^T_n, y^{jT}_{n+1}]^T$, $\tilde{y}^j_{n+1|n,f} = y^j_{n+1} - E(y^j_{n+1}|\vec{y}_n)$. The local filtration problem for the jth mode is to find

$$\hat{x}^j_{n+1|n+1,f} = E(x_{n+1}|\vec{i}^j_{n+1,f}) = E(x_{n+1}|\vec{y}^j_{n+1,f}). \tag{31}$$

For the system described by the eqn. (9) and (19) the random vectors $[x_{n+1}^T, \vec{y}_n^T, y_{n+1}^{jT}]^T$, $[y_{n+1}^{jT}, \vec{y}_n^T]^T$ and $[x_{n+1}^T, \widetilde{y}_{n+1|n,f}^{jT}]^T$ are gaussian. Thus the estimate

$$\hat{x}_{n+1|n+1,f}^j = E(x_{n+1}|\vec{y}_{n+1,f}^j) \tag{32}$$

results from the eqn. (6)-(7) and has the form

$$\hat{x}_{n+1|n+1,f}^j = \hat{x}_{n+1|n} + K_{n+1,f}^j \widetilde{y}_{n+1|n,f}^j = \hat{x}_{n+1|n} + K_{n+1,f}^j (y_{n+1}^j - \hat{y}_{n+1|n,f}^j) \tag{33}$$

where $\hat{y}_{n+1|n,f}^j = E(y_{n+1}^j|\vec{y}_n)$, $\widetilde{y}_{n+1|n,f}^j = y_{n+1}^j - \hat{y}_{n+1|n,f}^j = y_{n+1}^j - C_{n+1}^j \hat{x}_{n+1|n}$ and

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{y}_n) = A_n \hat{x}_{n|n} + \bar{w}_n. \tag{34}$$

The matrix gain $K_{n+1,f}^j$ can be found from (8) as

$$K_{n+1,f}^j = P_{x_{n+1}\widetilde{y}_{n+1|n,f}^j} P^{-1}_{\widetilde{y}_{n+1|n,f}^j \widetilde{y}_{n+1|n,f}^j}. \tag{35}$$

In (35)

$$\widetilde{y}_{n+1|n,f}^j = y_{n+1}^j - E(y_{n+1}^j|\vec{y}_n) = C_{n+1}^j \widetilde{x}_{n+1|n}. \tag{36}$$

It can be shown that the covariance matrices $P_{\widetilde{y}_{n+1|n,f}^j \widetilde{y}_{n+1|n,f}^j}$ and $P_{x_{n+1}\widetilde{y}_{n+1|n,f}^j}$ have the form

$$\begin{aligned} P_{\widetilde{y}_{n+1|n,f}^j \widetilde{y}_{n+1|n,f}^j} &= C_{n+1}^j P_{n+1|n} C_{n+1}^{jT} + R_{n+1}^j \\ P_{x_{n+1}\widetilde{y}_{n+1|n,f}^j} &= P_{n+1|n} C_{n+1}^{jT}. \end{aligned} \tag{37}$$

Thus the matrix gain $K_{n+1,f}^j$ results from (35) and (37) as

$$K_{n+1,f}^j = P_{n+1|n} C_{n+1}^{jT} (C_{n+1}^j P_{n+1|n} C_{n+1}^{jT} + R_{n+1}^j)^{-1}. \tag{38}$$

The covariance matrix $P_{n+1|n}$ is described by (14). Finally, the local Kalman filter is described by the eqn. (33)-(34) and (13).

Now, let us determine a local covariance matrix $P_{n|n,f}^j$ defined as

$$P_{n|n,f}^j = E(x_n - \hat{x}_{n|n,f}^j)(x_n - \hat{x}_{n|n,f}^j)^T. \tag{39}$$

Using (33) we have

$$x_{n+1} - \hat{x}_{n+1|n+1,f}^j = x_{n+1} - \hat{x}_{n+1|n} - K_{n+1,f}^j \widetilde{y}_{n+1|n,f}^j = \widetilde{x}_{n+1|n} - K_{n+1,f}^j \widetilde{y}_{n+1|n,f}^j \tag{40}$$

and

$$P_{n+1|n+1,f}^j \quad = E\widetilde{x}_{n+1|n+1,f}^j \widetilde{x}_{n+1|n+1,f}^{jT} = E\widetilde{x}_{n+1|n} \widetilde{x}_{n+1|n}^T - E\widetilde{x}_{n+1|n} \widetilde{y}_{n+1|n,f}^{jT} K_{n+1,f}^{jT} -$$

$$-EK^{j}_{n+1,f}\widetilde{y}^{j}_{n+1|n,f}\widetilde{x}^{T}_{n+1|n}+EK^{j}_{n+1,f}\widetilde{y}^{j}_{n+1|n,f}\widetilde{y}^{jT}_{n+1|n,f}K^{jT}_{n+1,f}. \tag{41}$$

Using (36), (35) and (37) gives

$$P^{j}_{n+1|n+1,f}=(\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1})P_{n+1|n}. \tag{42}$$

Let us notice that (33) can be written in the form

$$\begin{aligned}\hat{x}^{j}_{n+1|n+1,f}&=\hat{x}_{n+1|n}+K^{j}_{n+1,f}(y^{j}_{n+1}-C^{j}_{n+1}\hat{x}_{n+1|n})\\&=(\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1})\hat{x}_{n+1|n}+K^{j}_{n+1,f}y^{j}_{n+1}.\end{aligned} \tag{43}$$

Now we transform $(\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1})$ and $K^{j}_{n+1,f}$ to an appropriate form. We have

$$\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1}=\overbrace{\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1}P_{n+1|n}}^{P^{j}_{n+1|n+1,f}(42)}P^{-1}_{n+1|n}=P^{j}_{n+1|n+1,f}P^{-1}_{n+1|n}. \tag{44}$$

Denote by

$$O^{j}_{n+1}=C^{j}_{n+1}P_{n+1|n}C^{jT}_{n+1}+R^{j}_{n+1}. \tag{45}$$

Multiplying the both sides of the eqn. (38) by O^{j}_{n+1} gives

$$K^{j}_{n+1,f}(\overbrace{C^{j}_{n+1}P_{n+1|n}C^{jT}_{n+1}+R^{j}_{n+1}}^{O^{j}_{n+1}(45)})=P_{n+1|n}C^{jT}_{n+1}. \tag{46}$$

Thus

$$K^{j}_{n+1,f}R^{j}_{n+1}=(\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1})P_{n+1|n}C^{jT}_{n+1} \tag{47}$$

and

$$K^{j}_{n+1,f}=(\overbrace{\mathbf{1}-K^{j}_{n+1,f}C^{j}_{n+1}}^{P^{j}_{n+1|n+1,f}P^{-1}_{n+1|n}(44)})P_{n+1|n}C^{jT}_{n+1}(R^{j}_{n+1})^{-1}=P^{j}_{n+1|n+1,f}C^{jT}_{n+1}(R^{j}_{n+1})^{-1}. \tag{48}$$

Inserting (44) and (48) to (43) gives

$$\hat{x}^{j}_{n+1|n+1,f}=P^{j}_{n+1|n+1,f}P^{-1}_{n+1|n}\hat{x}_{n+1|n}+P^{j}_{n+1|n+1,f}C^{jT}_{n+1}(R^{j}_{n+1})^{-1}y^{j}_{n+1}. \tag{49}$$

From (49) it results that

$$(P^{j}_{n+1|n+1,f})^{-1}\hat{x}^{j}_{n+1|n+1,f}=P^{-1}_{n+1|n}\hat{x}_{n+1|n}+C^{jT}_{n+1}(R^{j}_{n+1})^{-1}y^{j}_{n+1}. \tag{50}$$

Now we determine a recursive form of the covariance matrix $(P^{j}_{n+1|n+1,f})^{-1}$.

The covariance matrix $P^j_{n+1|n+1,f}$ given by (42) can be written by

$$\begin{aligned} P^j_{n+1|n+1,f} = P_{n+1|n} - K^j_{n+1,f} O^j_{n+1,f} \overbrace{(O^j_{n+1})^{-1} C^j_{n+1} P_{n+1|n}}^{K^{jT}_{n+1,f}(38)} = \\ = P_{n+1|n} - K^j_{n+1,f} O^j_{n+1} K^{jT}_{n+1,f} \end{aligned} \tag{51}$$

The eqn. (51) has the form

$$\begin{aligned} P^j_{n+1|n+1,f} &= P_{n+1|n} - K^j_{n+1,f} (\overbrace{C^j_{n+1} P_{n+1|n} C^{jT}_{n+1} + R^j_{n+1}}^{O^j_{n+1}(45)}) K^{jT}_{n+1,f} = \\ &= P_{n+1|n} - K^j_{n+1,f} C^j_{n+1} P_{n+1|n} C^{jT}_{n+1} K^{jT}_{n+1,f} - K^j_{n+1,f} R^j_{n+1} K^{jT}_{n+1,f}. \end{aligned} \tag{52}$$

But from (44) we have

$$K^j_{n+1,f} C^j_{n+1} = \mathbf{1} - P^j_{n+1|n+1,f} P^{-1}_{n+1|n}. \tag{53}$$

Inserting (53) to (52) yields

$$\begin{aligned} P^j_{n+1|n+1,f} &= P^j_{n+1|n+1,f} P^{-1}_{n+1|n} P^j_{n+1|n+1,f} + K^j_{n+1,f} R^j_{n+1} K^{jT}_{n+1,f} = \\ &= P^j_{n+1|n+1,f} P^{-1}_{n+1|n} P^j_{n+1|n+1,f} + \overbrace{P^j_{n+1|n+1,f} C^{jT}_{n+1} (R^j_{n+1})^{-1}}^{K^j_{n+1,f}(48)} R^j_{n+1} \overbrace{(R^j_{n+1})^{-1} C^j_{n+1} P^j_{n+1|n+1,f}}^{K^{jT}_{n+1,f}} = \\ &= P^j_{n+1|n+1,f} P^{-1}_{n+1|n} P^j_{n+1|n+1,f} + P^j_{n+1|n+1,f} C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1} P^j_{n+1|n+1,f}. \end{aligned} \tag{54}$$

Multiplying two-times the both sides of the eqn. (54) by $(P^j_{n+1|n+1,f})^{-1}$ gives

$$\mathbf{1} = P^{-1}_{n+1|n} P^j_{n+1|n+1,f} + C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1} P^j_{n+1|n+1,f} \tag{55}$$

$$(P^j_{n+1|n+1,f})^{-1} = P^{-1}_{n+1|n} + C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1}. \tag{56}$$

Let us notice that from (50) and (56) we have

$$C^{jT}_{n+1} (R^j_{n+1})^{-1} y^j_{n+1} = (P^j_{n+1|n+1,f})^{-1} \hat{x}^j_{n+1|n+1,f} - P^{-1}_{n+1|n} \hat{x}_{n+1|n} \tag{57}$$

and

$$C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1} = (P^j_{n+1|n+1,f})^{-1} - P^{-1}_{n+1|n}. \tag{58}$$

Global optimal state estimate results from (20). Using (57) and (58) in (20) gives

$$P^{-1}_{n+1|n+1} \hat{x}_{n+1|n+1} = \sum_{j=1}^{M} (P^j_{n+1|n+1,f})^{-1} \hat{x}^j_{n+1|n+1,f} - (M-1) P^{-1}_{n+1|n} \hat{x}_{n+1|n}$$

$$\hat{x}_{n+1|n} = A_n \hat{x}_{n|n} + \bar{w}_n$$
$$P^{-1}_{n+1|n+1} = \sum_{j=1}^{M} (P^j_{n+1|n+1,f})^{-1} - (M-1) P^{-1}_{n+1|n}. \tag{59}$$

Equations (59) describe fusion Kalman filter that generates optimal global state estimate according to (20). Local node needs its own local measurement and global information from the central node with one step delay. to generate local state estimate. Thus communication from central node to the local nodes is need. That is why this algorithm is known as the fusion algorithm with feedback. The fusion equations (59) are suggested in [1], only.

6. The quality of the feedback fusion structure

Let us compare the centralized Kalman filtering (20) with fusion algorithms without feedback (30) and with feedback (59). The algorithms are exactly equivalent. Thus the feedback does not improve the performance at the central node (fusion center).

Does the feedback reduce local state filtering error?

Let us compare local covariance matrices described by the eqn. (27) and (56). Subtracting the both sides gives

$$(P^j_{n+1|n+1,f})^{-1} - (P^j_{n+1|n+1})^{-1} = P^{-1}_{n+1|n} - (P^j_{n+1|n})^{-1} \tag{60}$$

where $P_{n+1|n}$ and $(P^j_{n+1|n})$ are described by the eqn. (14) and (25). We apply the matrix inversion lemma ([8]) to find $P^{-1}_{n+1|n} - (P^j_{n+1|n})^{-1}$, which yields

$$\begin{aligned} &P^{-1}_{n+1|n} - (P^j_{n+1|n})^{-1} = \\ &\quad = W_n^{-1} A_n \{[(P^j_{n|n})^{-1} + A_n^T W_n^{-1} A_n]^{-1} - (P^{-1}_{n|n} + A_n^T W_n^{-1} A_n)^{-1}\} A_n^T W_n^{-1} \end{aligned} \tag{61}$$

Subtracting the both sides of the eqn. (20) and (27) and using (62) gives

$$\begin{aligned} P^{-1}_{n+1|n+1} - (P^j_{n+1|n+1})^{-1} &= P^{-1}_{n+1|n} - (P^j_{n+1|n})^{-1} + \sum_{i \neq j}^{M} C^{iT}_{n+1} (R^i_{n+1})^{-1} C^i_{n+1} = \\ &= W_n^{-1} A_n \{[(P^j_{n|n})^{-1} + A_n^T W_n^{-1} A_n]^{-1} - (P^{-1}_{n|n} + A_n^T W_n^{-1} A_n)^{-1}\} A_n^T W_n^{-1} + \\ &+ \sum_{i \neq j}^{M} C^{iT}_{n+1} (R^i_{n+1})^{-1} C^i_{n+1}. \end{aligned} \tag{62}$$

For $P_{0|0} = P^j_{0|0} = X_0$ we have

$$P^{-1}_{1|1} - (P^j_{1|1})^{-1} = \sum_{i \neq j}^{M} C^{iT}_{n+1} (R^i_{n+1})^{-1} C^i_{n+1} \geqslant 0 \tag{63}$$

and consequently

$$
\begin{aligned}
&(P^j_{1|1})^{-1} \leqslant P^{-1}_{1|1}\\
&(P^j_{1|1})^{-1} + A_1^T W_1^{-1} A_1 \leqslant P^{-1}_{1|1} + A_1^T W_1^{-1} A_1\\
&[(P^j_{1|1})^{-1} + A_1^T W_1^{-1} A_1]^{-1} \geqslant (P^{-1}_{1|1} + A_1^T W_1^{-1} A_1)^{-1}\\
&W_1^{-1} A_1 [(P^j_{1|1})^{-1} + A_1^T W_1^{-1} A_1]^{-1} A_1^T W_1^{-1} \geqslant W_1^{-1} A_1 (P^{-1}_{1|1} + A_1^T W_1^{-1} A_1)^{-1} A_1^T W_1^{-1}\\
&W_1^{-1} A_1 [(P^j_{1|1})^{-1} + A_1^T W_1^{-1} A_1]^{-1} A_1^T W_1^{-1} + \sum_{i\neq j}^{M} C_2^{iT} (R_2^i)^{-1} C_2^i \geqslant\\
&\qquad \geqslant W_1^{-1} A_1 (P^{-1}_{1|1} + A_1^T W_1^{-1} A_1)^{-1} A_1^T W_1^{-1}\\
&W_1^{-1} A_1 \{[(P^j_{1|1})^{-1} + A_1^T W_1^{-1} A_1]^{-1} - (P^{-1}_{1|1} + A_1^T W_1^{-1} A_1)^{-1}\} A_1^T W_1^{-1} +\\
&\qquad + \sum_{i\neq j}^{M} C_2^{iT} (R_2^i)^{-1} C_2^i \geqslant 0 \qquad (64)\\
&\quad P^{-1}_{2|2} - (P^j_{2|2})^{-1} \geqslant 0 \;\Rightarrow\; (P^j_{2|2})^{-1} \leqslant P^{-1}_{2|2}.
\end{aligned}
$$

Working recursively we obtain

$$(P^j_{n|n})^{-1} \leqslant P^{-1}_{n|n}. \tag{65}$$

From the above expression we have

$$(P^j_{n|n})^{-1} + A_n^T W_n^{-1} A_n \leqslant P^{-1}_{n|n} + A_n^T W_n^{-1} A_n \tag{66}$$

$$[(P^j_{n|n})^{-1} + A_n^T W_n^{-1} A_n]^{-1} \geqslant (P^{-1}_{n|n} + A_n^T W_n^{-1} A_n)^{-1} \tag{67}$$

$$W_n^{-1} A_n [(P^j_{n|n})^{-1} + A_n^T W_n^{-1} A_n]^{-1} A_n^T W_n^{-1} \geqslant W_n^{-1} A_n (P^{-1}_{n|n} + A_n^T W_n^{-1} A_n)^{-1} A_n^T W_n^{-1} \tag{68}$$

$$W_n^{-1} A_n \{[(P^j_{n|n})^{-1} + A_n^T W_n^{-1} A_n]^{-1} - (P^{-1}_{n|n} + A_n^T W_n^{-1} A_n)^{-1}\} A_n^T W_n^{-1} \geqslant 0. \tag{69}$$

Using (62) gives

$$P^{-1}_{n+1|n} - (P^j_{n+1|n})^{-1} \geqslant 0 \tag{70}$$

and according to the eqn. (60)

$$(P^j_{n+1|n+1,f})^{-1} \geqslant (P^j_{n+1|n+1})^{-1}. \tag{71}$$

Thus the feedback may reduce the local state filtering errors.

7. Conclusion

Two hierarchical fusion filtration algorithms are presented in the paper. They are based on two information structures. In the first structure local filters use local measurement, only, while in the second one, local nodes have global measurement information with one step delay. The first algorithm is called "without feedback", while the second one is called "with feedback". It is shown that the both algorithms are equivalent to the corresponding centralized Kalman filter. The advantage of the feedback is a possibility of a reduction of the local error state estimates.

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