Paper

# Multimedia Mathematical Communication in a Diverse Group of Students

Jolanta Brzostek-Pawłowska

*Research and Academic Computer Network (NASK), Warsaw, Poland*

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**Abstract—The article tackles the problem of improving mathematical communication in a group of students with different visual impairment levels, under the guidance of a group leader or a teacher. Visually impaired persons face a problem while learning mathematics. The said problem results from the specific nature in which mathematical content (formulas, function graphs, geometrical figures and projections of solids) is recorded and presented. The effectiveness of learning mathematics is boosted when students work in a group moderated by a leader. This requires them to share documents, with the leader being able to keep track of the individual work of each participant, and with the group discussing specific solutions. In order for a visually impaired student to be able to participate in and contribute to the work of the group, either remotely or locally, all participants must use universal IT tools that support visually impaired students without complicating the work of others. This paper presents interactive multimedia solutions developed under two research projects carried out by the author. The said solutions support communication in mathematics. Results of qualitative surveys on new solutions are presented, confirming their usefulness and the measurable impact they exert on the efficiency of the group's work concerning mathematical problems.**

***Keywords—efficiency of communication in learning mathematics, mathematical formula notations, semantic readout of formulas.***

## 1. Introduction

The problem of communication-related capabilities in mathematics that are necessary for transferring, acquiring, consolidating and using mathematical knowledge in everyday life, was tested on a group of secondary school students and is described in paper [1]. Low percentage results were obtained with regard to the following: (i) ability to described a situation, an idea or a mathematical correlation using algebraic expressions, graphics, images (35%), (ii) use of mathematical language in everyday life (35%), (iii) use of images or diagrams to express a mathematical concept (53,3%). The results confirmed existence of the low level of communication skills in the field of mathematics. The surveyed consisted of students without sight impairments. Visually impaired students (particularly blind ones) encounter even more serious problems concerning mathematical communication. This is mainly due to such spatial elements as formulas, function graphs, diagrams and geometric objects.

As shown by research presented in [2], one of the effective forms of teaching and learning mathematics is by working on a project, a problem or a mathematical task in a group of several people. The conclusions from these studies indicate a positive effect of, inter alia, the factor of mutual assistance of the group members, and of individualized assistance. Similar, individualized assistance is provided to a student in a two-person teacher-student group, e.g. during compensatory, additional classes or during remote, online consultations. A problem appears when a group learning mathematics, either on its own or with the help of a teacher, comprises visually impaired students. Such groups are, by definition, the norm in inclusive education. The problem of the students' poor mathematical communication skills overlaps with the problem of the efficiency of communication with a visually impaired member of the group who is often using a different user interface, e.g. Braille technology and other mathematical tools, such as equation editors.

For a diverse group to be able to cooperate efficiently and for the student's self-help factor or the teacher's assistance to work, the group must be equipped with IT tools that facilitate communication concerning mathematical problems. The tools that may be helpful in creating, presenting and exploring mathematical content, as well as in communicating by exchanging mathematical documents, include chats and remote voice conversations. A set of such software tools supporting the teacher or the group leader, as well as visually impaired and blind students, known under the name of PlatMat OPTY, has been developed under two research projects.

The paper presents selected solutions that have been developed and implemented in the tools comprising the PlatMat OPTY platform, used to enhance efficient communication concerning mathematical content containing formulas, within a group of students that is diverse in terms of the visual capabilities of its members. The users may create and share their math-related texts and audio content relying on audio-visual and tactile senses. The users may create and control a new way of communication using this system, in addition to conducting traditional, single medium-based conversations. In order to overcome interaction limitations, we have proposed, in PlatMat OPTY, the use of integrated senses. Solutions enabling quick recognition of mathematical graphics have also been developed[1], as have been tools allowing to perform arithmetic calculations in a written form [3]. They are intended for students with visual impairments but are not discussed in this paper.

## 2. Accessibility Problems in Mathematical Communication

The problems whose solutions have been sought through research and development activities include the following:

1. compatibility of various interfaces used for editing and presenting mathematical formulas, enabling a group made up of sighted and visually impaired people to cooperate while working on mathematical issues;

2. creating a single version of electronic mathematical documents – accessible both to sighted persons and to those with visual impairments, simplifying the process and shortening the time devoted by the author to creating e-documents, and avoiding stigmatization of visually impaired persons cooperating within the same group with sighted persons;

3. sharing information and e-mathematical documents in a group that is diversified in terms of visual acuity, working remotely via the Internet or locally, with or without access to Wi-Fi networks.

A special case of a group that is diversified in terms of visual acuity of the participants, is one made of students and a teacher of an integration class in a public school implementing the idea of inclusive education. Another case is a two-person group: a sighted teacher and a visually impaired pupil, working locally during compensatory classes, or remotely, via the Internet.

### *2.1. Various Mathematical Notations and User Interfaces*

The first problem concerns various notations (languages) used for writing formulas, and various user interfaces applied to save and read them, by means of which sighted, blind and low vision persons create and present formulas. Sighted people use spatial and graphical visualization of formulas, and usually write formulas with the help of popular formula editors available in MS Word, Open Office, Libre Office, offering spatial formula structures to be filled by the user, for example: $\frac{\sqrt{\square^{\square}}}{\square}$, $\sum_{\square}^{\square}\square^{\square}$.
While preparing scientific publications with a large number of formulas, academics use the LaTeX system [4]. For writing and reading formulas, blind and low-vision persons with serious visual disability can use the linear Braille mathematical notation, based on the 7-bit, 128 character ASCII standard available via a QWERTY keyboard, for example AsciiMath (AMS) notation [5], Wiskunde Notatie Dedicon (WND) [6], or an extended notation using multi-byte characters, such as Lambda notation [7], as well as UnicodeMath notation developed in PlatMat, based on AsciiMath. The difference between the latter and the AsciiMath notation is that the mathematical symbols not accessible from a QWERTY keyboard, and are entered using a keyboard shortcut or a mouse click on the symbol displayed in the formula editor. These symbols are saved using Unicode, not ASCII. In Poland, non-digitized, tactile convex printing Braille Mathematical Notation (BNM) is used [8]. In other countries, due to the lack of standardization, other mathematical Braille notations are used. It is worth mentioning, however, that such an approach hinders the interoperability of documents and prevents international cooperation of visually impaired people in the field of mathematics. In Poland, linear mathematical notations, other than BNM, are not widely used. The educational system for blind students is based on Braille notation and technology. The problem with linear notations, both of the Braille and ASCII variety, lies in the fact that the formulas in these notations are longer and require more characters than in spatially visualized forms.
An exemplary expression of a fraction with a root in the numerator, presented in a spatial form, looks as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

While the same expression presented in selected linear notations looks as follows:

| | | |
|---|---|---|
| BNM: | _x=;-b!-CbO;"-#d_ac 8 #b_a< | (29 characters), |
| Lambda: | x=⫽-b+-√b^2-4ac∕2a⑊ | (19 characters), |
| LaTex: | $$x=\frac{-b+-\sqrt{b^{2}-4ac}}{2a}$$ | (37 characters), |
| AsciiMath: | x=(-b+-sqrt(b^2-4ac))/(2a) | (26 characters), |
| WND: | x=(-b+-sqrt(b^2-4ac))/(2a) | (26 characters), |
| UnicodeMath: | x=(-b+-√(b^2-4ac))/(2a) | (23 characters). |

It is worth noting that in some linear notations, for example in Lambda, operating systems include fonts developed specifically for marking the structure of the formula. These fonts are accessible via a keyboard shortcut or a mouse click in the formula editor of a given notation. In Lambda, these characters are marked in red, indicating the beginning and the end of the fraction and the end of the numerator. As it may be noticed in the examples provided, the formulas are lengthy, both in Braille and in other line notations. The advantage that non-Braille linear notations (hereinafter: linear QWERTY notations) have over Braille, especially when the form of the formula is not, for printing purposes, converted to a mathematical Braille notation, consists in the possibility of editing and reading formulas using commonly available computer hardware with a QWERTY keyboard.
Formulas in QWERTY linear notations, unless otherwise programmed, are read character by character, using assistive software, such as screen readers, e.g. NVDA or Jaws

[1] Paper under preparation.

installed at the operating system layer. This method of reading is not friendly for a blind user and makes it difficult for them to recognize the structure of the formula and its transformation. However, it is often used by persons with seriously impaired vision, who find structural editors, for example MS Word, too difficult to use. Nevertheless, in the Netherlands, at schools teaching blind students, the WND linear notation (an example of a WND record is shown above), similar to the AsciiMath notation standard, has been used for 20 years, and the Braille mathematical notation is not used at all. Formulas are read by a screen reader, character by character, while convex printouts are created with Braille fonts whose codes correspond to the characters in the linear formula record. This way of teaching mathematics to visually impaired students does not require the teacher's and the pupil's knowledge of complex the Braille mathematical notation system, and does not require expensive specialist Braille equipment, such as Braille lines and notebooks. This technology is adapted to inclusive education purposes in accordance with the basic assumption stating that mainstream schools should be prepared to teach visually impaired students (and those with other disabilities). However, students using linear formula notations cannot work at the same pace as their sighted peers.

To recapitulate, quick conversion of formulas (e.g. during group discussions) commonly used by sighted persons into the Braille line notation used by blind persons is a problem that is partly solved by semantic formula reading and AsciiMath notation, used by low vision students.

### *2.2. Universality and Availability of Mathematical E-documents*

The second problem that we tried to solve, taking into account the first challenge of various user interfaces relied upon while creating and exploring formulas, is the rapid exchange of information and electronic documents containing formulas, occurring on a continuous basis during the work of a group made up of persons with various visual acuity. These needs have to be addressed in educational settings – in the classroom, during supplementary activities, remote e-consultations and e-tutoring, and in publishing houses during cooperation concerning mathematical documents, e.g. in the preparation of mathematical textbooks. Problems faced in the last of the scenarios above are described in detail in [9]. An electronic document containing formulas, projected spatially on a screen, is not accessible to blind people, unless it has been specifically processed. In general, formulas in electronic publications have the form of raster images, inaccessible to the blind, or are saved in the form of structural MathML notations, based on XML. The MathML standard may be supported by most browsers (based on Gecko and WebKit engines), which means that it displays the formula spatially (graphically), which is inadequate for people with a high level of visual impairment. A semantic readout of the formula may be of assistance here. This solution is easier to implement for formulas written in MathML than for linear notations. Semantic formula readers operating in many native languages are available, based on plug-ins for browsers, e.g. for Firefox and Internet Explorer (MathPlayer) but they do not operate in the Polish language.

As textbooks assume the form of apps and electronic exams replace traditional their paper predecessors, the demand for ICT tools is growing. We observe a continuous development of multimedia e-publications, also those of mathematical character, but no measurable development of supporting IT technologies that facilitate the absorption of mathematical content users with a serious visual impairment may be observed in Poland. Educational publications and auxiliary materials, especially in the field of mathematics, require user's interactivity in solving problems and mathematical tasks. This applies, for instance, to notebooks, work sheets, examples in textbooks and tests. The level of interactivity offered by e-books is insufficient in relation to the needs of mathematical education, not only in the case of visually impaired students, but also in the case of sighted users. Interactivity of e-books boils down to navigating the document, selecting fragments of content, making tabs and notes. Few e-publications use MathML notation, due to the poor rendering of formulas saved in MathML by browsers other than Firefox. An example of misinterpretation of entries in MathML is presented below:

- the cube root in the expression x+1: $\sqrt[3]{x}+1$,
- the root of minus one-third degree of x+1: $\sqrt[-13]{x+1}$.

It is safer for the publisher to place a raster image of a specific formula in an e-book. The effect is that it is not possible to edit the formula or navigate the structure of the formula to better understand it or to reach the element of the structure that is to be modified.

To ensure that a group of visually impaired people may operate efficiently, facilities are necessary that increase the accessibility of formulas, such as semantic reading of a formula or of its selected fragments, quick selection of a fragment of the formula and its edition by means of a preferred method (Braille or QWERTY notation), in combination with a readout. The possibility of creating a mathematical e-document in one universal version which may be used by both sighted and visually impaired persons, and which may also be relied upon for transferring their e-work between them, is a condition that needs to be fulfilled for ensuring efficient work, within a group, of its sighted, visually impaired and blind members. All that is related to the need of solving a third problem, namely efficient exchange of universal mathematical e-documents and information.

### *2.3. Efficient Exchange of Mathematical e-documents and Information*

The exchange of information and documents within the group should be ensured regardless of the environmental conditions related to the availability of the network or its lack. The first example may be the work of students in the classroom, performed under the guidance of a teacher, where it is necessary to monitor both the workflow of each student and the exchange of documents between the teacher and students. The teacher should be able to remotely monitor the work of each pupil, in order to be able to react quickly to any mistakes made by students. Students should be able to pass, to the teacher, their work along with any comments. The teacher should be able to send back, to the student, the corrected work with mistakes pointed out and described, and with guiding and explanatory comments included. The teacher should be able to show selected solutions to the students and to discuss them. But this is where specific problems need to be tackled.

In a classroom which is not, for example, an IT room, access to a network and/or the Internet may not be available, as the author of the paper has experienced while pursuing various projects. Teachers should have the tools to carry out these educational operations without barriers caused by the diversity of the group (visual acuity of the participants or limitations concerning access to the network). In order to implement these educational operations, the teacher needs to be able to organize an ad-hoc Wi-Fi network, locally in the room, and to connect all computers used by the teacher and students, including an interactive whiteboard. In the case of an interactive whiteboard, there is a need of connecting the whiteboard to the teacher's computer, as this will provide the teacher and the students with access to tools required for creating and exchanging universal mathematical e-documents.

Online assistance provided to students working at home, in a dormitory or in hospital, ensuring that they are not left behind in their schoolwork or are able to make up for any losses as soon as possible, is another example. To offer this type of help, in addition to the Internet, there is a need for a remote method of viewing the pupil's work, exchanging e-documents and conversing via written or spoken mediums.

The problems discussed above are related to the provision of efficient classwork communication means, i.e. those between the teacher and students. Internet-based systems facilitating the provision of teacher-student communication exist and are used, as it transpires from research conducted within the EuroMath project under the Erasmus+ program [10]. Google Classroom [11], Impero [12] and Desmos Classroom Activities [13] are systems used in Ireland and the Netherlands, for example. According to research conducted, they have not been used in Polish schools for teaching visually impaired students. The authors' analysis of the accessibility of these systems intended for visually impaired users, the results of which were included in paper [14], showed that only Google Classroom is fully available to this group of users. However, in relation to the three problems discussed above, it only partially solves the third problem by enabling the teacher and the students to share documents.

## 3. Development of Accessible Communication Media

This section of the paper presents solutions that have been developed within PlatMat OPTY, addressing the three problems discussed in the previous section: the use of various languages and mathematical interfaces; creation of universal and accessible mathematical e-documents; efficient distribution and exchange of e-documents and mathematical information.

### *3.1. Media for Editing, Reading and Exploring Formulas*

Each participant of the group may edit a given formula using their preferred formula editor, and may use an accessible user interface. Five different formula editors that may be deployed by teachers and students have been developed:

- for teachers and sighted students – a structured editor similar to the MS Word (Fig. 1a),
- for teachers and sighted students – a customized version of a Windows-based editor for manual edition of formulas (Fig. 1b),
- for low vision students as well as for teachers and sighted students – UnicodeMath editor, based on the AsciiMath notation, with a shortened linear visualization of the formula, e.g. instead of "sqrt", the square root symbol $\surd$ is displayed (Fig. 1c),
- for blind students – AsciiMath notation editor (Fig. 1d),
- for blind students – BNM notation editor (Fig. 1e).

The interfaces which may be used to edit formulas include the following:

- QWERTY keyboard and mouse,
- keyboard shortcuts for entering mathematical symbols,
- keyboard shortcuts for navigating the formula structure,
- touch gestures with one and two fingers to navigate the formula structure while editing the formula (at the beginning/end of the formula, by one character to the right/left, one step up/down in the hierarchy structure),
- Braille keyboard emulated on a QWERTY keyboard (f, d, s, l, j, k, l keys for the six-point edition in BNM),
- physical Braille keyboard and keyboard shortcuts,
- aural reading of the formula characters (and text characters) entered.

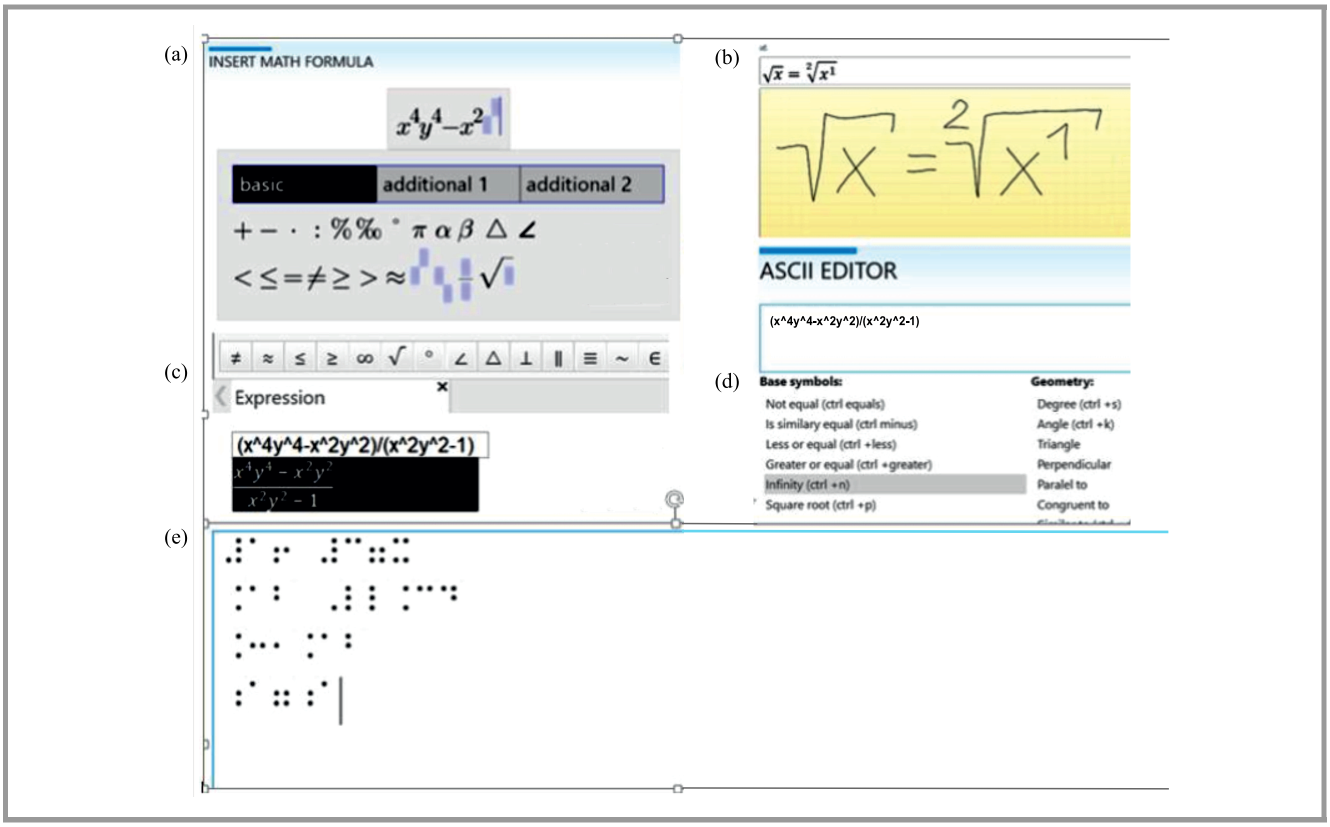


*Fig. 1.* Five editor formulas available in PlatMat OPTY for sighted users and users with a sight dysfunction.

The interfaces that may be used while reading formulas include the following:

- a screen where formulas are always visualized spatially for the teacher's needs and, depending on the user's needs and preferences, also linearly in AsciiMath, UnicodeMath and BNM,
- Braille line (also known as Braille monitor), on which BNM or AsciiMath characters are activated by touch, by moving pins,
- semantic reading of a formula or of a selected fragment of a text including the formula, in Polish,
- keyboard shortcuts and touch gestures with one and two fingers for reading and detailed exploration of a given formula.

The record of the formula obtained in a notation that is different than the one used by the original user must be – without a noticeable delay – converted into the notation which he/she uses. The following five converters cooperating with each other on an on-going basis via programming interfaces (API) have been developed:

- BNM into MathML,
- MathML into BNM,
- MathML into AsciiMath,
- UnicodeMath into AsciiMath,
- AsciiMath into UnicodeMath.

In addition, a ready-made library in the JavaScript language for converting AsciiMath to MathML was used.

In order for a formula, regardless of the editor and notation it was written in, to be spatially displayed by a browser for the needs of sighted users and to be semantically read out for the needs of visually impaired users, it must be saved in MathML notation. Other notations (AsciiMath, UnicodeMath, BNM) are converted to MathML. For the purpose of editing formulas, the inverse conversion from MathML to the required notations is performed. As mentioned above, some browsers have trouble with correctly displaying formulas saved in MathML. The MathML notation is correctly interpreted by Firefox. It is worth noting that this set of converters cooperating with each other on an on-going basis, has been developed for the first time; although many attempts have been made in relation to conversion of various mathematical notations, and despite the fact that their results are available in the form of online services [15], [16] and local solutions [17], [18], they fail to include the Polish version of BNM notation.

Automatic translation covering four methods of expressing mathematical language (BNM, AsciiMath, UnicodeMath and MathML) is a rather complex problem. The development of BNM converters poses a particularly major research challenge due to the problem of semantic inter-

Table 1
Mathematical communication based on exchanging .epub files between users relying on various mathematical notations and user interfaces

| Participants | Direction of communication | Participant interface | Participant's notation | Mathematical notation | Conversion |
|---|---|---|---|---|---|
| W, S | W→S | W | Structural editor | MathML | MathML→epub |
| | | S | UnicodeMath editor | UnicodeMath | epub→MathML/AsciiMath/UnicodeMath |
| | S→W | S | UnicodeMath editor | UnicodeMath | UnicodeMath/AsciiMath/MathML→epub |
| | | W | Structural editor | MathML | epub→MathML |
| W, N | W→N | W | Structural editor | MathML | MathML→epub |
| | | N | Ascii editor | AsciiMath | epub→MathML/AsciiMath |
| | N→W | N | Ascii editor | AsciiMath | AsciiMath/MathML→epub |
| | | W | Structural editor | MathML | epub→MathML |
| | W→N | W | Structural editor | MathML | MathML→epub |
| | | N | BNM editor | BNM | epub→MathML/BNM |
| | N→W | N | BNM editor | BNM | BNM/MathML→epub |
| | | W | Structural editor | MathML | epub→MathML |
| S, N | S→N | S | UnicodeMath editor | UnicodeMath | UnicodeMath/AsciiMath/MathML→epub |
| | | N | BNM editor | BNM | epub→MathML/BNM |
| | N→S | N | BNM editor | BNM | BNM/MathML→epub |
| | | S | UnicodeMath editor | UnicodeMath | epub→MathML/AsciiMath/UnicodeMath |
| | S→N | S | UnicodeMath editor | UnicodeMath | UnicodeMath/AsciiMath/MathML→epub |
| | | N | Ascii editor | AsciiMath | AsciiMath/MathML→epub |
| | N→S | N | Ascii editor | AsciiMath | AsciiMath/MathML→epub |
| | | S | UnicodeMath editor | UnicodeMath | epub→MathML/AsciiMath/UnicodeMath |
| Key: W – sighted user, S – low vision user, N – blind user | | | | | |

Table 2
Examples of rules applied while translating formula elements into texts of their semantic readout

| Item | Formula elements | Readout rule |
|---|---|---|
| 1 | Integers | The integers are read as they are pronounced in Polish, e.g. 121 'sto dwadzieścia jeden' (English: *hundred twenty one*) |
| 2 | Floating-point numbers | Numbers with a coma (*point in English notation*) e.g. 5,26, are read as 'pięć przecinek dwadzieścia sześć' (English: *five coma twenty six*) |
| 3 | Letters – vowels | The vowels are read unchanged |
| 4 | Letters – consonants | The consonants are read phonetically, e.g. b as beh. Big letters are preceded by the word capital |
| 5 | Greek letters | Greek letters are read phonetically, e.g. $\alpha$ as alpha |
| 6 | Subscripts | The subscripts are read, e.g. '… with subscript end of subscript'. If the subscript contains only one element, then the text 'end of the subscript' is omitted. Area subscripts such as Pb are read (in Polish) as pe be (English pronunciation: *peh beh*) and Pp as pe pe (English pronunciation: *peh peh*) |
| 7 | Superscripts | The superscripts are read, e.g. '… to the power … end of superscript'. If the superscript contains only one element, then the text 'end of the superscript' is omitted. If the superscript is 2, and it is the only element, then the text read is: „… kwadrat (English: *square*). If the index is 3 and it is the only element, then the text read is '… sześcian' (English: *cube*) |
| 8 | Fractions | The fractions are read as follows: a fraction numerator … denominator … end of a fraction. Numeric fractions are read literally, e.g. 1/6 – one sixth |

pretation of particular symbols, which often depends on the context (appearing in connection with other symbols, a space place before or after the symbol etc.).
The set of converters developed enables instant, on-going communication between people using various mathematical notations and different user interfaces. The term "on-going communication" relates to three communication scenarios:

- an .epub file being generated (in the EPUB 3 standard discussed below) by one of the users and sent to another user or to other users, containing formulas recorded in MathML notation, regardless of the notation in which the formulas were edited,
- transferring them to the interactive whiteboard, displaying them spatially on the group leader's screen, regardless of the notation used in the edition phase,
- displaying on the screen of the leader, and possibly on the interactive whiteboard, the content of the remote screen of the selected group member, editing the formulas in the preferred notation, with the said formulas being then saved in MathML and visualized spatially.

Table 1 shows the conversion processes taking place when exchanging .epub files, depending on the notations preferred by the the sender and recipient of mathematical content. In the remaining two situations, the conversions are similar, as in each case the conversion leads to creating a MathML notation, or a user-preferred notation based on MathML notation. It is worth noting that in order for a blind user to see the content on the interactive whiteboard receiving data from another user's screen, the screen contents must be sent to the blind user as an .epub file that will be unpacked, and then the MathML notation of the formulas included in the content will be converted to BNM or AsciiMath.
In order to increase the chances of blind and low-vision students working in a group at a pace that would not interfere with the work of the entire, in addition to the ability to read formulas by sight or by touch on the Braille line, another option to improve mathematical communication has been introduced. It allows a formula or its fragments to be read in Polish, semantically, in a manner adopted in Poland for that specific purpose. To offer this functionality, Poland's semantic formula reader relying on synthetic speech has been developed. The basic module of the reader has the form of a translator of the formula structure recorded in MathML notation, converting it into the text to be read out. The MathML notation is based on XML tag language [17]. With the introduction of HTML 5, MathML has superseded, in the browsers, the AsciiMath notation, which was used on the Internet at that time. The structured, hierarchical nature of MathML enables reliable spatial visualization of formulas. An example of a fraction with the root in the numerator, which was shown above in several linear notations, saved in MathML, is presented in Listing 1.

**Listing 1.** MathML example

```
<math mode="display"
xmlns="http://www.w3.org/1998/Math/MathML">
 <semantics>
  <mrow>
    <mi>x</mi>
    <mo>=</mo>
    <mfrac>
      <mrow>
        <mo form="prefix">&#x2212;<!-- ? --></mo>
        <mi>b</mi>
        <mo>&#x00B1;<!-- &PlusMinus; --></mo>
        <msqrt>
          <msup>
            <mi>b</mi>
            <mn>2</mn>
          </msup>
          <mo>&#x2212;<!-- ? --></mo>
          <mn>4</mn>
          <mo>&#x2062;<!-- &InvisibleTimes; --></mo>
          <mi>a</mi>
          <mo>&#x2062;<!-- &InvisibleTimes; --></mo>
          <mi>c</mi>
        </msqrt>
      </mrow>
      <mrow>
        <mn>2</mn>
        <mo>&#x2062;<!-- &InvisibleTimes; --></mo>
        <mi>a</mi>
      </mrow>
    </mfrac>
  </mrow>
  <annotation encoding="TeX">
     x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}
  </annotation>
  <annotation encoding="StarMath␣5.0">
     x={-b plusminus sqrt {b^2 - 4 ac}} over {2a}
  </annotation>
 </semantics>
</math>
```

A table used for translating formula templates into texts to be read out has been developed based on predetermined readout rules. Texts and readout rules have been developed for formulas introduced by primary and secondary school curricula. The readout texts have been agreed upon with mathematics teachers. The translator, the rules developed and the problems encountered during translation were presented in [20]. Some examples of translation rules are presented in Table 2.
For example formula

$$y^{a^{b_c}} \neq y^{b^b \cdot c}$$

is read as 'igrek do potęgi a do potęgi be z indeksem dolnym ce koniec wykładnika koniec wykładnika nie równa się igrek do potęgi a do potęgi be koniec wykładnika ce koniec wykładnika' (In English: *wai to the power ei to the power bi end of superscript with a subscript si end of subscript end of superscript is not equal to wai to the power ei to the power bi end of superscript si end of superscript*).
Figure 2a shows the structure of this formula (and the text of the readout) visualized in the formula navigator, while Fig. 2b,c shows the structure of selected fragments of the formula during navigation and the texts of their readouts. The texts of the readouts which do not appear in real conditions on the screen, have been included in the drawings for illustrative purposes.
The texts of readouts from the translation tables are passed to the NVDA screen reader synthesizer, which PlatMat cooperates with in its part intended for blind users.

### 3.2. Universally Accessible Medium for Recording Mathematical Content

In search of an electronic medium recording mathematical content and enabling its distribution and exchange within a group of participants with diverse visual acuity, the increasingly popular EPUB e-publication standard has been chosen over its competitor – Amazon's commercial AZW standard (an extension of the MOBI standard). EPUB is an open standard for the distribution and exchange of digital publications and documents that is based on the structured XML markup language [21].

EPUB version 3.1 (EPUB3) relies on the following standards: HTML5 – content, CSS3 – style, SVG – vector graphics, MathML – formulas, MPEG4 – video and sound, OTF – fonts, JS – scripts. The EPUB3 container, which is a ZIP file, integrates various types of files that can create multimedia, interactive, mathematical content. The features and properties of individual formats may be effectively used to increase the accessibility of EPUB3 content. In particular, MathML for writing formulas and SVG vector graphics standards, based – just as EPUB3 – on the structural language of XML markups, are essential for improving the accessibility of mathematical content. The structure of the recording allows detailed exploration of an EPUB3 document and of the formulas and graphics contained therein, in a manner accessible to people with visual impairments. Active elements – JS scripts, which may be included in the EPUB3 container, can be used to increase the accessibility of a mathematical document, e.g. to read, using synthetic speech, a part of the formula and also to support the user's interactivity, for instance by means of a touch gesture or a keyboard shortcut. JS scripts improve the interactive capability of a user working with an EPUB3 document, beyond the typical interactivities available in e-publications, such as note taking, selecting text fragments, creating tabs or navigating the document structure.

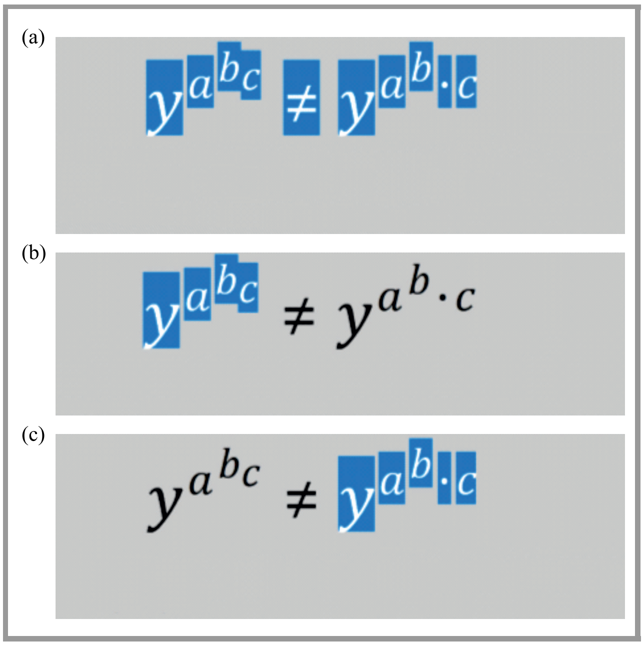


*Fig. 2.* Visualization of the formula structure and texts of its readouts (a), and formula fragments while navigating the formula (b) and (c).

The ability to enter formulas into an e-document, or to edit formulas contained in the text using the user-preferred notation, as well as to convert them to MathML notation, with the prospect of subsequent conversion when the document reaches a user whose preferred notation differs from that of the original one, is a good example of enhanced interactivity. These types of interactivities are very much needed, for example in mathematical education (solving tasks/tests, improving student's work) or in cooperation on publishing mathematical e-publications. Scripts may also be used for additional navigation, beyond navigation functionalities offered by browsers, among such elements of the mathematical e-document as formulas, text or voice comments, test questions, and mathematical graphics.

Conversion of mathematical notations specified in Table 1, as well as exploration of the entire e-document and the formulas contained therein, handled by JS scripts, enables the creation of a universal, mathematical EPUB3 e-document that is accessible to every participant of the cooperating group, regardless of the level of their visual acuity. It should be emphasized that universality is a useful feature for the creator of a mathematical e-document, because it means that the document may be created once, in one version, for all group members, and accessibility is a useful feature for the e-document recipient who receives the same e-document as other group members and may read, modify and explore it in a manner tailored to his/her needs.

Theoretically, browsers would be the most convenient way to handle EPUB3 documents. There are browser add-ons (plug-ins) extending their functionalities, so as enable them to tackle e-documents, e.g. EPUBReader for Firefox, but these are typical readers of documents not containing mathematical content and they do not or only partially support the MathML standard. They play the role of typical e-publication readers and offer, as mentioned above, a limited scope of interactive support.

Due to the lack of interactive software supporting mathematical content stored in EPUB3, the installable PlatMat software that is based on the Gecko engine, was and still remains the only software for creating, reading, modifying and exploring mathematical EPUB3 e-documents. It is also a tool for disseminating, exchanging and collecting e-documents and mathematical information – functionalities that are discussed in the following part of this paper.

## 4. Media for Sharing Mathematical E-documents and Information

The third problem discussed earlier, which was taken up in the works on PlatMat, is the question of an efficient, ongoing exchange of EPUB3 mathematical e-documents and information in a group that is diversified in terms of visual acuity of its members, in various environmental conditions concerning the availability of local Wi-Fi and Internet connections.

### *4.1. Mathematical Communication in a Wi-Fi Network*

Each member of the group, equipped with the PlatMat software, can create an EPUB3 document and save it locally. The leader can also save the document in a remote repository on the www.platmat.pl portal, so as to have it generally available. In order for the document placed in the repository to be published, it must undergo a verification process. The verification process takes place between the verifier and the author of the document. After successful verification, the portal administrator places a positive verification flag in the document's metadata. Then, the document becomes public, visible to the users and available for download. Each group participant may download the document published in the repository to his/her own local disk. In the local network the leader may send a document to each member of the group. Each group member may send/send back a document to the leader.

The applications of the leader and those of group members communicate with each other via the local Wi-Fi network. The group is connected forming the topology of a star, with the leader's computer serving as the central node. Group members do not communicate with each other directly. The exchange of documents between group members is possible through the leader. The star topology was adopted, in consultation with teachers, as the most suitable for the working conditions of a school group – with the teacher and the students in a classroom.

To connect the group members' applications with the leader and to enable them to exchange documents, it must be made sure that both applications operated on the same subnet. The subnet is identified by the first 3 digits of the IPv4 address – they are identical within the same subnet. After launching the application on the leader's computer and starting up the applications of the group members, the latter begin to actively search for the leader's application. When the searching process comes to an end (after about 10–20 s), the application of each group member will connect to leader's application it has identified, or it will ask the user to select the appropriate leader if more than one leader application has been found in a given network. If no leader has been found, the search will continue indefinitely. The same will occur in the event of a loss of connection.

Where no local Wi-Fi network is available, it is possible to set up a hot spot on the leader's computer, provided that the network card supports such a functionality (it must operate in the AccessPoint mode). Some classrooms have no access to a Wi-Fi network, and the option of becoming unrestricted by this limitation while communicating within the group is very useful. In addition to exchanging documents, a local Wi-Fi network allows the leader to activate the remote desktop of a selected group member, for example a student in a classroom, and to monitor the student's work by displaying a picture of the student's screen on the leader's computer to support the student if any problems in solving tasks are encountered. The leader may display, on the interactive whiteboard, the content of their own screen

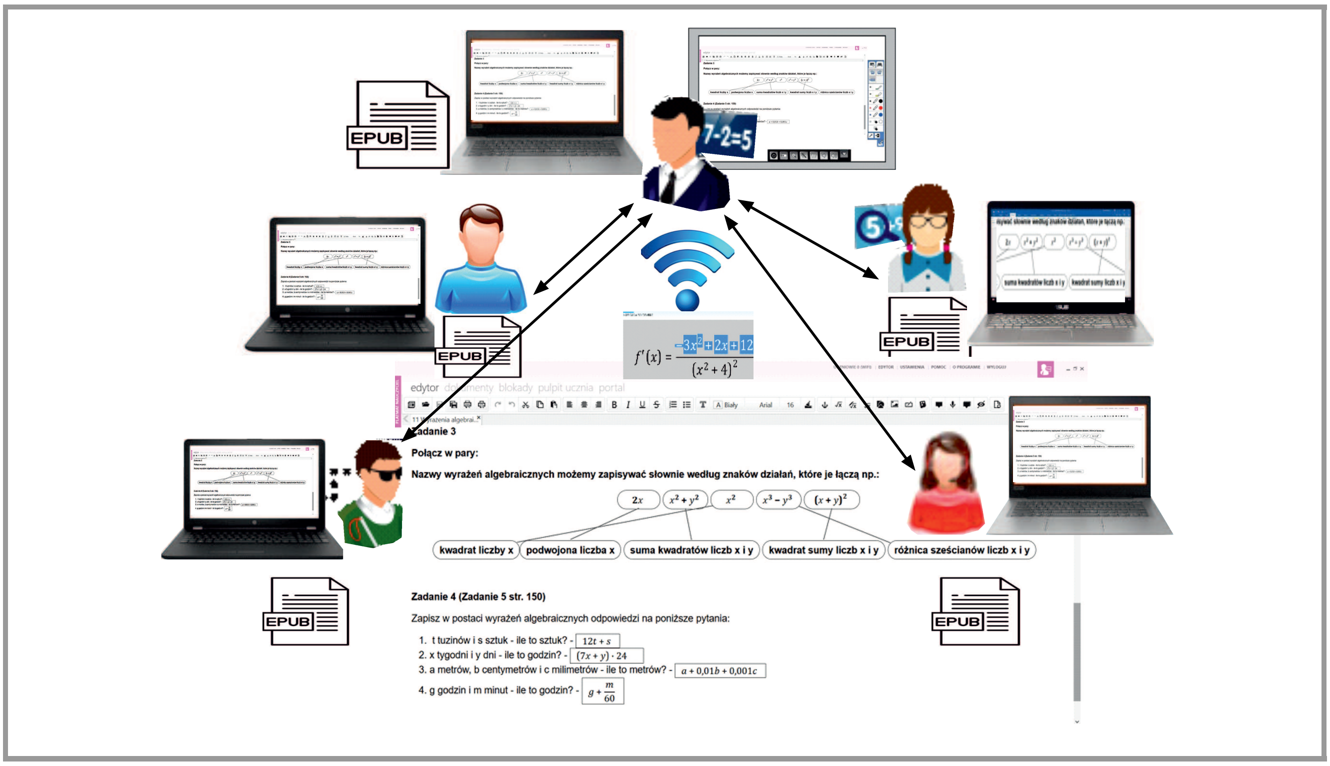

*Fig. 3.* Mathematical communication in a diverse group of participants, relying on a local Wi-Fi network.

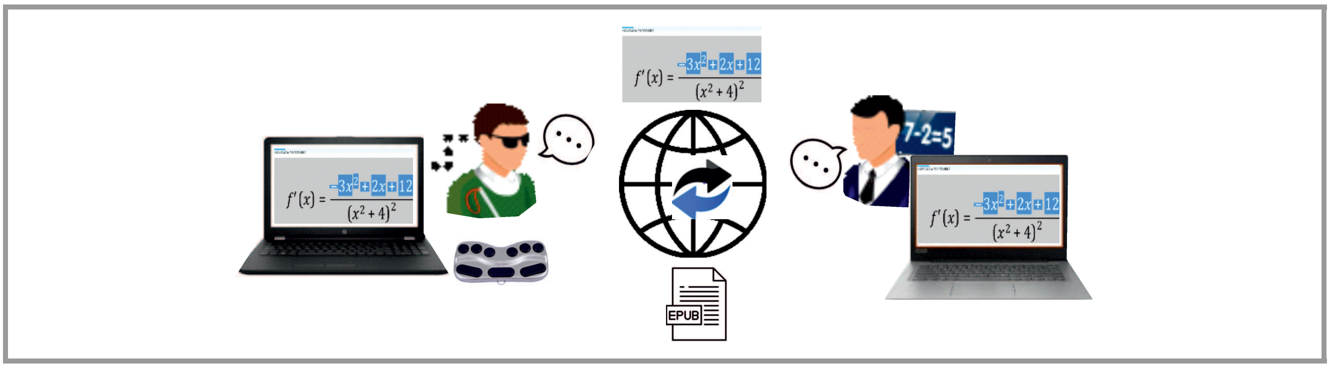


*Fig. 4.* Mathematical communication of a leader and a group member via the Internet (chat, Braille, voice, EPUB3 documents exchange, remote desktop).

Table 3
Mathematical communication via a Wi-Fi network and the Internet

| | | Role in the group | | Network | |
|---|---|---|---|---|---|
| Item | Operation | Group leader | Group member | Local Wi-Fi | Internet |
| 1 | Creating/modifying EPUB3 e–documents | + | + | – | – |
| 2 | Collecting locally EPUB3 e–documents | + | + | – | – |
| 3 | Publishing EPUB3 e–documents in a remote repository for publication | + | – | – | + |
| 4 | Downloading published EPUB3 e–documents from a remote repository | + | + | | + |
| 5 | Sending an EPUB3 e–document to a group member | + | – | + | + |
| 6 | Sending an EPUB3 e–document to the group leader | – | + | + | + |
| 7 | Initiating a chat | – | + | – | + |
| 8 | Chat with a group member | + | + | – | + |
| 11 | Remote desktop of a group member viewed by the leader | + | – | + | + |

or of the screen of a group member, received via the remote desktop function. The Wi-Fi based communication model is illustrated in Fig. 3.

### *4.2. Mathematical Communication via the Internet*

PlatMat-based online communication was developed for the purpose of remote consultations and is useful for mathematics as it offers the following functionalities: chat (also in Braille), voice calls, exchange of documents, remote desktop monitoring. Examples of educational activities that require remote assistance include the following: helping a student who is behind in his/her work, one who is poor at maths or is home-schooled. Another example involves editorial cooperation concerning a mathematical document, with the blind person using Braille technology. Chat typed in six-point Braille font is received by the sighted person in the form of plain text, a mathematical .epub document, while formulas edited in mathematical Braille notation are received by a sighted person in MathML notation and are displayed spatially (graphically). On the remote desktop of a blind person working in Braille technology (BNM editor) or in ASCII technology (Ascii editor), the formulas are visualized spatially thanks to the conversion process presented in Table 1. Such an approach ensures there are no problems in communication between two people using different mathematical languages, whether they cooperate via the Internet or within a group, using a local network. Mathematical communication via the Internet is presented in Fig. 4.

Table 3 presents communication-related operations carried out in PlatMat by the leader and by group members in a local Wi-Fi network and via the Internet.

## 5. Surveys on the Usefulness of New Solutions

Research on the impact and usefulness of the solutions proposed in PlatMat was conducted at three different points during the process of developing the new technology. We shall focus on the most recent surveys conducted in 2017 among math teachers and their students at 3 educational institutions in Warsaw, Kraków and Siedlce (Poland) attended by sighted, low-vision and blind students. 5 mathematics teachers and 11 students took part in the qualitative surveys that were combined with individual interviews. The aim of the survey was to determine the measurable benefits enjoyed by mathematics teachers, blind students and low-vision students, resulting from the use, both in the classroom and at home, of the PlatMat tools with new ICT solutions. The research was carried out after pilot phase under which PlatMat tools were implemented at those facilities over a period of several months. Because not all students had access to laptop computers in the classroom, the pilot-phase classes were conducted primarily as compensatory lectures, one a one-on-one basis or in small groups with 2–3 students. The compensatory activities were also carried out via the Internet. Similarly to the compensatory classes, on-going consultations concerning homework were also offered remotely. The questionnaires were divided into three parts, concerning: the teacher's work, the blind student's work and the low-vision student's work. The fourth part presented the criteria and assessment methodologies used. The questionnaires covered all new solutions implemented in PlatMat, including those supporting the work performed by a group of students diversified in terms of their visual acuity, as described in this paper.

The opinions about formula editors were not focused on any particular editor. Teachers, depending on their preferences and needs, used a structured editor, a UnicodeMath editor or a handwriting editor. Low vision students preferred the UnicodeMath editor. Blind students preferred the BNM Braille editor. No attempt was made to work on the AsciiMath editor, which is a functional option available in software for blind students. Students participating in the survey studied in upper grades of technical schools and high schools, were well skilled and experienced in using Braille. An opinion has been reached that the new technologies replacing Braille must be introduced at the very early stages of education.

The primary measurable benefits offered by the presented solutions, as identified in the survey, are as follows:

- improvement of IT efficiency of younger students, which shortens the time required for performing mathematical operations (mastery of keyboard shortcuts),
- less time spent on mathematical operations by low-vision students,
- increased self-reliance of students, inter alia thanks to the semantic reading of formulas, recognizing the structure of a formula, own records that are clear for their author (low-vision students),
- the number of mistakes made by students was not larger, and it was clearly lower in 'pairing' tasks and tasks with a narrative containing parameters for calculations,
- better ability to communicate, grater level of students' comfort while working and higher effectiveness of the teacher's support, as the student's screen could be monitored by the teacher,
- greater capacity for providing support to a student and for making it more precise,
- time needed by teachers to prepare worksheets, especially with tasks involving fractions, roots, equation systems and special characters, the writing of which is quite troublesome while using Word.

The research – surveys, criteria, measures and results – is described in detail in the Research Report submitted to PFRON [22].

# 6. Conclusions

The paper presents a selected range of new solutions proposed in the PlatMat system. Their aim is to support work in the field of mathematics, performed in a group of participants with diverse visual acuity. The groups for which PlatMat is primarily intended include groups of students at regular or special needs schools, attended by blind and low-vision students. The solutions discussed, aimed at facilitating the work and communication in the field of mathematics include the following: a sequence of conversions, performed on an on-going basis, of various mathematical notations (languages), enabling the use of mathematical notation and formula editing tools preferred by users, as well as on-going cooperation; universal, accessible and interactive mathematical documents in the EPUB3 e-publication standard, enabling the exchange of mathematical content and information; setting up the exchange of mathematical documents and information via a Wi-Fi network and via the Internet, in various environments and between users using different math languages and interfaces. Qualitative research/surveys listing the opinions of mathematics teachers and their students on the usefulness of the presented solutions, were conducted among groups of 2–3 students having personal computers. The ability to create a single version of lesson materials in the form of an epub document – an interactive document for all students, and the ability to monitor students' work via a Wi-Fi network or via the remote desktop function, was very well received by teachers. Among students, the greatest satisfaction was expressed by low-vision students who could now edit formulas using the UnicodeMath editor. Previously, they had been using the MS Word formula editor – a feature that is difficult to operate due to the precision of movements required when entering values into formula template fields. In order to assess the usefulness of the discussed solutions in large groups (meaning, under real-world conditions, that a lesson needs to be conducted in a classroom), the first requirement that has to be met is that all students need computers. Research performed in 2018 as part of the abovementioned EuroMath project and concerned with ICT tools used to support teaching maths, showed that few students use laptops or any other computer equipment (apart from smartphones) in the classroom. It would be worthwhile to organize, for experimental purposes, a class of students equipped with laptops and PlatMat tools for teaching mathematics, to carry out research on the effects of computerization of mathematical education and to disseminate the positive results that are likely to be obtained. This will create an incentive for other schools and will contribute to the promotion of PlatMat technologies.


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2. "OPTY: Studies on the effectiveness of computerization of mathematical education of students with visual impairments using the optimized PlatMat" tools, no. BEA/000027/BF/D, co-financed by PFRON, 2016–2017.


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**Jolanta Brzostek-Pawłowska** Ph.D., graduate of the Faculty of Electronics and Information Technology at the Warsaw University of Technology, assistant professor at NASK – PIB. She has been conducting research and development work on e-learning, assistive technologies and alternative user interfaces for many years. She managed research projects that resulted in the creation of new technologies such as TeleEdu LMS, PlatMat platform supporting mathematical education for visually disabled students. She is currently conducting the EuroMath (EU Program Erasmus+) project adapting the best solutions of the PlatMat to educational conditions in other European countries.

https://orcid.org/0000-0002-9870-0480

E-mail: jolanta.brzostek-pawlowska@nask.pl

Research and Academic Computer Network (NASK)

Kolska 12

01-045 Warsaw, Poland