Analysis of the dynamical performance of the two-mass drive system with the modified state controller

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In this publication a modified state controller applied in the speed control loop of two-mass drive system is shown. A characteristic feature of this type of a drive system is an existence of oscillations of electromagnetic state variables, resulting from the finite stiffness of the connection between driven motor and the load machine. Effective damping of torsional vibrations in such a system is achieved by introducing additional feedback in the control structures of the two-mass drive. In the described control system the reduction of the feedbacks number is achieved by elimination of the one coming from the torsional torque. At the same time, it was assumed that the beneficial dynamic properties of the two-mass drive obtained in a case of the application of all possible additional feedbacks should be kept. Calculation of the controller gains was done using the pole placement method. Application of the pole placement method gives a possibility for setting the parameters of the control structure for given values of damping coefficient and resonance frequency of the close-loop system, so it is possible to influence the dynamic properties of the drive.

1. Introduction

In the recent years requirements for the perfect action in steady and dynamic states of industrial electrical drives are becoming more stringent. The aim of such systems is to minimize the duration time of the transient process, the ideal tracking of the given trajectory of the speed (or position), robustness to parameter change of the controlled systems. The requirements mentioned above lead the engineers to develop new methods and control algorithms of the drives. In addition, they require accurate modeling and working conditions to obtain high precision of control, which is often connected with elimination of the simplifying assumptions.

The mathematical model of the two-mass system presents phenomena occurring in many electrical drives, which can be encountered in practical industrial applications. Examples of drive systems characterized by these properties can be found in:

- mill drives,
- mechanisms of textile and paper machinery,
- conveyors,
- radio telescopes,
- servo drives,

- drives of the robots,
- positioning systems of hard disks.

A characteristic feature of this type of drive is the presence of elastic connections between the motor and the load machines. The existence of such connections in the mechanical part of the drive is the cause of generation of dynamic oscillations of the drive state variables. Such a phenomenon makes it impossible to precisely control the speed or angular position of the shaft in the drive system with elastic joint. In extreme cases, the oscillations may cause loss of stability of the controlled drive system [1].

In order to obtain a damping of torsional vibrations in the drive system with elastic joints following solutions are implemented:

- modification of the mechanical part,
- application of modified setting of the controllers,
- extension of the classical control structures by introduction of additional feedbacks,
- implementation of advanced control techniques.

The first method mentioned above relies on an introduction of additional elements in the mechanical part of the drive (i.e. introduction of mechanical dumpers); such a solution increases the cost of the drive and also its dimensions what is especially nowadays very important in industrial applications [3]. It should also be noted that the realization of such additional components is not easy and requires additional preparation. There is also possibility of modification of the control structure gains, which however cause the reduction of the electrical drive dynamics. Such a solution introduces a significant delays in the control structure, the time of speed setting in reference value is large, while system response to dynamic change of the load torque is slow.

One of the most effective and the simplest in realization methods for damping of torsional vibrations in the drive system with elasticity is introduction of additional feedbacks in the classical control structure (i.e. feedbacks from torsional torque and load speed, and/or their derivatives) [1]. Suppression of state variables oscillations caused by elastic shaft between motor and load machine can be also achieved after implementation of the advanced control structures, but it should be noted, that they are computationally complex algorithms and difficult during development of the project. They are based on:

- sliding mode control [4],
- elements of artificial intelligence: fuzzy logic and neural networks [5],
- adaptive control structures [6],
- predictive controllers [7].

In this work, in order to ensure proper work of the electric drive with elastic connection, which allows high accuracy of the speed control and robustness in presence of parameter changes, a structure based on a state controller is proposed. The values of gain coefficient in this controller are calculated using pole placement method [2]. A characteristic feature of such an analysis of the controller is possibility of the assumed placement of all poles of the closed structure, what allows obtaining the required dynamic properties of the controlled system. In the case of classical methods of selecting controller settings (Kessler criteria), only the dominant poles are taken into account, therefore, there are inconsistencies between reference transients and those achieved in the drive structure. In the literature exist many of examples of state controller application in drives with elastic coupling [8], [9], [10].

However, the beneficial properties of the speed control structure with the state controller are possible to obtain when information about all state variables is available. In a case of the two-mass system the most often used additional feedbacks are taken from thee torsional torque and the load-side speed. In real application full information about state variables is difficult to obtain, it requires extension of the drive and implementation of additional sensors, therefore increases the cost of a whole project. Additionally, in industrial application problems with installation of sensor systems and measurement noises can appear. For solving this problem, the special structures for estimation of the required state variables must be used. This issue is currently dominated by two trends of development:

– implementation of algorithmic model for state variables reconstruction (i.e. the Kalman filter or the Luenberger observer) [10],
– neural estimators [11].

The algorithmic methods, based on the observer theory, allow obtaining very good results. However, in this case, there is a need for knowledge of the mathematical model and parameters identification of the object. On contrary, neural networks do not need any knowledge about the mathematical model and its parameters, but are characterized by high computing power requirements during operation, and time-consuming (algorithms for weight coefficient calculations) and complex design process (in a case of their structure optimization). Therefore, reasonable and very beneficial is elimination of feedbacks number used in the control structure and use minimal number of measured or estimated variables. In the case of control structure with the state controller, described below, a feedback from the torsional torque is eliminated.

Several design stages of modified state controller are presented in this publication. The results of simulation and experimental verification of the described control structures implemented for speed control of the drive system with elastic joint are presented.

2. Mathematical model of the two-mass drive and control structure

Mechanical part of the drive system consists of a motor machine – represented by electromagnetic torque m_e , motor speed ω_1 and mechanical time constant T_1, connected with a load machine – represented by load torque m_L, load-side speed ω_2

and mechanical time constant T_2, using an elastic shaft represented by a torsional torque m_s and a time constant T_c. During the analysis of such a model a simplified mathematical description based on the following equations is used:

$$\frac{d}{dt}\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\\ m_s(t)\end{bmatrix}=\begin{bmatrix}0 & 0 & \frac{-1}{T_1}\\ 0 & 0 & \frac{1}{T_2}\\ \frac{1}{T_c} & \frac{-1}{T_c} & 0\end{bmatrix}\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\\ m_s(t)\end{bmatrix}+\begin{bmatrix}\frac{1}{T_1}\\ 0\\ 0\end{bmatrix}[m_e]+\begin{bmatrix}0\\ \frac{-1}{T_2}\\ 0\end{bmatrix}[m_L] \tag{1}$$

Though the power converter which supplies the motor as well as a control system are very fast, it is possible to take into account the delays of the electromagnetic torque control loop in the design process. The influence of the time constant of electromagnetic part of the drive on transients of state variables of drive is presented in [12]. This analysis shows that delays in electromagnetic control loop in the control structure described in this paper can be neglected. So in the further analysis and in the block diagram of the drive structure presented in Figure 1, an ideal electromagnetic torque control loop was assumed (there is no additional delays), which may be represented by the transfer function:

$$G(s)=1 \tag{2}$$

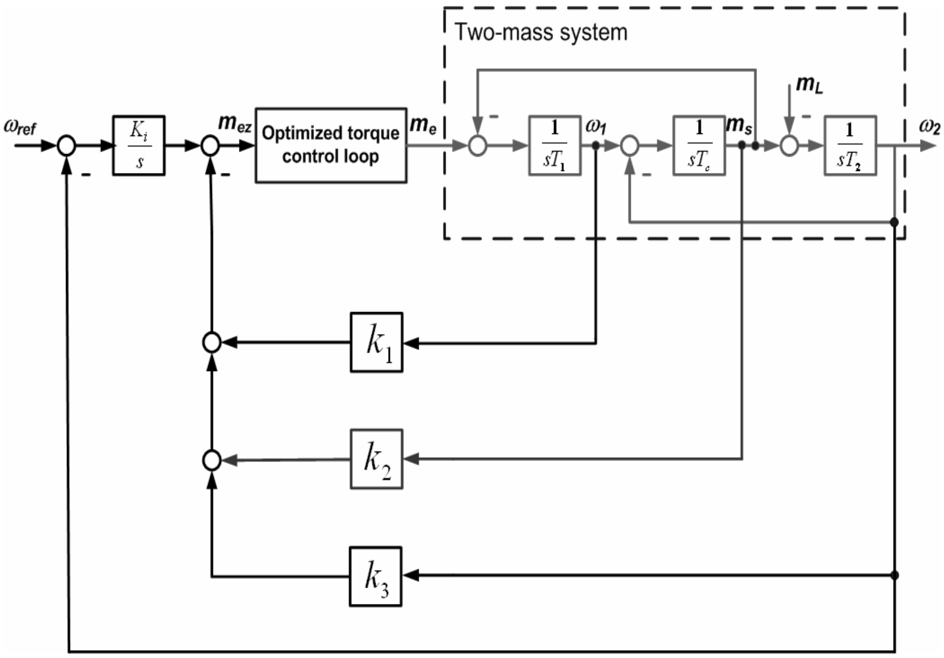

Fig. 1. The speed control structure of the two-mass drive system based on the state controller

In order to determine the transfer function of the closed structure presented in Fig. 1 the following equations were used:

$$T_1 \frac{d\omega_1}{dt} = m_e - m_s \tag{3}$$

$$T_2 \frac{d\omega_2}{dt} = m_s - m_L \tag{4}$$

$$T_c \frac{dm_s}{dt} = \omega_1 - \omega_2 \tag{5}$$

$$m_e = K_i \int (\omega_r - \omega_2) dt - k_1\omega_1 - k_2 m_s - k_3\omega_2 \tag{6}$$

Introducing the Laplace operator we obtain:

$$T_1 s\omega_1 = m_e - m_s \tag{7}$$

$$T_2 s\omega_2 = m_s - m_L \tag{8}$$

$$T_c s m_s = \omega_1 - \omega_2 \tag{9}$$

$$m_e = R(\omega_r - \omega_2) - k_1\omega_1 - k_2 m_s - k_3\omega_2 \tag{10}$$

where:

$$R = \frac{K_i}{s} \tag{11}$$

After some calculations the following equation is obtained:

$$\begin{aligned} &\omega_2\left(T_1T_2T_cs^3 + T_1s + T_2s + R + k_1T_cT_2s^2 + k_1 + k_2T_2s + k_3\right) = \\ &R\omega_r - k_1T_csm_L - k_2m_L - m_L - T_1T_cs^2m_L \end{aligned} \tag{12}$$

which enables to determine the transfer function of the closed control system:

$$\frac{\omega_2}{\omega_r} = \frac{R}{T_1T_2T_cs^3 + T_1s + T_2s + R + k_1T_cT_2s^2 + k_1 + k_2T_2s + k_3} \tag{13}$$

Taking into account expression (11) we have:

$$\frac{\omega_2}{\omega_r} = \frac{K_i}{s^4T_1T_2T_c + s^3k_1T_cT_2 + s^2(T_1 + T_2 + k_2T_2) + s(k_1 + k_3) + K_i} \tag{14}$$

The characteristic equation of the closed system has the following form:

$$H(s) = s^4 + s^3\frac{k_1}{T_1} + s^2\left(\frac{1}{T_2T_c} + \frac{1}{T_1T_c} + \frac{k_2}{T_1T_c}\right) + s\left(\frac{k_1}{T_1T_2T_c} + \frac{k_3}{T_1T_2T_c}\right) + \frac{K_i}{T_1T_2T_c} \tag{15}$$

In order to calculate the equations defining gains of the state controller, the characteristic equation of the system (15) have to be compared to the reference polynomial of the same order. For the considerations in this project the following polynomial was taken into account:

$$H_{ref}(s) = \left(s^2 + 2\xi_r\omega_o s + \omega_o^2\right)\left(s^2 + 2\xi_r\omega_o s + \omega_o^2\right) = \\ s^4 + s^3\left(4\xi_r\omega_o\right) + s^2\left(2\omega_o^2 + 4\xi_r^2\omega_o^2\right) + s\left(4\xi_r\omega_o^3\right) + \omega_o^4 \tag{16}$$

where: ξ_r, ω_o – are required dumping factor and resonance frequency of the closed-loop system.

Comparing the elements with the same power of the Laplace operator, equations presented below can be obtained:

$$4\xi_r\omega_o = \frac{k_1}{T_1} \tag{17}$$

$$2\omega_o^2 + 4\xi_r^2\omega_o^2 = \frac{1}{T_2T_c} + \frac{1}{T_1T_c} + \frac{k_2}{T_1T_c} \tag{18}$$

$$4\xi_r\omega_o^3 = \frac{k_1}{T_1T_2T_c} + \frac{k_3}{T_1T_2T_c} \tag{19}$$

$$\omega_o^4 = \frac{K_i}{T_1T_2T_c} \tag{20}$$

and finally after a few calculations following expressions describing the gain coefficients of the state controller are obtained:

$$k_1 = 4\xi_r\omega_o T_1 \tag{21}$$

$$k_2 = T_1T_c\left(2\omega_o^2 + 4\xi_r^2\omega_o^2 - \frac{1}{T_2T_c} - \frac{1}{T_1T_c}\right) \tag{22}$$

$$k_3 = T_1T_2T_c 4\xi_r\omega_o^3 - k_1 \tag{23}$$

$$K_i = \omega_o^4 T_1T_2T_c \tag{24}$$

In order to reduce the number of additional feedback in the structure, attempt of elimination of torsional torque feedback was made. Therefore in equations (21) - (24), the following assumption was introduced:

$$k_2 = T_1T_c\left(2\omega_o^2 + 4\xi_r^2\omega_o^2 - \frac{1}{T_2T_c} - \frac{1}{T_1T_c}\right) = 0 \tag{25}$$

Based on (25) the formula specifying the damping coefficient ξ_r of the system with assumed resonance frequency was calculated:

$$\xi_r = \sqrt{\frac{T_1 + T_2 - 2T_1T_2T_c\omega_o^2}{4T_1T_2T_c\omega_o^2}} \tag{26}$$

Obtained value is taken into account during calculation of the remaining gains of the state controller (k_1, k_3 and K_i, according to (21)-(24)). For maintaining the correctness of calculations and in order to obtain real value of the damping factor, the limit value of resonance frequency ω_o was introduced:

$$\omega_o < \sqrt{\frac{T_1 + T_2}{2T_1 T_2 T_c}} \tag{27}$$

According to the presented approach, the dynamics of the drive system with elasticity is modeled using assumed value of parameter ω_o. It should be noticed that the impact of this parameter is increased, in contrary to classical state controller, because at the same time increasing the resonance frequency results in decreasing the value of the damping coefficient.

3. Simulation results

For the drive system with elastic joint, working in the control structure with the state controller based on the parameters calculated in accordance with the equations presented in previous section, simulation tests in Matlab/Simulink were prepared. Following parameters of the two-mass system were assumed: T_1=T_2=203ms and T_c=2.6ms. For the considered parameters of the drive system the limitation of the resonance frequency, calculated according to the formula (27) takes value: ω_o<43.5277.

In the following figures results of simulation tests are presented. All tests were done for reference speed equal 25% of the nominal speed, to demonstrate the behavior of the drive in the linear operation region, when the limitation of the electromagnetic torque is not active. After stabilization of the velocity at a reference level, at time t_1=0.5s and t_2=1.5s, the load torque was switched off. The control structure has been studied for different values of the resonance frequency. The results of simulation are presented in Fig. 2.

Obtained transients demonstrate the ability to influence on the system dynamics in a wide range. Increasing of the resonance frequency leads to a smaller values of the damping factor (according to the formula (26)); in result for pulsation ω_o=40s^{-1} oscillation in transients of electromagnetic torque and both system speeds are observed (Fig. 2a,b). It is important that theses oscillations are diminishing, but they prolong the setting time of state variables at a reference level. For pulsation value ω_o=30s^{-1} oscillations are well damped and the setting time of the system speed is short (Fig. 2e,f). For greater values of ω_o, in the transient states of the system, bigger values of electromagnetic torque are observed, which forces the highest dynamics (Fig. 2c,d).

In the Fig. 3 analysis of the pole placement of the described control structure is presented. Changes of the design parameters (ω_o and ξ_r) cause specific changes of the pole location on the complex plane. It should be noted that, in the control structure with state controller, where one feedback from the torsional torque is eliminated, increasing of the pulsation ω_o moves the poles of the system in the direction of the imaginary axis. This situation is in contrary with the case when this feedback exists, and the damping factor ξ_r is constant and chosen independently.

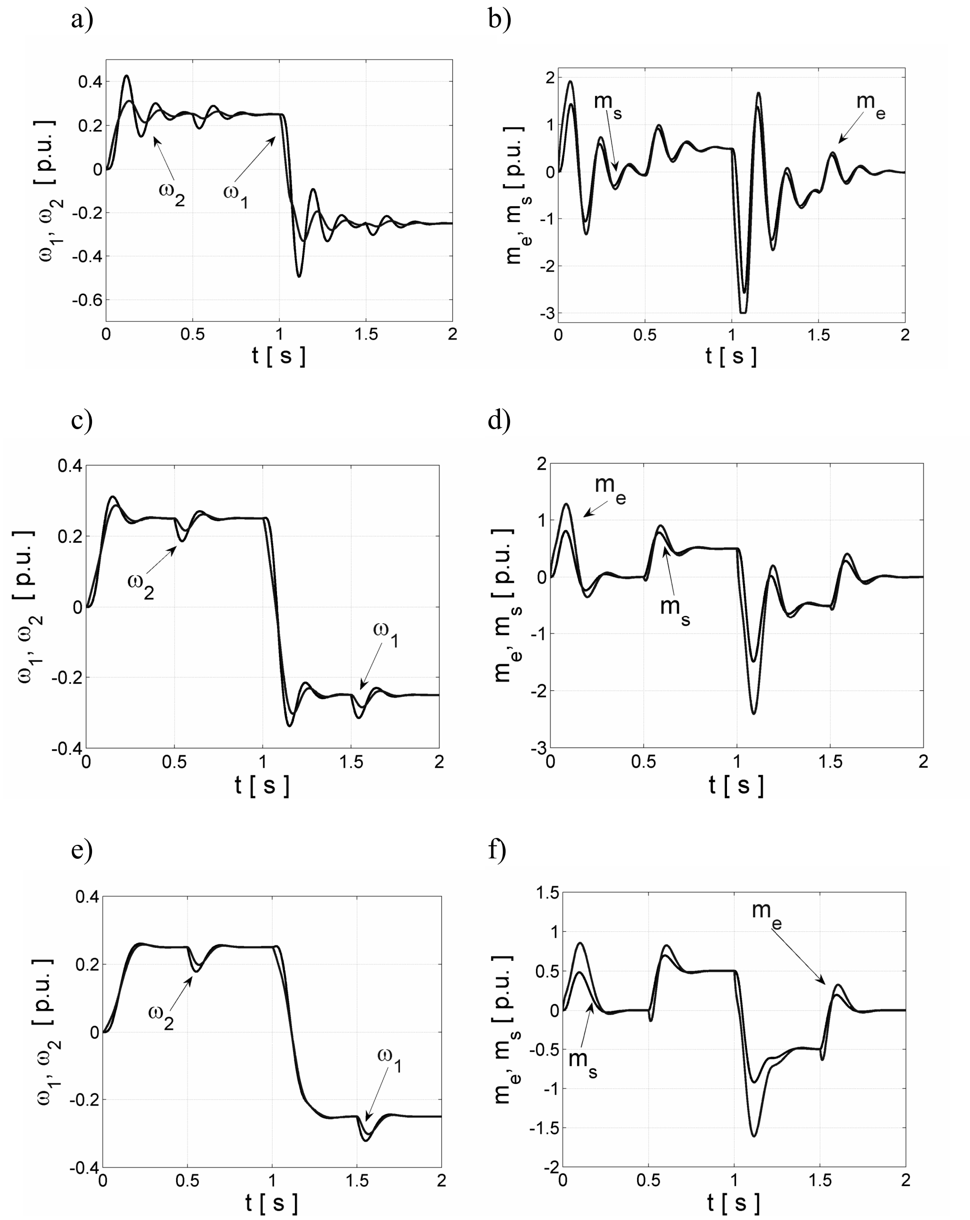

Fig. 2. Transients of motor and load speeds (a,c,e) and torques: electromagnetic and shaft (b,d,f) for two-mass driver system with modified state space controller, for different values of resonant frequency and damping factor $\omega_o = 40\mathrm{s}^{-1}$, $\xi_r = 0.30$ (a,b), $\omega_o = 35\mathrm{s}^{-1}$, $\xi_r = 0.52$ (c,d) and $\omega_o = 30\mathrm{s}^{-1}$, $\xi_r = 0.74$ (e,f) – simulation results

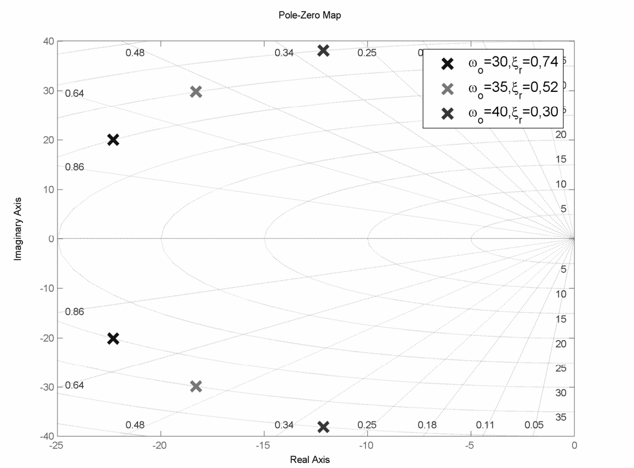

Fig. 3. Location of poles for two-mass driver system with modified state space controller

A comparison of transients obtained for classical state controller (with feedback form shaft torque) and modified state controller (with reduced number of feedbacks) are presented in Fig. 4. Tests were prepared for the same values of damping coefficient and resonance frequency: ω_o = 30s^{-1} and ξ_r = 0.74. It is possible to observe that elimination of the analyzed feedback does not deteriorate the dynamic properties of the control structure for tested values of settings.

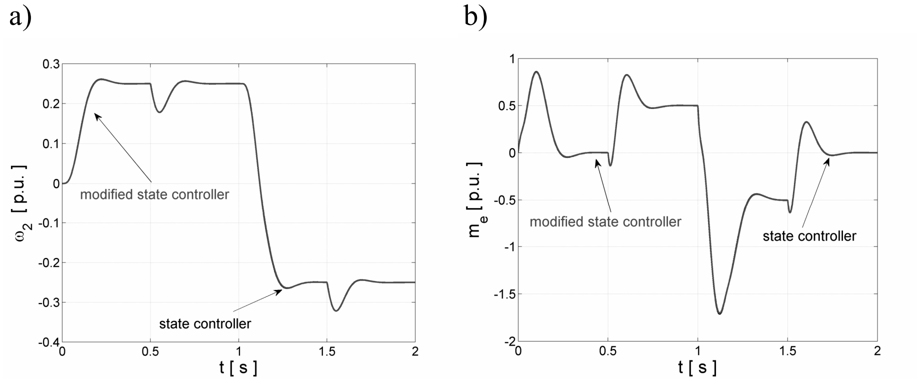

Fig. 4. Comparative transients of the load speed (a) and electromagnetic torque (b) in the control structure with classical and modified state controller – simulation results for ω_o=30s^{-1} and ξ_r=0.74

All the above results were obtained for the two-mass system, where the mechanical time constants of the motor and load machine have the same value. In the last stage of simulation studies the tests of the designed control structure are made for the case of twice bigger value of the mechanical time constant ($T_2=2T_{2N}$).

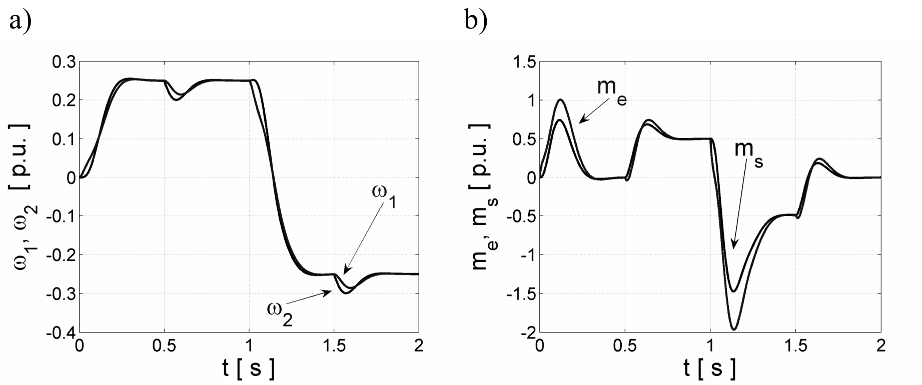

Fig. 5. Simulation results for the drive with state controller for $T_2=2T_{2N}$

Results of simulation are presented in Fig. 5 (for the modified state controller without feedback from shaft torque, where ω_o=25s^{-1} and ξ_r=0.80). This change of T_2 parameter was not taken into account during the calculations of the gains in the tested controller. For changed value of T_2 the proposed control structure also enables the effective vibration suppression in the two-mass system. For the same value of ω_o, bigger values of electromagnetic torque (and also motor current) is observed during system transients. Due to increase of the moment of inertia of the load machine, the settling time is little longer. This test presents possibility of the modified state controller and the proposed design procedure also in case of parameter changes of the drive system.

4. Experimental results

For verification of the presented control structure in the real drive the laboratory set-up shown in Fig. 6 was used. The laboratory set-up is composed of two machine motors, connected with an elastic shaft. The stiffness of connection depends on the shaft diameter. The speeds of machine and load motor are measured by incremental encoders (36000 pulses per rotation). On the laboratory set-up the LEM sensors for current measurement are implemented. Measurements data and control signals are connected with digital and analog inputs/outputs of the dSPACE 1102 card. The motor is driven by a transistor power converter.

There is no torque shaft sensor in the laboratory set-up. In order to observe the shaft torque transients in the analyzed drive, an estimator based on one of two-mass system equations was used and applied in DSP:

$$m_s = m_e - T_1 \frac{d\omega_1}{dt} \tag{28}$$

The idea of such estimator was presented in the application with RRC (*Resonance Ratio Control*) control structure [13], [14]. The model used in the studies is slightly modified in comparison with the original prototype. In the equation (28) a derivative of the motor machine speed exists, so the measurement noises occurred in this signal can be intensified. For reduction of this phenomenon additional low-pass filters are used in both paths of processing, according to the diagram presented in Fig. 7.

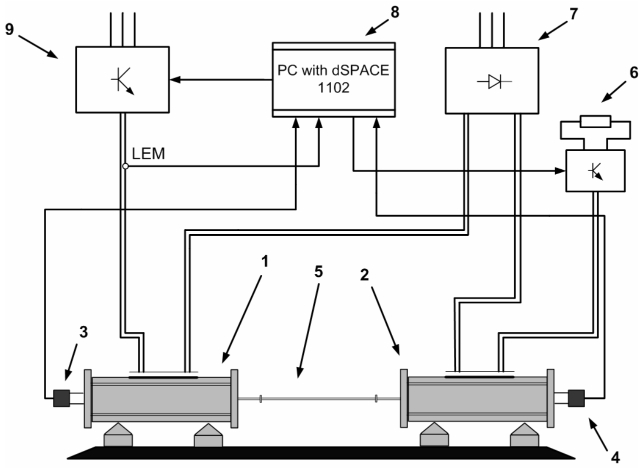

Fig. 6. Schematic diagram of the experimental set-up, where: 1 - motor machine, 2 - load machine, 3,4 - encoders, 5 - shaft, 6 -resistor, 7 - rectifier, 8 control structure, 9 - power converter

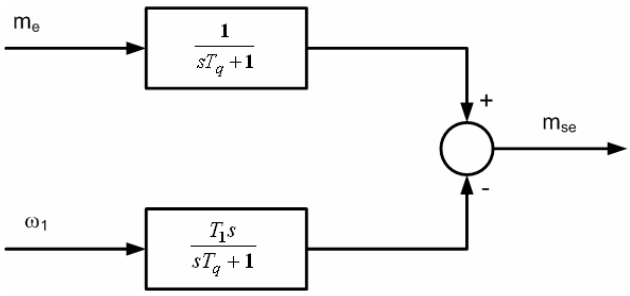

Fig. 7. Schematic diagram of the torque shaft estimator

Output signal of the estimator is described with following equation:

$$m_{se} = \frac{m_e}{sT_q + 1} - \frac{sT_1\omega_1}{sT_q + 1} \tag{29}$$

In the structure presented on Fig. 7 information about time constant of the motor machine T_1 and also measurement of electromagnetic torque m_e and speed ω_1 is needed. Moreover, the problem with value of time constant T_q of the filter is appearing. But the advantage of the applied torsional torque estimator is its simplicity, which leads to small calculation power requirements.

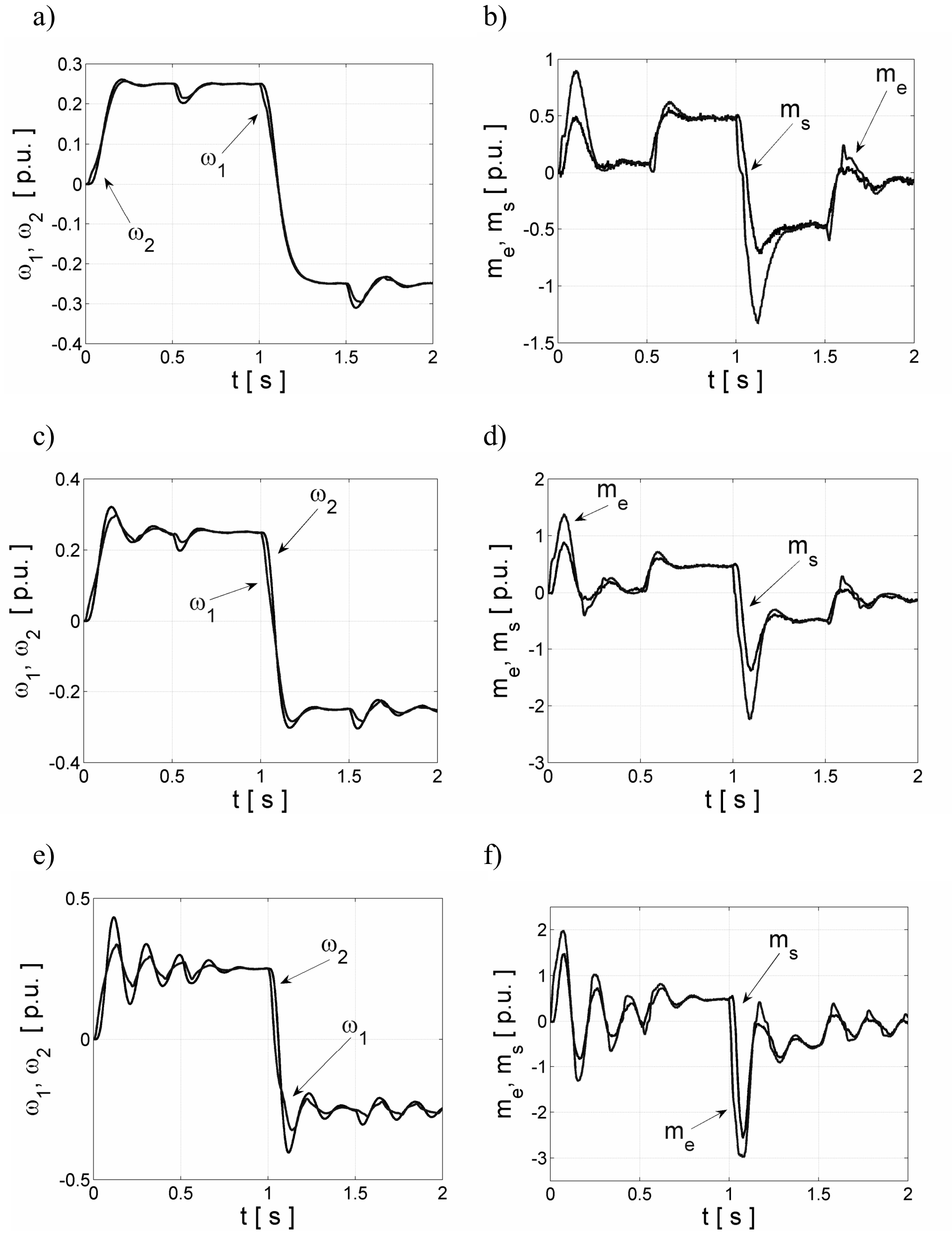

Fig. 8. Transients of motor and load speeds (a,c,e) and torques: electromagnetic and shaft (b,d,f) for two-mass driver system with modified state space controller, for different values of resonant frequency and damping factor ω_o=30s^{-1}, ξ_r=0,74 (a,b), ω_o=35s^{-1}, ξ_r=0,52 (c,d) and ω_o=40s^{-1}, ξ_r=0,30 (e,f) – experimental results

As in the previous case correct work of the speed control structure with modified state controller presented in simulation were confirmed in experimental tests (Fig. 8). All researches were prepared assuming the parameters and working conditions similar as in the simulation tests. After calculations of the controller gains the large values of K_i (especially for bigger values of resonance frequency), strengthening the value of the error in the system, are obtained (according to (24)). It is advantageous because it increases the value of the output signal of the controller and therefore contributes positively to the control quality. However, in real implementation the negligible noises in the control structure can also be strengthened and interfered with the system, which feature additionally limits the maximal value of the pulsation ω_o.

Changes of the resonant frequency and damping factor give effective changes of the dynamics in the controlled structure. Forcing a very high dynamics of the drive (large value of ω_o and small ξ_r) leads to oscillations of state variables (Fig. 8e,f). After setting correct values of ω_o and small ξ_r it is possible to achieve a short time of speed response of the drive system, without readjustment and state variables oscillation (Fig. 8a,b).

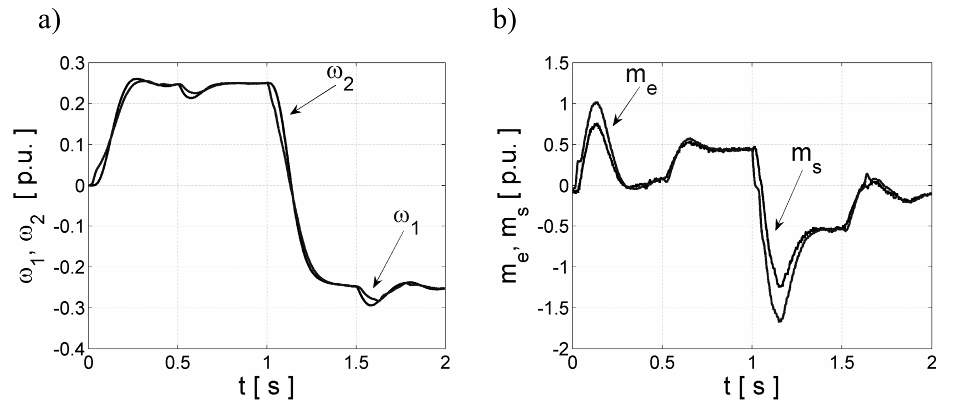

Fig. 9. Experimental results for the drive with state controller for $T_2=2T_{2N}$

Also in the case of experimental tests correctness of the control structure operation for the two-mass system with doubled time constant of the load machine is presented in Fig. 9. Obtained results are similar to results from simulation tests (see Fig. 5).

5. Conclusion

The design of speed control structure with a modified state controller for the two-mass drive system simplifies the realization of such a structure in the real solution and decreases financial cost of electrical drive. The additional feedback from shaft torque was eliminated, so there is no necessity of measurement or estimation of this state variable. At the same time, proposed control structure gives similar properties as a classical state controller. Comparative tests present possibility of obtaining good dynamics and damping of torsional vibrations similarly as in a case of full state controller. All presented tests were made for the natural resonance frequency of the two-mass drive equal 9.8kHz, which was caused by possibility of experimental verification of obtained results. The simulation results are confirmed by laboratory experiments.

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