Technical Sciences, 2016, 19(2) 129–138

UNIWERSYTET WARMIŃSKO-MAZURSKI W OLSZTYNIE

Quarterly peer-reviewed scientific journal
ISSN 1505-4675
e-ISSN 2083-4527
TECHNICAL SCIENCES
Homepage: www.uwm.edu.pl/techsci/

MATHEMATICAL MODELS FOR SPECIALIZED AND SENSORY NETWORKS OF WIRELESS ACCESS

***Oleksandr Tymchenko*[1], *Michał Zelyanovsky*[2], *Karolina Szturo*[1,3], *Oleksandr O. Tymchenko*[2]**

[1] Department of Safety Engineering
University of Warmia and Mazury
[2] Ukrainian Academy of Printing, Lviv
[3] Lodz University of Technology

Received 17 January 2015, accepted 24 May 2016, available online 25 May 2016.

Key words: sensor networks, mathematical model, topology control.

Abstract

This article reviews and compares the special features of specialized wireless and sensor networks. The components of a mathematical model of existing specialized wireless and sensor network are also available for review, particularly: wireless channel models, signal propagation models and communication graph models, etc. The need for a topology control mechanism in wireless and sensor networks is also explained.

Introduction

Specialized wireless networks include networks in which there is no fixed infrastructure and whose network topology may change with time. A wireless sensor network is a particular case of a specialized network that all the devices in such a network are homogeneous. Multilink transmission is used in both types of networks (HAENGGI et al. 2009, JERUCHIM et al. 2000, JOHNSON, MALTZ 1996, RAPPAPORT 1996). The following table summarizes the features of two related types of networks.

Correspondence: Oleksandr Tymchenko, Katedra Inżynierii Bezpieczeństwa, Uniwersytet Warmińsko-Mazurski, ul. Oczapowskiego 11, 10-719 Olsztyn, phone: 89 524 61 25.

Table 1

Features of specialized wireless networks and wireless sensor networks

Specialized wireless networks	Wireless sensor networks
Heterogeneous devices	homogeneous devices
Mobile nodes	stationary (quasi-stationary nodes)
Multilink transmission is optional	multilink transmission is needed in most cases
Geographically located network	

Problems

One of the most difficult aspect in determining the model of wireless network is that, yet complete enough. The simplicity will allow simulating and displaying the theoretical results and completeness should be provided so that such a model could be applied in practice. Let us consider the components of mathematical models for the specialized and sensor wireless networks (RAPPAPORT 1996).

The wireless channel

We introduce the following notation:

– u,v – a pair of wireless nodes,
– P_r and P_t – power of received and transmitted signals, respectively,
– β – sensitivity threshold,
– PL(u, v) – transmission losses (losses on tract).

A direct wireless connection exists if, and only if, $P_r \geq \beta$. The value of β depends on many factors, including the parameters of the transceiver and the data rate. The higher the data rate is, the higher β is:

$$P_r = \frac{P_t}{PL(u, v)} > \beta.$$

The presence of a wireless channel between u and v can be foreseen, if PL(u, v) are known. A modeling of losses on a tract is the most difficult task. The mechanisms of regulation of a signal's propagation can be divided into three categories: reflection, diffraction and dispersion.

Distribution of signal and losses on a tract

Losses on a tract are modeled in accordance with distribution of signal. There are a few models of distribution of signal, thus for every partial case it is possible to create a unique model.

Let's denote:

- G_t, G_r – amplification factors of the transmitting and receiving antennas, respectively,
- λ – wavelength,
- L – losses are on a tract.

1. Direct visibility. This model is the simplest and is used when there are no obstacles between a transmitter and receiver. Consequently:

$$P_r(d) = \frac{P_t \times G_t \times G_r \times \lambda^2}{(4\pi)^2 \times d^2 \times L},$$

using symbol

$$\frac{G_t \times G_r \times \lambda^2}{(4\pi)^2 \times L} = C_f,$$

we have:

$$P_r(d) = C_f \times \frac{P_t}{d^2}.$$

From the last equality, we see that the coverage area in the model of distribution on direct visibility is a circle of radius $d = \sqrt{C_f \times P_t}$.

2. Dual-beam model

Accept h_t, h_r heights of receiver and transmitter antennas from the ground (Fig. 1).

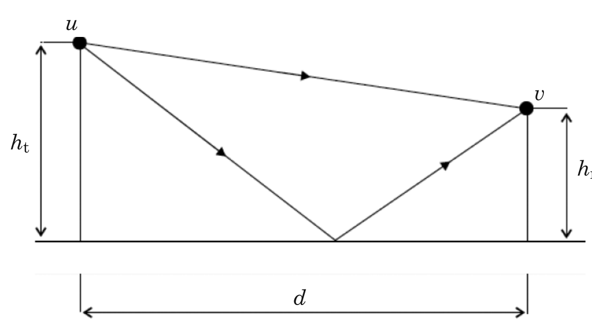

Fig. 1. In the dual-beam model, the signal spreads in two ways – the direct and the reflected beam from the ground

Then, assuming that horizontal distance $d = \sqrt{h_t \times h_r}$, have: $P_r(d) = C_f \times \frac{P_t}{d^4}$.

3. A model with logarithmic dependence on distance (a model for a heterogeneous or anisotropic environment).

Let's consider a model for a heterogeneous environment. It can be seen from above that the radius of coverage area $r \propto \sqrt[\alpha]{P_t}$. The value of α is defined experimentally for the different type environments, some partial cases are shown in the following table.

Table 2

Value of h defined for the different type of environments

Environment	Value of α
Open space	2
City	2.7–3.5
Inside the building, direct visibility	1.6–1.8
Inside the building, direct visibility is missing	4–6

Source: SANTI (2012).

Such a model assumes only the average value of the accepted power which may significantly differ from the peak values. Therefore, to predict the variability of the wireless channel, so-called probabilistic models of distribution are used. They are divided into 2 classes: large-scale (large range) and small-scale (small range). The latter are also called models of the multi-beam signal fading or simply fading.

Important among the large-scale models is the model with the logarithmic normal shading, where losses on a tract are modeled by changing with a random value which has logarithmic normal distribution in circumscription $\frac{P_t}{d^\alpha}$. The most important model of a signal's fading is Rayleigh's model (TYMCHENKO, ZELYANOVSKIY 2008).

There are also models for ultra-broadband connection, narrowband connection, and unlicensed frequency bands for industry, science and medicine (ISM, Industrial, Scientific and Medical band), where for losses on tracts the models of control points are used: $L(d) = \begin{cases} 40.2 + 20 \times \log(d) & d \le 8m \\ 58.3 + 33 \times \log(d/8) & d > 8m \end{cases}$

For networks which are intended for the use inside of a building, there is a model of losses on a tract considering the walls and ceilings, for networks which work in open terrain and also in forest, the investigated losses effect is in a letter.

Communication graph

A communication graph determines the topology of the network, namely, the set of connections, which nodes could be used for communication.

Let us introduce the denotation in the topology graph:
– N – the set of wireless nodes,
– d – number of considered dimensions, in our case d = 1, 2, 3,
– l – side of square region,
– R – connected region in which the nodes are placed, $R = [0, l^d]$,
– $L(u)$ – reflection u in R, given in d-measurable coordinates,
– T – time interval of realization (a set of modeling moments),
– t -time flow.

Function L: $N \rightarrow R$ connects each node with its location inside R. In the case of mobility of wireless nodes, we have the function L: $N \times T \rightarrow R$ defines the plurality of d-measurable coordinates, giving the location of $\forall\ u \in R$ and $\forall\ u \in T$. Thus, the d-measurable wireless specialized mobile network is represented by a pair Md = (N, L).

In the function of determination of coverage area for M_d there is a certain function RA, that indicates for every element u in N a value of transmission range $RA(u) \in (0,r_{\max})$, $r_{\max}$ – the maximal distance of the node's transmission u. Determination of part R, whose information can be correctly accepted is possible on the basis of information about $r \in (0,r_{\max})$ and the amount of measurement of network d. For a one-dimensional network, it is a segment with a length centered at u, for a two-dimensional network it is a circle of radius r with center u and in the case of three-dimensional networks (e.g. a network of underwater sensors located at different depths) – the ball of radius r centered at u (Fig. 2).

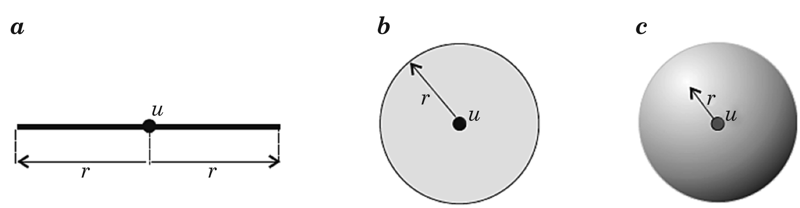

Fig. 2. Measurements for a wireless network in which a modeling is conducted: a – it is a one-dimensional area, b – two-dimensional, c – three-dimensional areas accordingly

Having a network $M_d = (N, L)$ and function RA, we introduce the notion of communication graph, displayed with the function RA on M_d at the moment of time t. It is an oriented graph $G_t = (N,E(t))$, where a directed rib $[u, v]$, that belongs to the plural of ribs $E(t)$, exists then and only after

$RA(u) \geq \delta(L(u,t),L(v,t))$, $\delta(L(u,t),L(v,t))$ – Euclidean distance between u and v at time t. In other words, a connection between u and v is possible when, and only after, the distance between nodes at most RA(u) at the moment of time t. In the case of the existence of rib $[u, v]$, v is the neighbor of the first link for u. Wireless connection is considered symmetric, if $(u,v) \in E_t$ and $(v,u) \in E_t$. In this case u, v are symmetric neighbors.

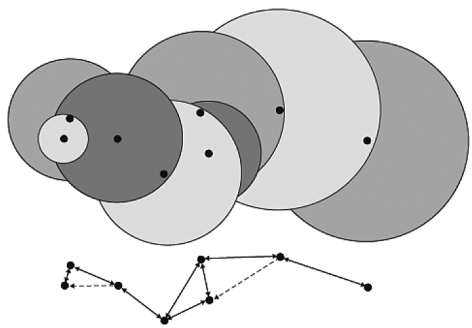

Fig. 3. Example of a wireless network communication graph

Determination of the coverage area at maximal measurement power, such that RA(u) = $r_{\max}$ for $\forall u \in N$ means that every node transfers at maximal power. We call the resulting communication graph, the graph of maximal power. Such a graph presents all the possible connections between nodes in a network.

Determination of the coverage area depends on time t, if a communication graph is coherent in time t, i.e. if there is at least one oriented route between any two nodes.

Determination of the coverage area at which all the nodes have the same radius of transmission r, $r \in (0,r_{\max})$ called r-homogeneous or simply homogeneous, if a value *of* r is not important. It should be noted that the communication graph, generated by homogeneous determination of coverage area for every node, is non-oriental, as $(u,v) \in E_t \Leftrightarrow (v,u) \in E_t$.

Energy consumption is one of the most important metrics in wireless sensor networks. Let α – a degree of losses on a tract. Having some determination of coverage area RA for a network $M_d = (N, L)$, we can say that expenses of energy c on providing RA determined as:

$$c(RA) = \sum_{u \in N} RA(u)^{\alpha}$$

Models of mobility of the specialized wireless networks

Most models of mobility provide movement without obstacles, as it simplifies modeling considerably.

1. RWP – Random Waypoint model – a node randomly selects some point and moves to it on a line. When a mobile node arrives at the destination point, it makes a pause, then elects the coordinates of a new destination point and continues movement (VERDONE et al. 2008).

2. RDM – Random direction model – a node randomly selects a direction and rate of movement. Once the node reaches the boundary of spreading of R, there are a few variants of subsequent actions (ZELYANOVSKIY, TYMCHENKO 2008):

– to choose a new direction and speed and continue movement to the bound R;

– to „reflect" from the bound R and continue movement;

– modification: a node moves in select direction some casual time t, and then changes direction and speed of movement.

3. Model with Brownian movement – in such a variant, movement in steps is used which are chosen randomly, thus the parameters of every next step depends on the previous. An example of a model of Brownian motion is a model using three parameters:

– P_{stat} _ the probability that movement is absent all of the time,

– P_{move} – the probability that a node will move during a current interval of time,

– m – model speed of a node at the present moment of time.

If a node moves during some interval of time i, then its position at the moment of time $i+1$ is determined randomly in a square with a side $2 \times m$ centered at the current node position.

4. Model considering geography – in this case nodes move along predefined ways. Previous nodes are placed in such ways and begin to move on a given scheme. At the crossroads (if such are present) nodes randomly choose the direction and speed of further movement.

5. A model with mobility of groups. From all set of nodes N, a certain subset of leaders is elected $N_l \subset N$, thus $|N_l| \subset |N|$. All other nodes randomly choose a leader and move after him. The leader may use one of the above models.

Topology control of wireless networks

Topology control aims to control the set of connections between pairs of nodes in the network to facilitate and enable the general one-or two-way messaging between all nodes in the network (SANTI 2006).

The theory of topology control is closely connected with the theory of the connectivity of a communication graph network. The primary goal of topology control in the protocol stack is the search for optimal coverage area for each node in the network to provide the required properties of all networks (energy consumption, total coverage area, the lifetime of the network). Optimality may be, for example, the full connectivity of nodes with minimal radiated power of nodes. If all nodes have the same parameters of radiation power (in the case of homogeneous networks – sensor networks), then we can talk, in particular, about the critical radius of coverage area (CTR).

Critical coverage area

Critical coverage area *rc* for providing of compendency of communication graph, is the minimum value of radius of coverage area r, $r \in (0, r_{\max})$ such that at r-homogeneous determination of the coverage area, a communication graph is coherent. P. Santi in JERUCHIM et al. (2000) proved that *rc* in some network $M_d = (N, L)$ corresponds to the longest rib of Euclidean carcass of communication graph G (EMST – Euclidean Minimum Spanning Tree). The difficulty is that for the distributed system (which is a specialized or sensory wireless network) transmission of global (from the point of view of the whole network) knowledge about the coordinates of every node to each of the nodes is surplus and requires plenty of messages, that, in the same queue, leads to losses of power. The latter affects such parameters as the lifetime of the network.

Topology control consists of determining and deleting nodes of the network from a communication graph which are ineffective in terms of energy consumption.

In Figure 4 the two-dimensional communication graphs of network (d=2) are represented. On the left all possible values of r_c are shown. On the right we can see the real value *of* r_c, which provides the compendency of graph.

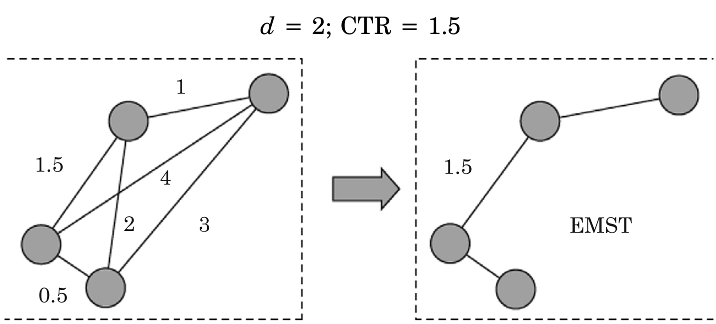

Fig. 4. Communications graph and critical coverage area

In JERUCHIM et al. (2000) it is shown that, in terms of energy consumption, it is appropriate to use multilink messaging over short distances rather than trying to convey a message without the relaying. Multilink transmission of information can also increase the network capacity, i.e. the number of simultaneously working devices in it as message transmissions over long distances creates interference for nodes that are located in a coverage zone, which leads to the retransmission of messages or even makes the communication impossible in the area of such difficult conditions.

Conclusions

Recently, scientific resources have focused on modeling signal transmissions in cellular systems or broadcast communications (JOHNSON, MALTZ 1996). But the models for cellular communication cannot be applied in the case of sensor networks because, first of all, it is typical of the placement of a pair of nodes (base station, mast antenna) at a high distance from the ground, while in the case of the sensory network, all nodes can be located on the ground (for example, right in the grass) or be attached to the walls of the building. Therefore, the direction of modeling of specialized and sensor wireless networks, as well as personal range networks is actively developing and only beginning to emerge a complete model for networks of this class. The considered components are rapidly improving, and there are new specifications to them. The correct creation of the model and its verification ensure successful operation of the network, reducing the time expenses of modeling and debugging and allows the efficiency of the existing network to be improved.

References

HAENGGI M., ANDREWS J.G., BACCELLI F., DOUSSE O., FRANCESCHETTI M. 2009. *Stochastic geometry and random graphs for the analysis and design of wireless networks*. IEEE Journal on Selected Areas in Communications, 27(7): 1029–1046.

JERUCHIM M.C., BALABAN P., SHANMUGAN K.S. 2000. *Simulation of Communication Systems*. Second Edition. Kluwer Academic/Plenum, New York.

JOHNSON D.B., MALTZ D.A. 1996. *Dynamic source routing in ad hoc wireless networks*. In: *Mobile Computing*. Eds. T. Imielinski, H. Korth. Chapter 5, pp. 153–181. Kluwer Academic Publishers, Invited paper.

RAPPAPORT T.S. 1996. *Wireless Communications*. 1st edn. Upper Saddle River, Prentice Hall, New York.

SANTI P. 2006. *Topology control in wireless ad hoc and sensor networks*. ACM Computing Surveys (CSUR), 37(2): 164–194.

SANTI P. 2012. *Mobility models for next generation wireless networks: ad hoc, vehicular and mesh networks*. John Wiley & Sons Ltd., New York, p. 23.

Tymchenko O., Zelyanovskiy Yu. 2008. *Methods and protocols of data exchange of sensory networks of*. The Collection of Scientific Works, IPME NAS of Ukraine, 46: 176–183.

Verdone R., Dardari D., Mazzini G., Conti A. 2008. *Wireless Sensor and Actuator Networks: Technologies, Analysis and Design*. Elsevier, London.

Zelyanovskiy Yu., Tymchenko O. 2008. *Intellectual system for wireless specialized sensory and networks of the personal radius of action: software and hardware platform of node of wireless network of*. Design and information technologies. The Collection of Scientific Works, IPM NAS of Ukraine, 49: 185–193.