The Impact of the Load Characteristics on the Assessment and Optimization of the Shunt Compensation of Distribution Feeders

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***Summary*: The paper gives the results of applying an alternative procedure for the economical assessment and optimization of the shunt compensation of radial power distribution feeders with concentrated and/or distributed loading, taking into account the feeder copper losses as well as the installation cost of capacitors. In addition to the energy and capacitor specific costs, the objective function depends on the size and location of the capacitor, the feeder resistance, the voltage level, and the load current. The results of a detailed parameter study on the impact of the load parameters are presented. The study focuses primarily on the impact of the load factor, the load power factor as well as the feeder's loading profile.**

Key words:
Radial Feeders,
Shunt Compensation,
Losses,
Cost Minimization,
Capacitor Allocation,
Load Characteristics

1. INTRODUCTION

The loss reduction achieved through the shunt compensation of feeders depends primarily on the sizes and locations of the shunt capacitors [1–9]. Its effectiveness depends on the specific costs of energy and that of the compensating capacitors [2, 5]. Expressions are derived in [6] for the capacitors' optimal sizes and locations for feeders with uniformly distributed and/or lumped load. Reference [7] presents the corresponding expressions for feeders having increasing or decreasing distributed load densities or lateral lengths. Shunt compensation should however be assessed by comparing the reduction in the annual demand and energy charges due to the smaller amount of copper losses, with the annual cost of the required shunt capacitors. This concept was introduced in [5, 8] for radial feeders supplying exclusively concentrated loads. Analytical solutions for idealized situations are given in [3, 6], considering only the reduction in the demand and energy charges. In a previous paper by the author [9], the potential reduction in the feeder total annual cost and its affecting parameters were analyzed. Expressions were derived for the feeder's total annual cost comprising its annual energy loss cost plus the annual cost component of the selected compensating capacitors. This was followed by a procedure for identifying the optimal size and location of the capacitors leading to the least total annual cost, as well as the value of this minimal annual cost. Since this cost reduction depends on the loading profile of the feeders (e.g. the concentrated part of the feeder's load), on the load curve and the load power factor, in addition to the size and location of the capacitors, the following detailed study on the impact of these load characteristics on the compensation's feasibility assessment and optimization was conducted.

2. METHOD OF ANALYSIS

List of Symbols

c — per unit reactive power compensation
c_{opt} — optimal per unit reactive power compensation
I_c — capacitor current
I2 — the lumped part of the feeder's reactive current
$i(x)$ — the feeder's longitudinal reactive current
k_c — specific cost of the compensating capacitor
k_e — specific energy loss cost
opt — a suffix that denotes optimal values
P_{cu} — feeder's copper loss due to the reactive currents
Q_C — reactive power of the compensating capacitor
Q_{load} — feeder's total reactive power
R — feeder's resistance
S — annual saving
T — the load's annual duration
x — distance measured from the feeder's sending end
x_1 — capacitor location
λ — the per unit concentrated part of the feeder's current
μ — a parameter that depends on: k_c, k_e, T, V, R, I, φ

Figure 1 shows the considered radial feeder of length 1 per unit, and total resistance R. The distance x is measured from the substation at which $x = 0$ pu. The feeder load includes a concentrated (lumped-sum) load at $x = 1$ pu and a uniformly distributed component. Only the reactive current distributions and their associated copper losses will be considered in the following analysis.

a) the Current Distribution

Assuming I_1 to be the total feeder reactive current at the supply end, the lumped part of the load is denoted λI_1. Then the feeder's longitudinal current before shunt compensation is given by [6]:

$$i(x) = [1 - (1-\lambda)\, x]\, I_1 \tag{1}$$

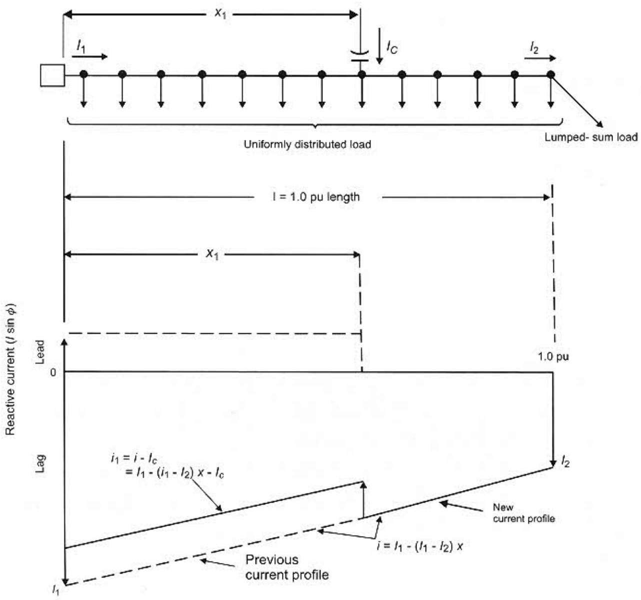


Fig.1. The distribution longitudinal feeder reactive current $i(x)$ before and after adding the shunt capacitor, adopted from [6].

The addition of a shunt capacitor at the point of coordinate x_1 will affect only the distribution in the feeder section between $x=0$ and $x=x_1$ as indicated in Figure 1. In this section, $i(x)$ will be:

$$i(x) = [1-(1-\lambda)\,x]\,I_1 - cI_1 \tag{2}$$

where c denotes the per unit reactive power compensation.

b) The Loss Calculation:

The feeder's copper losses P_{cu} (in the three phases) due to the reactive current can be obtained from:

$$P_{cu} = 3\int_{x=0}^{x=l_1} i(x)^2 R dx \tag{3}$$

The loss reduction ΔP_{cu} due to shunt compensation is:

$$\Delta P_{cu} = -3x_1[(x_1-2)c - x_1 c\lambda + c^2]R(I\sin\varphi)^2 \tag{4}$$

Assuming that the feeder is operated T hours per year, the corresponding reduction in the annual copper loss cost will be ($k_e T \Delta P_{cu}$), where k_e is the specific energy cost.

c) the Annual Saving *S*

With a c per unit compensation, the required 3-phase reactive power rating of the capacitors will be:

$$Q_c = c\sqrt{3}\,VI\sin\varphi \tag{5}$$

with V denoting the feeder's line voltage.

The annual saving due to applying shunt compensation is accordingly:

$$S = (k_e T\Delta P_{cu}) - (QcQc) -$$

$$-3k_e TRI^2 \sin^2\varphi.[x_1^2\, c - 2x_1 c - \lambda c x_1^2 + c x_1^2 + \mu \tag{6}$$

where:

$$\mu = \frac{k_c\sqrt{3}V}{3k_e TR(I\sin\varphi)} = \frac{k_c Q_{Load}}{k_e P_{cu,original} T} \tag{7}$$

The factor $3k_e TRI^2 \sin^2\varphi$ in Equation 6 represents the annual cost of the copper losses (due to the reactive current) before compensation. It follows that the per unit saving in the annual cost due to the shunt compensation is $-[x_1^2\, c - 2x_1 c - \lambda c x_1^2 + c x_1^2 + \mu c]$, based on the original annual energy loss cost.

The parameter μ includes most of the system data. It is proportional to the load reactive power Q_{load} and the specific cost of the compensating capacitor k_c. On the other hand, it decreases inversely with the specific energy cost k_e, the original feeder losses as well as the load connection time T. It follows that μ is inversely proportional to the load factor (if a rectangular annual load duration curve is assumed).

d) The Optimization of S

The annual saving S will be the objective function of the optimization. It is a function of the compensation level c and the capacitor's location x_1. Equating the corresponding partial derivatives to zero for a maximum S, it results:

$$c_{opt} = 2 + 2\lambda x_{1,opt} - 2x_{1,opt} \tag{8}$$

and:

$$x_{1,opt}^2 - 2x_{1,opt} - \lambda x_{1,opt}^2 + 2c x_{1,opt} + \mu = 0 \tag{9}$$

from which:

$$x_{1,opt} = \frac{1}{3(1-\lambda)}[1+\sqrt{1+3\mu(1-\lambda)}] \tag{10}$$

The corresponding c_{opt} and the maximum annual saving S_{opt} can then be easily obtained.

Two constraints should be taken into account. First, the optimal distance should satisfy $0 \le x_{1,opt} \le 1$. If the above expressions yield $x_{l,opt}>0$, the value $x_{l,opt}=1$ is assumed. Second, if a negative value of c_{opt} results, a zero compensation level is adopted.

3. RESULTS OF A CASE STUDY

A 3-phase 4.16kV feeder of a total resistance $R=0.5$ ohm is considered. Its total current I is assumed 212A at $\cos\varphi=08$. lagging, The specific costs are $k_c=\$0.01$/kWh and $k_c=\$1.5$/kVAr/year for the energy loss and the compensating capacitors, respectively [6]. A rectangular load duration curve is assumed.

a) Feeders with uniformly-distributed load only ($\lambda = 0$)

Figure 2a shows the effect of both the annual specific capacitor cost k_c in \$/VAr and the load factor on the optimal compensation level c_{opt}. Based on the optimization constraints, the region in which the plot is indicating $c_{opt}=0$ implies that the shunt compensation is economically infeasible. If feasible, the optimal compensation level decreases with k_c and increases with the value of the load factor. The plot 2-(b) shows that the optimal location $x_{1,opt}=0$ will be closer to the feeder's supply point as the load factor increases and/or the capacitor become less expensive. The flat plateaux $x_{1,opt}=1$ in 2-(b) corresponds to economically uninteresting regions, described by $c_{opt}=0$ in the plot 1-(a). As indicated in plot 2-(c), the optimal annual saving will increase with the load factor, and will decrease for more expensive capacitors. Because of the assumed rectangular shape of the annual load duration curve, the optimal annual saving (for any given value of k_c) increases linearly with the load factor, as shown by Equation 6.

The impact of the specific energy cost k_e is depicted in the plots 2-(d), 2-(e) and 2-(f). The 3-D plot 2-(d) indicates that, depending on k_e, there is a lower limit for the load factor above which the shunt compensation will be of economical advantage. If feasible, the optimal per unit compensation level c_{opt} increases with the specific energy cost. The optimal capacitor location $x_{1,opt}$ exhibits exactly the opposite behaviour as seen in plot 2-(e). It becomes closer to the feeder's supply point as the load factor and/or the energy cost increases. As noticed in plots 2-(d) and 2-(e), the feasibility region of compensation starts with small ratings of the shunt capacitors at the feeder's remote end ($x_{1,opt}=1$). Plot 2-(f) indicates that, within the feasibility region, the achieved optimal annual saving increases rapidly with both the specific energy cost k_e and the load factor.

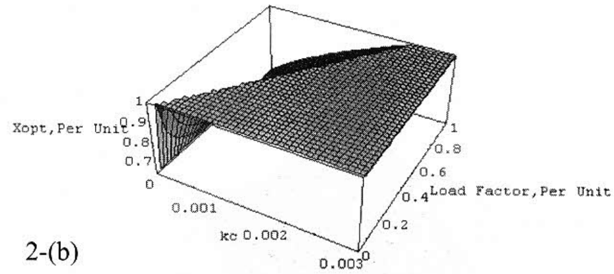
b) Feeders with 50% concentrated load **(λ = 0.5)**

The plots in Figure 3 illustrate the results for the same feeder described earlier but with $\lambda = 0.5$. The presence of the concentrated load component extends the feasibility of the shunt compensation over a wider range of k_c and the load factor. This can be realized if the plot 3-(a) is compared with the corresponding plot 2-(a) for exclusively distributed feeder load. It can also be noticed that, for the same values of k_c and the load factor, the optimal level of compensation is higher for $\lambda = 0.5$.

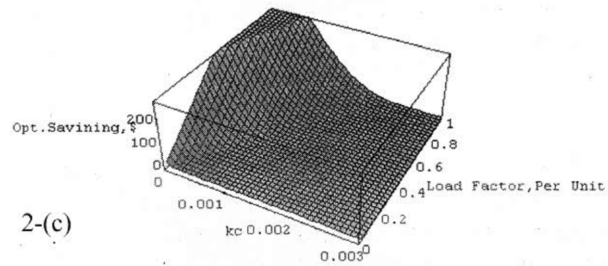
Results have also indicated that, for the interesting range of k_c and load factor, the optimal capacitor location will be at the feeder's remote end if half the feeder's load is concentrated at $x=1$. As plot 3-(b) shows, the optimal achievable annual saving for any combination of the capacitor's specific cost and the load factor will be higher for $\lambda = 0.5$. This can be seen by comparing plot 3-(b) with the corresponding one 2-(c). For the theoretical case $k_c = 0$ and a load factor of 1, plot 3-(b) gives an optimal annual saving of approximately \$1000, at an optimal compensation level of about 0.60 per

$$\begin{bmatrix} i_{d\_ref} & i_{q\_ref} \end{bmatrix}^T = \begin{bmatrix} \frac{2P_{ref}}{3|v|} & \frac{2Q_{ref}}{3|v|} \end{bmatrix}^T, P_{ref}, Q_{ref}$$

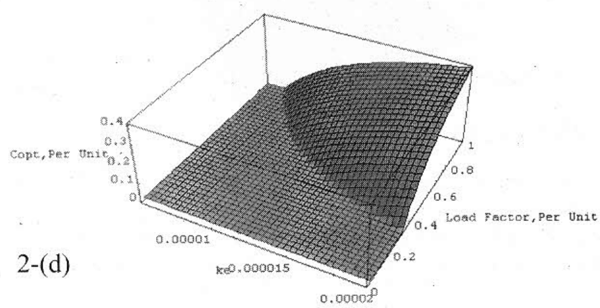
2-(a)



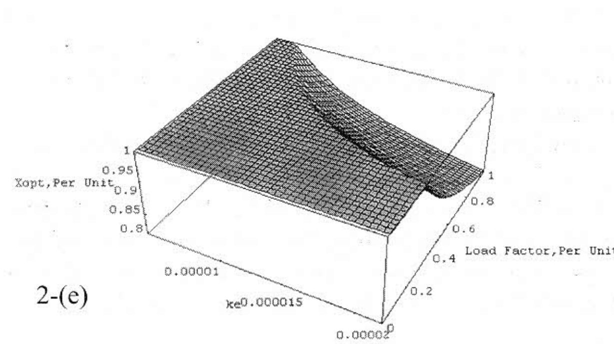
2-(b)



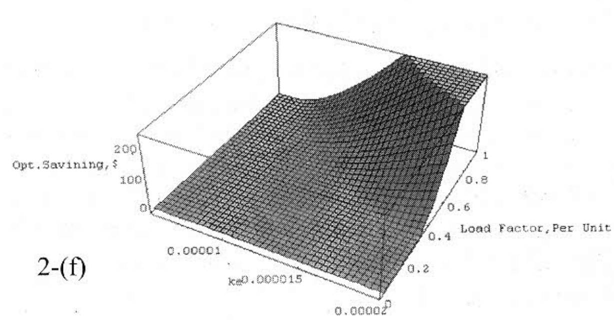
2-(c)



2-(d)



2-(e)



2-(f)

Fig. 2. Results of a parameter study on feeders with exclusively distributed load (λ=0). The load power factor is 0.80 lagging.

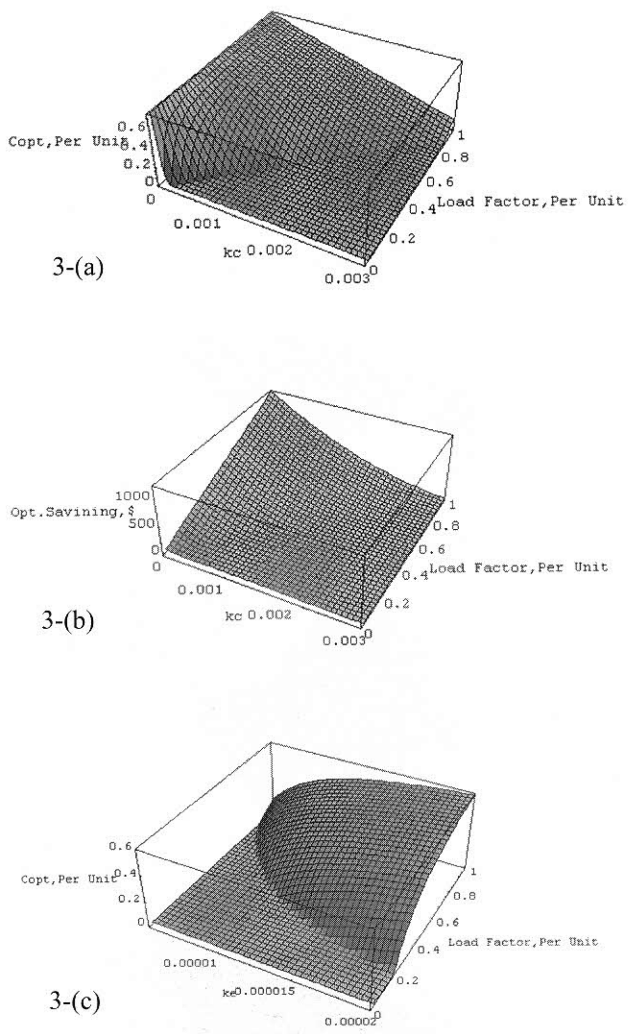


Fig. 3. Results for the case λ = 0.5, i.e. 50% of the feeder's load is concentrated at its end. The load Power factor is 0.8 lagging

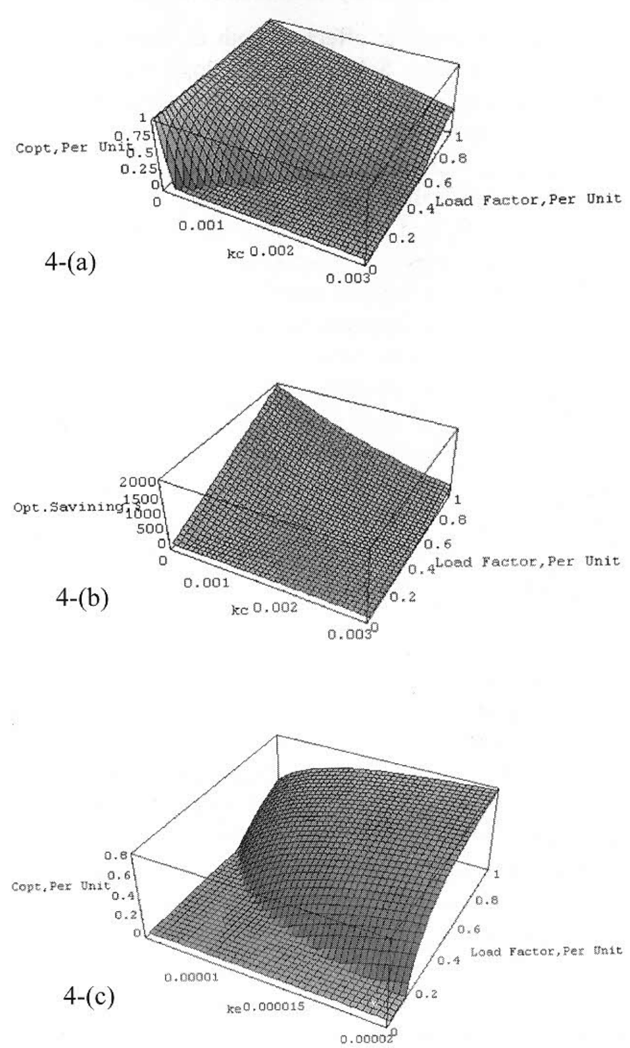


Fig. 4. The results for a feeder with a concentrated load at its remote end, λ =1.

unit. Plot 3-(b) depicts also that, similar to the case $\lambda = 0$, the optimal saving due to compensation, if feasible, increases linearly with the load factor, and decreases with the specific compensation cost k_c.

A comparison of plot 3-(c) with the corresponding one 2-(d) for λ =0 indicates that the feasibility of compensation for $\lambda = 0.5$ covers wider ranges of both k_e and the load factor. For the same values of these two parameters, the optimal compensation level in the case of $\lambda = 0.5$ is higher than in the previous case of an exclusively distributed feeder load, $\lambda = 0$.

c) Feeders with concentrated load only (λ =1)

This part discusses the result of the special case described by λ =1, i.e. if the feeder has only a lumped load at its remote end. As expected, the results indicated that the optimal capacitor location is at the feeder's free end ($x_{1,opt}$= 1), i.e. immediately across the load. For any combination of k_c and the load factor, the values of the optimal compensation levels c_{opt} are always higher than those in the two previous cases of λ=0 and λ =0.5. It is also seen that the feasibility range of the shunt compensation (in terms of the specific capacitor cost and the load factor) will increase. It is also interesting to notice that the full compensation c_{opt} =1 will result as an optimal solution for any load factor.

The higher optimal annual saving in comparison with the two previous cases is apparent from the plot 4-(b). Moreover, the relatively more extended feasibility range of compensation is shown in plot 4-(c). It reaches about c_{opt} =0.8 per unit at a load factor of unity and a relatively more expensive energy rate, k_e.

The three plots in Figure 5 illustrate the impact of both the feeder's per unit concentrated load and the load factor on the parameters describing the optimal compensation. Plot 5-(a) indicates that the compensation will be economically feasible only for load factors in excess of about 0.70 per unit, if the load is entirely distributed, λ = 0. On the other hand, the feasibility already starts for load factors above about 0.30 per unit if feeders with exclusively lumped load (λ=1) are considered. If the network's other technical and economical data remain unchanged and equal to those of the base case, plot 5-(b) shows that the optimal capacitor location will be

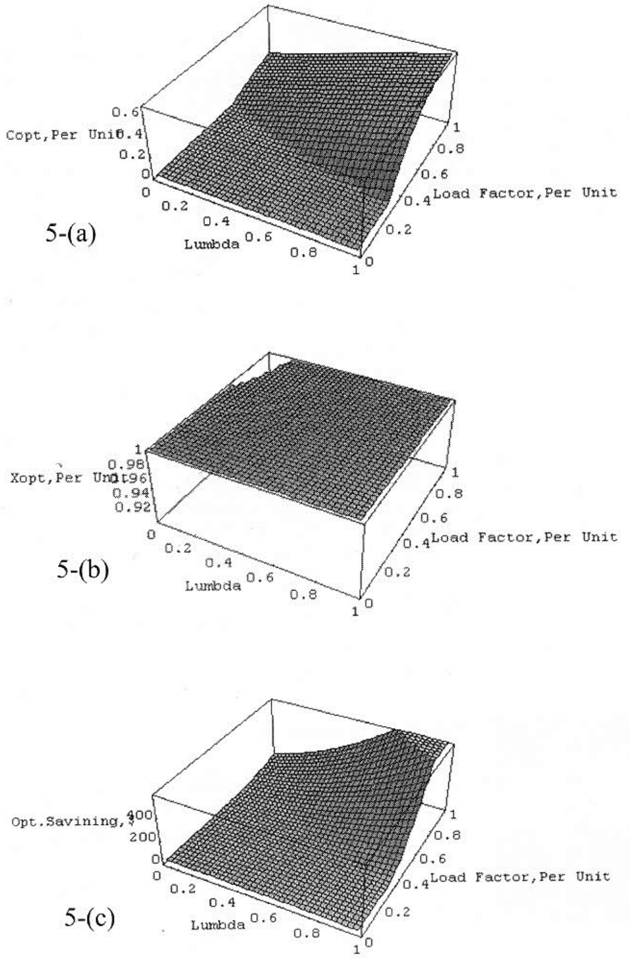


Fig. 5. Effect of both parameters (the per unit feeder's concentrated load) and the load factor on the optimal compensation.

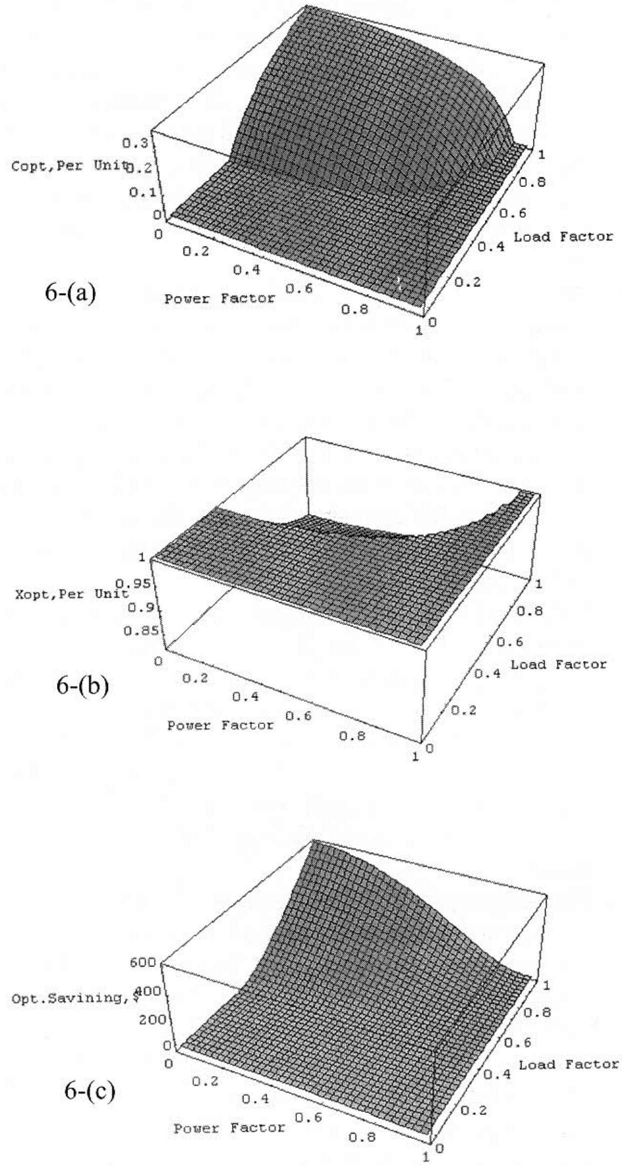


Fig. 6. Effect of the feeder's load original power factor on the optimal compensation, ($\lambda = 0$).

very close to the feeder's end unless the load factor is close to unity and λ is close to zero. The optimal annual saving is seen in plot 5-(c) to increase rapidly with both λ and the load factor.

Figure 6 illustrates in the plot 6-(a) the effect of both the original power factor of the feeder's load and the load factor on the optimal level of compensation. The case of entirely distributed feeder loading is considered. The plot indicates the feasibility region of compensation. For high values of the load factor, the shunt compensation will still be economically attractive even if the original power factor is within the usually accepted limits.

Figure 6-(b) shows that in the considered ranges of the load's initial power factors and load factors, the optimal capacitor location will be within the last 20% of the feeder's length. Relatively low power factors and/or high load factors will shift the capacitor location closer to the supply point.

4. CONCLUSIONS

1. A suggested technique for the assessment and optimization of the shunt compensation of distribution feeders is applied to a case study comprising a 3-phase 4.16kV feeder of a total resistance $R = 0.5$ ohm. Its total current I is assumed 212A at $\cos\varphi = 0.8$ lagging, The specific costs $k_e = \$0.01$/kWh and $k_c = \$1.5$/kVAr/year for the energy loss and the compensating capacitors, respectively. It takes into account both the achieved reduction in the feeder's energy copper loss cost as well as the cost of the compensating capacitors.
2. For feeders with an entirely uniformly-distributed load, the optimal compensation level decreases with the specific capacitor cost and increases with the value of the load factor. The optimal location of the capacitor will be closer to the feeder's supply point as the load factor increases and/or the capacitor become less expensive. Moreover, the optimal annual saving will increase with the load factor, and decrease for more expensive capacitors.
3. Depending on the specific energy cost, there is a lower limit for the load factor above which the compensation will be of economical advantage. If feasible, the optimal

per unit compensation level increases with the specific energy cost. The optimal capacitor location becomes closer to the feeder's supply point as the load factor and/or the energy cost increases. The feasibility region of compensation starts with small ratings of the shunt capacitors at the feeder's remote end ($x_{1,opt} = 1$). Within the feasibility region, the optimal saving increases rapidly with both the specific energy cost and the load factor.

4. The presence of a concentrated component in the feeder's load extends the feasibility range of the shunt compensation over a wider range of the specific capacitor cost and the load factor. The optimal capacitor location will be at the feeder's remote end if half the feeder's load is concentrated at $x = 1$. The optimal saving, if feasible, increases linearly with the load factor, and decreases with the specific compensation cost. The feasibility of shunt compensation for $\lambda = 0.5$ covers wider ranges of both k_e and the load factor.
5. If the feeder has only a lumped load at its remote end, the optimal capacitor location is at the feeder's free end ($x_{1,opt} = 1$), i.e. immediately across the load. The optimal compensation levels are always higher than those in the two cases of $\lambda = 0$ and $\lambda = 0.5$. The feasibility range of compensation (in terms of the specific capacitor cost and the load factor) will increase. The full compensation $c_{opt} = 1$ will result as an optimal solution for any load factor.
6. The compensation will be economically feasible only for load factors in excess of about 0.70 per unit, if the load is entirely distributed, $\lambda = 0$. The feasibility already exists for load factors above about 0.30 per unit if feeders with exclusively lumped load ($\lambda = 1$) are considered. The optimal capacitor location will be very close to the feeder's end unless the load factor is close to unity and λ is close to zero. The optimal saving increases rapidly with both λ and the load factor.
7. In the case of entirely distributed feeder loading and high values of the load factor, the compensation will still be economically attractive even if the original power factor is within the usually accepted limits. In the considered ranges of the load's initial power factors and load factors, the optimal capacitor location will be within the last 20% of the feeder's length. Relatively low power factors and/or high load factors will shift the capacitor location closer to the supply point.

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