Transactions on Aerospace Research eISSN 2545-2835, VOL. 267, NO. 2/2022, 80-95
DOI: 10.2478/tar-2022-0012

DIAGNOSTIC MODEL OF AIRCRAFT TURBINE ENGINE GOVERNOR PUMP

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Abstract

This paper presents a mathematical model for a hydromechanical fuel governor pump, to be used in parametric diagnostics. The design and operation of the governor are described. The main requirements of the model are formulated, its structure is determined, corresponding to the specifics of the diagnostic task, and assumptions to make the model simpler are presented (single-dimensional flow and absence of heat exchange). The presented model consists of idealized elements with lumped parameters (such as pressure and mass consumption of the working fluid), accounting for the compressibility of the substance and the design arrangement of the governor (presence of mechanical rests, metering orifices of complex shapes, relay switchers, etc.). Equations of elements with lumped parameters, linked by hydraulic channels in one node, are presented. The model – a system of first-order differential-algebraic equations – is solved and the parameters of the governor pump are determined for different steady-state and transient operation modes. We compare our results to the requirements for the corresponding parameters outlined in the Engineering Specifications. The model is matched to the specifications by correcting setting parameters (tightening of elastic springs, areas of throttles, etc.), and a method of initial model linearization is developed. Based on the results, we conclude that our model can be used as a diagnostic algorithm for a governor pump, at the testing and development stages, during manufacturing, repair and maintenance.

Keywords: aircraft turbine engine, hydromechanical governor, governor pump, diagnostics, model, influence coefficients
Type of the work: research article

1. INTRODUCTION

The processes of designing and manufacturing units for various aircraft systems are becoming more complex every year. The main reason is the increasing complexity of the structure of the units which must ensure high standards of operation of aircraft systems and the growing requirements for operating conditions and reliability. A high quality of operation of all aircraft systems must be maintained at all stages of their life cycle. Due to the complex design of the units, there are strict requirements for the accuracy of maintaining the working process parameters at the production and operation stages, and malfunctions may occur even with minor deviations from the requirements of the technical documentation for product design parameters.

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ARTICLE HISTORY **Received** 2022-01-24 **Revised** 2022-03-02 **Accepted** 2022-04-01

The flight readiness of an entire fleet of aircraft directly depends on the speed of troubleshooting. In order to quickly find causes of malfunctions, a range of diagnostic methods are used. Selecting different diagnostic methodologies allows us to develop high-quality products in the shortest possible time, which is extremely important for mass production.

This paper discusses the fuel governor of a helicopter turboshaft engine. The task of such a fuel governor is to ensure all the specified power ratings of the engine. The safety of the entire flight depends directly on the quality of engine control. Therefore, it is essential to identify and eliminate any fault states of the fuel governor in a timely manner. To solve this problem, various state recognition methods have been proposed and applied in engineering technology fields, including statistical, metric, logical and other methods [1, 2, 3]. However, for practical implementation of these methods, it is necessary to develop a large database of diagnostic parameters of the governor obtained during field experiments. Subsequently, the information obtained must be grouped into classes depending on the fault involved. Since the number of possible faults caused by deviations of the state parameters from the technical requirements is huge, developing databases based on field experiments is a practically impossible task. Much more versatile is the simulation method [4], based on a mathematical model of the fuel governor. Under this method, carrying out full-scale experiments is replaced by calculations of the working process parameters using a mathematical model with predetermined deviations of the diagnostic parameters of the state.

This paper discusses the main stages of developing a diagnostic mathematical model of the fuel governor pump of a turboshaft helicopter engine, which is the basis of the algorithm for parametric diagnostics [1–4]. The aircraft engine fuel governor pump is an electro-hydromechanical system with many functions and a complex design consisting of hundreds of parts. Therefore, a rational approach to forming a diagnostic algorithm is based on deconstructing the object for autonomous diagnosis of its individual functional parts. The mathematical models of these parts are much simpler than the model of the entire object, which simplifies their development and ultimately improves the quality of diagnostics. For example, we considered the functional part of the governor pump, which controls the free turbine rotor speed. The first stage of developing a mathematical model is to determine its requirements corresponding to the problems of diagnostics. Based on these requirements, a number of assumptions were confirmed, making it possible to simplify the mathematical model without significantly changing the quality of the diagnosis. This means it possible to use a mathematical model as part of a diagnostic algorithm. The paper discusses the stage of its linearization by determining the matrices of the influence coefficients. The analysis of linearization errors, which ultimately affect the quality of diagnostics, is presented.

2. BRIEF DESCRIPTION OF THE GOVERNOR PUMP OPERATION

The structure of the fuel governor pump is shown in Fiure 1. It consists of a pump, a pressure drop valve (PDV), a main metering valve (MMV), a pressurizing valve for the first circuit of injectors (PVc1), a valve for opening the second circuit of injectors (PVc2), and a cut-off valve for the secondary circuit of injectors (CVc2). The pump generates a flow Q_P proportional to the rotational speed n_{TC} of the regulator shaft driven by the turbocompressor rotor. The PDV generates a constant differential pressure ΔP across the MMV, comparing the pressure P_P and P_{MMV}. With a constant differential pressure, the flow through the MMV Q_{MMV} is determined by the flow area F_{MMV}, which is adjusted by the servo piston (SP) according to the command pressure P_{contr} coming from the governors. Some of the MMV flow rate goes to the main fuel line, and the remainder Q_{SP}, Q_{MPVTC} enters through the constant area orifice F_{SP} to ensure the operation of the servo piston and the governors, respectively.

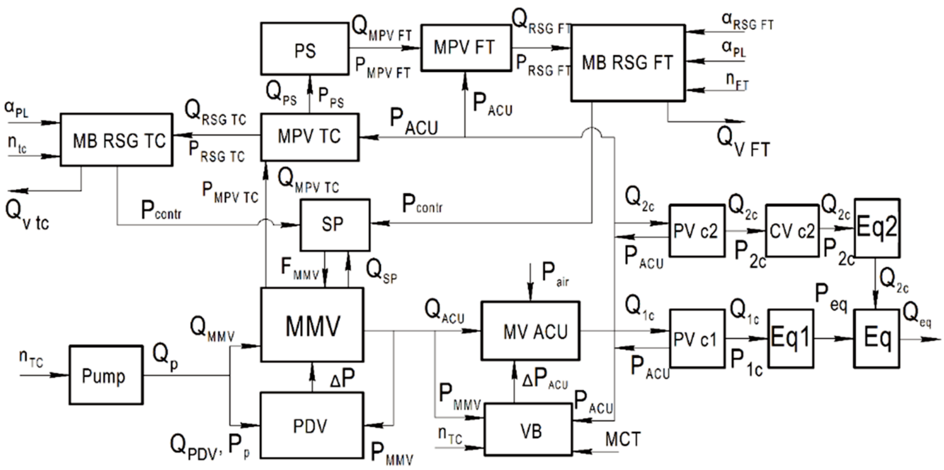

Figure 1. Schematic diagram of the fuel governor pump.

Combined with sensitive elements, MMV and PDV form the two main governors: the rotation speed governor of the turbocompressor (RSG TC) and the rotation speed governor of the free turbine (RSG FT). The sensitive elements include the mechanical block of the turbocompressor rotation speed governor (MB RSG TC) and the mechanical block of the free turbine rotation speed governor (MB RSG FT). MB RSG TC converts the rotational speed of the turbocompressor n_{TC} into a mechanical signal, which is compared with the input signal – a mechanical signal formed by setting the position of the power lever α_{PL}. Suppose the actual value of the turbocompressor rotation speed n_{TC} exceeds the given input value. In this case, the MB RSG TC increases the flow Q_{VTC} through the valve into the drain line, thus generating control pressure P_{contr}. The mechanical block of the rotation speed governor of the free turbine (MB RSG FT) has a similar structure. The only difference is that the free turbine rotational speed n_{FT} is compared in the MB RSG FT with another input signal. The input signal for the RSG FT is a mechanical signal formed by setting the power lever position α_{PL} and by setting the position of the readjustment lever of the free turbine rotation speed governor α_{RSGFT}. In the circuit with the MB RSG FT, the power synchronization governor (PS) is connected in series, which comes into operation by changing the flow area F_{PS} when the adjacent engine of the power plant has more power under the same operating conditions of both engines. The minimum pressure valves MPV TC and MPV FT, which are connected in series in their circuits, operate to limit the minimum pressure P_{ACU} in the system at the operating modes of the RSG TC and RSG FT governors. Since PS, MPV FT, and MB RSG FT are connected in series, the values Q_{SP}, Q_{MPVFT} and Q_{RSGFT} are equal.

The third governor, the acceleration control unit (ACU), determines the required fuel consumption of the engine by parameters such as n_{TC} and the air pressure P_{air} behind the compressor. The ACU includes a metering valve MV ACU and a block of valves (BV) which form a pressure drop ΔP_{ACU} across the MV ACU, proportional to the square of the turbocompressor rotation speed n_{TC}. In turn, pressure P_{air} determines the MV ACU area. Some of the flow Q_{VB} is fed through a hydraulic divider, consisting of orifices with areas F_{VB1} and F_{VB2} to ensure the VB operation. The ACU operates in transient modes when the RSG TC and RSG FT are disabled, and adjusts the acceleration time when switching from one engine power rating to another. The ACU can be readjusted to a higher value ΔP_{ACU} by the MCT relay-type solenoid valve command. Pressurizing valves of the first and second circuits of the injectors form pressure P_{ACU} in the system, which is necessary for the functioning of all hydraulic elements. At the unit's outlet, the flow rate Q_{eq} is formed, which is the sum of the flow rates through the first circuit of the injectors Q_{1c} and the second circuit of the injectors Q_{2c}.

3. REQUIREMENTS FOR THE MATHEMATICAL MODEL AND ACCEPTED ASSUMPTIONS

As discussed above, the fuel governor pump, as a unit of the engine fuel system, functionally and structurally consists of several governors of the engine's working process parameters. Each of these governors functions individually in certain engine modes or when changing from mode to mode. Therefore, the operation of one specific governor is considered one of the modes of operation of the fuel governor pump as a whole. When the engine changes from one power rating to another, the fuel governor pump also switches from the mode of operation of one governor to the mode of operation of another; thus, the governor mode is reconfigured. Changing the fuel governor pump modes is a transient process. The basic requirements of the diagnostic mathematical model of the fuel governor pump are as follows:

1) The first requirement of the mathematical model of the fuel governor pump is to ensure the simulation of all operating modes in which diagnostics of its technical condition are carried out. There are three such modes for the governor pump under consideration: the operating mode of the free turbine rotation speed governor, the operating mode of the rotation speed governor of the turbocompressor, and the operating mode of the acceleration control unit. The mode is determined by the values of the input parameters: the turbocompressor rotational speed n_{TC}, the free turbine rotational speed n_{FT}, the power lever position α_{PL}, the position of the readjustment lever of the free turbine rotation speed governor α_{RSGFT}, the compressor discharge pressure P_{air}, and the command of the MCT relay type electromagnetic valve.
2) The presence of a fault at a certain mode is determined by a deviation of the monitored parameters of the working process from the values specified in the technical documentation. The faults can be caused by deviations from the technical documentation requirements of such state parameters as the diameters of the orifices, the stiffness of the springs, clogging of the channels, etc. Therefore, the second requirement for the mathematical model can be formulated as follows: the mathematical model of the fuel governor pump must include all diagnostic state parameters and must provide the ability to calculate all measured working process parameters when the diagnostic state parameters change. According to the technical documentation, a deviation of the fuel consumption Q_{eq} in the operating mode of the free turbine rotation speed governor at the outlet from the governor by more than 1.5% is considered a faulty state. It is known from maintenance experience that a deviation of the same diagnostic parameter by 10% is considered as an inoperative state. The boundaries of the faulty state for other modes of operation of the fuel governor are also defined in the technical documentation as tolerances for the controlled parameter of the working process. The task of diagnosing this unit is to localize the current state between the faulty and inoperative states. Thus, the diagnostic mathematical model of the unit should make it possible to simulate a healthy state and faulty states, corresponding to the specified limits of the state parameters variation.
3) Another important requirement for the mathematical model is the simulation of the dynamics of changes in the working process parameters. This requirement is due to the presence of controlled parameters of the working process which reflect the dynamics of various transient processes in the governor pump.

It should be noted that, provided that all of the above requirements are met, designing a mathematical model of a fuel governor with such a complex structure and design is a difficult task. Therefore, it is necessary to make several assumptions about the operation of the governor pump, which will have little to no effect on meeting the specified requirements but will significantly simplify the structure of the mathematical model.

The fuel governor pump consists of hydraulic and mechanical structural elements, which are combined into several functional units that perform one or more common functions. Based on such a representation of the object, its mathematical model can be represented as a hydromechanical circuit consisting of idealized active and passive elements with lumped parameters. The parameters for the hydraulic elements of the circuit are pressure and fuel flow rate, while for the mechanical elements of the circuit they are displacement, speed and acceleration.

Based on the operating conditions of the fuel governor pump, it is possible to neglect the thermal processes in the system. This is because the working fluid is aviation kerosene whose properties change insignificantly in the operating temperature range from –60°C to 80°C. Therefore, the thermal conductivity of the working fluid is taken to be zero. The geometry and length of the channels and the values of the flow consumptions of the working fluid suggest that the fluid flow is single-dimensional and turbulent. The proposed mathematical model considers the compressibility but does not consider the inertia of the working fluid flow due to its insignificant effect on the dynamics of the processes occurring in the fuel governor pump.

The mathematical model consists of the following state parameters: k_i – spring stiffness of the i-th elastic element, x_{ti} – initial tightening of the spring of the i-th elastic element, S_i – area of the hydraulic cylinder (elastic elements include such units as PDV, MMV, VB, SP, MPV TC, MPV FT, MB RSG TC, MB RSG FT, PVc1, PVc2 and CVc2), and F_i is the flow area of the i-th metering element (nodes of the PDV, MMV, VB, SP, MPV TC, MPV FT, MB RSG TC, MB RSG FT, PVc1, PVc2 and CVc2). The calculated parameters of the working process of the mathematical model include x_i, v_i – movement and speed of the i-th elastic element, Q_i – flow rate of the working fluid through the i-th throttling element (namely, Q_P, Q_{eq}, Q_{PDV}, Q_{MMV}, Q_{ACU}, Q_{1c}, Q_{2c}, Q_{SP}, Q_{MPVTC}, Q_{PS}, Q_{MPVFT}, Q_{RSGTC}, Q_{RSGFT}, Q_{VTC}, Q_{VFT} according to Figure 1), and P_i – pressure of the working fluid in the i-th node (pressure P_{MMV}, P_{ACU}, P_{contr}, P_{1c}, P_{2c}, P_{RSGTC}, P_{PS}, P_{MPVTC}, P_{MPVFT}, P_{RSGFT} according to Figure 1).

4. DESCRIPTION OF THE ELEMENTS OF THE GOVERNOR PUMP MATHEMATICAL MODEL

Pump. The source of the flow rate of the working fluid Q_P is the active element, i.e. the pump. The flow rate is determined as a linear function of n_{TC} with a constant coefficient D:

$$Q_P = D \cdot n_{TC}, \tag{1}$$

Passive elements. Passive elements include capacitive elements and resistance elements.

The capacitive element [5, 6] takes into account the compressibility of the liquid only, and is described by the equation

$$\left(\frac{V_{comp} + V_{var}}{E} \right) \cdot \frac{dP}{dt} = Q_{comp}, \tag{2}$$

where V_{comp} is the constant volume of the capacitive element, m^3; V_{var} is the variable volume, changing due to the movement of the mechanical elements, m^3; E is the bulk modulus of elasticity of the working fluid, Pa; P is the pressure in the capacitive element, Pa; Q_{comp} is the flow rate of the working fluid forming by its compression, m^3/s.

In this model of the capacitive element, the bulk modulus has a constant value and does not depend on the presence of air in the working fluid, the amount of which is considered to be negligible.

The ***resistance element*** [7] is described using equation (3). The compressibility of the fluid in such an element is ignored.

$$Q_i = \mu \cdot F(x) \cdot \sqrt{\frac{2}{\rho}} \frac{\Delta P}{\left[\Delta P^2 + \left(\frac{\rho}{2}\left(\frac{\mathrm{Re}_{cr} \cdot \upsilon}{\mu \cdot \sqrt{\frac{4 \cdot F(x)}{\pi}}}\right)\right)\right]^{\frac{1}{4}}}, \tag{3}$$

where Q_i is the volumetric flow rate of the liquid in the i-th element, m^3/s; μ is the flow rate coefficient; ρ is the density of the working fluid, kg/m^3; ΔP is the pressure drop, Pa; x is the movement of the mechanical elements, m; F is channel cross-sectional area, m^2; Re_{cr} is the critical Reynolds number; υ is the kinematic viscosity of the working fluid, m^2/s.

Equation (3) makes it possible to consider the change in the fluid flow regime by changing Re_{cr}. The value of Re_{cr} can be established experimentally. However, for a hydromechanical chain with circular channels with a diameter of 3 mm or more, we will take $\mathrm{Re}_{cr} = 2000$.

Capacitive element with an elastic element. The fourth type of element can be described as a capacitive element with an elastic element [5]. This element does not take into account the inertia of the fluid flow and its compressibility. The equation of the mechanics of the elastic element (6) has the form

$$m \cdot \frac{d^2 x}{dt^2} + \beta \frac{dx}{dt} + k \cdot x + S \cdot \Delta P = 0, \tag{4}$$

where ΔP is the pressure drop, Pa; x is the movement of the mechanical element, m; S is the area of the hydraulic cylinder, m^2; k is the stiffness of the spring, N/m; m is the mass of the mechanical element, kg; β is the damping coefficient.

The volume of liquid displaced by the moving part of the elastic element per unit of time is

$$Q_{mv} = S \cdot \frac{dx}{dt}. \tag{5}$$

The general equation of the hydromechanical chain is based on the balance of fluid flow rates [5]:

$$\sum_{i=1}^{n} Q_i = 0, \tag{6}$$

where n is the number of channels included in the nodal point of the hydromechanical circuit.

As an example, let us compose a flow rate balance equation for the hydraulic nodal connection shown in Figure 2 for the following elements:

– pump 1;
– capacitive element 2, in which the volume of hydraulic channels and cavities of elastic elements is concentrated;
– resistances 3 and 4, describing variable cross-section valves MMV and PDV, respectively;
– elastic element with a piston area 5, spring 6 with stiffness k_{PDV}, and initial tightening x_t.

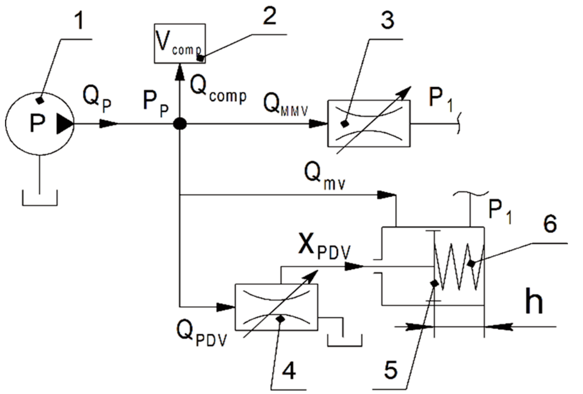

Figure 2. Example of a diagram of elements interconnected by hydraulic lines in one unit.

The condition of the flow rate balance is calculated as follows:

$$Q_P - Q_{MMV} - Q_{PDV} - Q_{mv} - Q_{comp} = 0. \tag{7}$$

The formulas expressing the fluid flow rates included in this equation, proceeding from the assumption that the hydromechanical system consists of simple elements with lumped parameters, are as follows:

$$Q_P = D \cdot n_{TC}, \tag{8}$$

$$Q_{MMV} = \mu \cdot F_{MMV}\left(x_{MMV}\right) \cdot \sqrt{\frac{2}{\rho}} \frac{\left(P_P - P_{MMV}\right)}{\left[\left(P_P - P_{MMV}\right)^2 + \left(\frac{\rho}{2}\left(\frac{\mathrm{Re}_{cr} \cdot \upsilon}{\mu \cdot \sqrt{\frac{4 \cdot F_{MMV}\left(x_{MMV}\right)}{\pi}}}\right)\right)^2\right]^{\frac{1}{4}}}, \tag{9}$$

$$Q_{PDV} = \mu \cdot F_{PDV}\left(x_{PDV}\right) \cdot \sqrt{\frac{2}{\rho}} \frac{\left(P_P\right)}{\left[\left(P_P\right)^2 + \left(\frac{\rho}{2}\left(\frac{\mathrm{Re}_{cr} \cdot \upsilon}{\mu \cdot \sqrt{\frac{4 \cdot F_{PDV}\left(x_{PDV}\right)}{\pi}}}\right)\right)^2\right]^{\frac{1}{4}}}, \tag{10}$$

$$Q_{mv} = S_{PDV} \cdot v_{PDV}, \tag{11}$$

$$Q_{comp} = \left(\frac{V_{comp} + S_{PDV} \cdot x_{PDV}}{E}\right) \cdot \frac{dP_P}{dt}. \tag{12}$$

For this system, the drain pressure is assumed to be zero. If the flow area of the resistance element is specially profiled and depends on the movement of the elastic element (for example, $F_{PDV}(x_{PDV})$), this dependence is modeled using linear interpolation [6].

The movement (balance of forces) of the elastic element of the PDV is described by a second-order differential equation. In order to apply the standard procedures for solving systems of first-order differential equations, we will express the first derivative concerning displacement through a separate velocity parameter v_{PDV}, and we will model this element with two equations:

$$v_{PDV} = \frac{dx_{PDV}}{dt}, \tag{13}$$

$$m \cdot \frac{dv_{PDV}}{dt} = -\beta \cdot v_{PDV} + S_{PDV} \cdot P_P - k_{PDV}\left(x_t + x_{PDV}\right) - H\left(x_{PDV}, v_{PDV}\right), \tag{14}$$

where $H(x_{PDV}, v_{PDV})$ is a function that simulates the operation of the hard stop.

This function is specified as a condition under which a force is generated equal to the reaction force from the compression of the rigid element of the stop:

$$\text{if } x_{PDV} > h$$

$$H = k_{hs} \cdot \left(x_{PDV} - h\right) + \beta_{hs} \cdot v_{PDV} \tag{15}$$

Otherwise

$$H = 0. \tag{16}$$

The value k_{hs} is proportional to the bulk modulus of the material from which the stop is made. The damping coefficient of the stop β_{hs} is selected to exclude oscillations in a rigid system.

Equations for the remaining nodes of the system are drawn up in the same way.

5. RESULTS OF THE FUEL GOVERNOR PUMP SIMULATION

The mathematical model of the fuel governor pump, consisting of equations (1)–(16), is a system of differential-algebraic equations [8]. A special implicit solver ode23t [9] was used to solve such a system. The initial conditions for solving the mathematical model are such that at the time $t = 0$, all the calculated parameters are equal to zero. The relative accuracy of the calculation was set to be 10^{-6}.

After debugging the mathematical model and checking the functioning of all its elements, the input parameters were selected in such a way that the calculated parameters for the operating modes established in the technical documentation corresponded to the values established in the technical documentation. The selection of input parameters was carried out within limits specified by the technical documentation for the fuel governor.

The working process parameters were calculated for the three main operating modes of the fuel governor pump, specified in the technical documentation to analyze the results.

The first mode (operating mode of the RSG TC) was set by a stepwise change of the input parameters at the initial time $t = 0$ from zero to the following values: $n_{TC} = 4085$ rpm, $n_{FT} = 0$ rpm, $\alpha_{PL} = 120°$, $\alpha_{RSGFT} = 66°$, $P_{air} = 8$ kgf/sm^2, MCT – switched on. The results are shown in Figure 3.

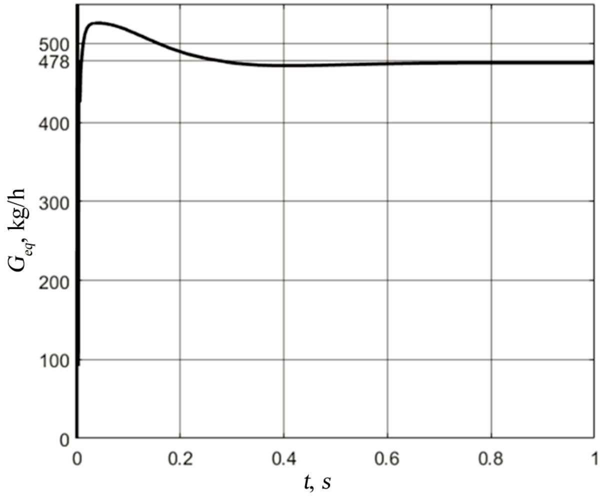

Figure 3. Dependence of the fuel mass flow rate G_{eq} on time in the operating mode of the RSG TC.

During the period from 0 to 0.6 s, a transient process of fuel flow rate regulation by the RSG TC governor is observed. An overshoot of 10% is characteristic. However, in the technical specifications, this indicator of the quality of control is not regulated. The specification defines a steady-state value of mass flow rate G_{eq} = 480 ± 10 kg/h. Fig. 3 shows that the calculated steady-state value G_{eq} = 478 kg/h meets this requirement.

The second mode (the mode of the RSG FT operation, Fig. 4) was set by a stepwise change of the input parameters at the initial time t = 0 from zero to the following values:

n_{TC} = 3740 rpm, n_{FT} = 4080 rpm, α_{PL} = 85°, α_{RSGFT} = 66°, P_{air} = 6.5 kgf/sm^2, MCT – switched on.

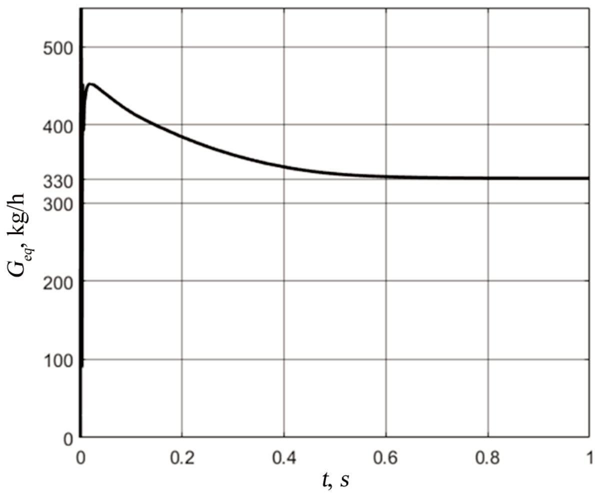

Figure 4. Dependence of the fuel mass flow rate G_{eq} on time in the operating mode of the RSG FT.

During the period from 0 s to 0.6 s, a transient fuel flow rate regulation process by the RSG FT governor is observed. An overshoot of 30% characterizes this transient. In the technical specifications, this indicator of the quality of control is not regulated. However, it should be noted that a significant step-change causes such a significant overshoot in n_{FT}, not foreseen in the specifications. The steady-state value G_{eq} = 330 ± 10 kg/h is required by the technical specification. Figure 4 shows that the calculated steady-state value G_{eq} = 330 kg/h corresponds to the requirement of the technical documentation.

The third mode – the mode of operation of the ACU – was set by a stepwise change of the input parameters at the initial moment of time t = 0 s from zero to the following values:

n_{TC} = 3220 rpm, n_{FT} = 0 rpm, α_{PL} = 120°, α_{RSGFT} = 66°, P_{air} = 2.25 kgf/sm^2, MCT – switched on.

During the period from 0 s to 0.2 s, a transient process of fuel flow rate regulation by the ACU regulator is observed. The steady-state value G_{eq} = 180 ± 10 kg/h is required by the technical specification. Figure 5 shows that the calculated steady-state value G_{eq} = 180 kg/h corresponds to the requirement of the technical documentation.

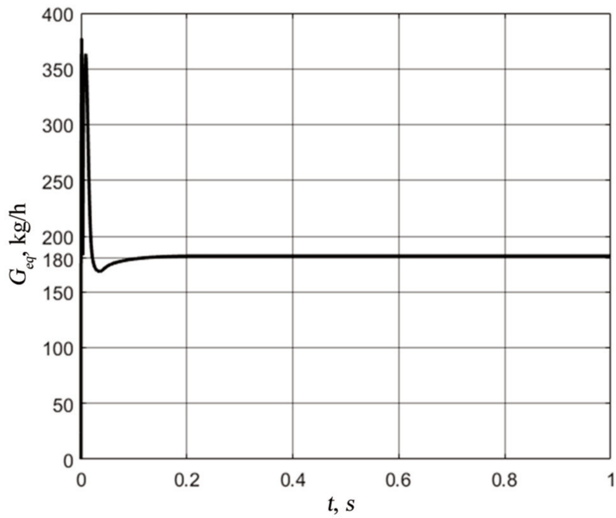

Figure 5. Dependence of the fuel mass flow rate G_{eq} on time in the operating mode of the ACU.

The process of a jump-like transition from one mode to another was modeled (Fig. 6) to display the dynamics of changes in the calculated parameters of the mathematical model. At the initial moment of time t = 0, a stepwise change in the input parameters from zero to the following values was set:

n_{TC} = 3100 rpm, n_{FT} = 0 rpm, α_{PL} = 0°, α_{RSGFT} = 66°, P_{air} = 2.25 kgf/sm^2, MCT – switched on.

At the moment of time t = 2 s, there is an abrupt increase α_{PL} to 120 and settling the ACU operation mode. During the period from 0 to 0.5 s, a transient fuel flow rate regulation process by the RSG TC governor and the establishment of fuel flow rate G_{eq} = 90 kg/h are observed. The change of G_{eq} when switching from the RSG TC operation mode to the ACU operation mode (shown in Fig. 6) occurred between 2 s to 4.2 s. An insignificant jump in fuel flow rate in the time interval 2 – 2.1 s is caused by a short-term pressure drop in the control cavity of the servo piston after the RSG TC valve is fully opened.

The time of the transient process was 2.2 s, which meets the requirements of the technical specifications. Also, this transient is characterized by an overshoot of 8%. The amount of the overshoot is not regulated by the technical documentation.

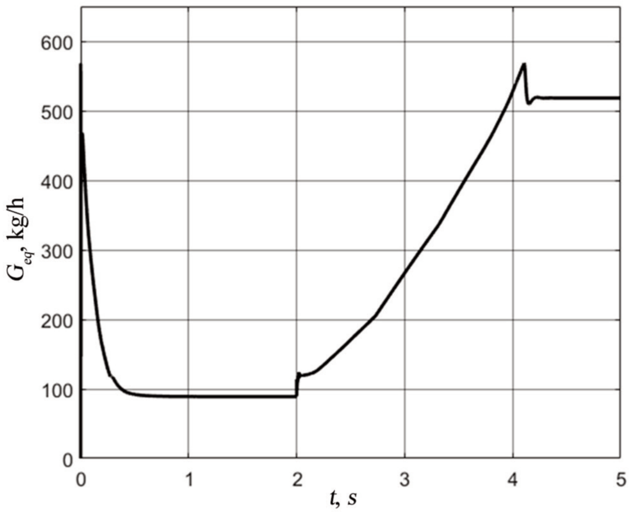

Figure 6. Dependence of the fuel mass flow rate G_{eq} on time during the transition from the RSG TC operation mode to the ACU operation mode with an abrupt change α_{PL}.

6. LINEAR MATHEMATICAL MODEL OF THE FUEL GOVERNOR. INFLUENCE COEFFICIENT MATRIX

To speed up the operation of the diagnostic algorithm, it is advisable to linearize the mathematical model, significantly reducing the mathematical operations required for calculations and ensuring the implementation of the algorithm on various computing platforms. However, it is necessary to justify replacing the basic model with a linear one.

For further research, the operating mode of the rotation speed governor of the free turbine RSG FT was selected. Ten state parameters are considered, the deviations of which have the most significant influence on the controlled parameter of the working process Q_{eq}. When choosing the composition of the measured parameters of the working process, 12 parameters available for measurement were considered, which are associated with the design features of the regulator. They include the fuel flow rate at the outlet of the regulator Q_{eq} and the pressure in various cavities. For the considered operating mode of the RSG FT, the boundaries of the faulty and inoperative states of the product were determined by the deviation of the fuel flow rate Q_{eq} from the value determined in the technical documentation for the regulator. The list of state parameters is shown in Table 1, where the following designations are used:

1. F_{PS} is the flow area of the orifice of the power synchronizer regulator, m^2;
2. F_{E1} is the flow area of orifice E1 of the first injectors circuit, m^2;
3. F_{eq} is the area of the flow area of the equivalent orifice Eq, m^2;
4. k_{PDV} is the PDV spring stiffness, N/m;
5. k_{PV1} is the spring stiffness of the pressurized valve of the first circuit of the injectors, N/m;
6. k_{PV2} is the spring stiffness of the pressurized valve of the second circuit of the injectors, N/m;
7. k_{CV2} is the spring stiffness of the cut-off valve of the second circuit of the injectors, N/m;

8. F_{VB} is the area of the inlet orifice of the valve block of the ACU, m^2;
9. F_{RSGFT} is the constant flow area of the RSG FT valve, m^2;
10. $X_{t\,RSGFT}$ is the pre-tightening of the spring of the MB RSG FT, m.

The list of workflow parameters is shown in Table 1.

Table 1. List of diagnostic signs.

№	**Designation of the state parameters**	**Working process parameter description**
1.	Q_{eq}	Volumetric fuel flow rate at the outlet of the regulator, m^3/s
2.	P_{1c}	Fuel pressure at the outlet of the pressurized valve of the first circuit of the injectors, Pa
3.	P_{2c}	Fuel pressure at the outlet of the cut-off valve of the second circuit of the injectors, Pa
4.	P_{ACU}	Fuel pressure at the outlet of the ACU, Pa
5.	P_{MMV}	Fuel pressure at the outlet of the MMV, Pa
6.	P_P	Fuel pressure at the outlet of the pump, Pa
7.	P_{VB}	Fuel pressure controlled by the VB of the ACU, Pa
8.	$P_{n_{TC}}$	Fuel pressure proportional to the square of the rotational speed n_{tc} supplied to the VB, Pa
9.	P_{contr}	Fuel control pressure in the servo piston cavity, Pa
10.	P_{PS}	Fuel pressure before the throttle of the power synchronizer regulator, Pa
11.	P_{MPVFT}	Fuel pressure behind the throttle of the power synchronizer regulator, Pa
12.	P_{CPV}	Fuel pressure at the outlet of the constant pressure valve, Pa

A linear model is created by calculating the matrix of influence coefficients (MIC) H [6]. The elements of the MIC determine the influence of the deviation of each state parameter on the calculated parameters Y and are calculated by solving the equations of the mathematical model with a given alternative deviation of the state parameters from the initial values. In the first approximation, it is assumed that the state parameters change with the same scale of variation $\alpha = 0.02$. The elements of the MCI were calculated using a two-sided variation of the state parameters as follows:

$$H_{ij} = \frac{\dfrac{Y_i^+ - Y_i^-}{Y_{0i}}}{\dfrac{\theta_j^+ - \theta_j^-}{\theta_{0j}}}, \tag{17}$$

where H_{ij} is the coefficient of the influence of the deviation of a certain state parameter on the working process parameter; Y_i^+ is the value of the working process parameter, calculated by the mathematical model with the deviation of the state parameter in the positive direction; Y_i^- is the value of the parameter of the working process, calculated according to the mathematical model with the deviation of the state parameter in the negative direction; Y_{0i} is the initial value of the workflow parameter; $\theta_j^+ = (1 + \alpha)\ \theta_{0j}$ is the value of the state parameter corresponding to its deviation in the positive direction from the initial value θ_{0j}; and $\theta_j^- = (1 - \alpha)\ \theta_{0j}$ is the value of the state parameter corresponding to its deviation in the negative direction from the initial value. Substituting these values into formula (17), we obtain

$$H_{ij} = \frac{Y_i^+ - Y_i^-}{2Y_{0i}\alpha}. \tag{18}$$

Table 2 shows the coefficients of the influence of the state parameters θ_j on the working process parameters Y_i, calculated by the formula (18) for the operating mode of the RSG FT governor.

Since the linearized model operates with state parameters in relative values, the parameters of the workflow must also be converted to a relative coordinate system:

$$Z_i = \frac{Y_i - Y_{0i}}{Y_{0i}}, \tag{19}$$

where Z_i is the relative deviation of the i-th parameter of the workflow from its initial value. Relative deviations of the measured parameters of the working process Z are diagnostic features of the governor.

Table 2. Matrix of influence coefficients.

	F_{PS}	F_{E1}	F_{eq}	k_{PDV}	k_{PV1}	k_{PV2}	k_{CV2}	F_{VB}	F_{RSGFT}	$X_t\ _{RSGFT}$
Q_{eq}	-0.29298	0.10528	0.32791	0.02360	-0.02116	-1.49001	-0.02820	-0.00800	-0.08103	-0.17845
P_{1c}	-0.10353	-0.03899	-0.08346	0.00729	-0.13799	0.40023	-0.00324	-0.00130	-0.02874	-0.05996
P_{2c}	-0.77437	-0.39160	-0.23076	0.06380	-0.00666	-4.29518	-0.07638	-0.01994	-0.21211	-0.47306
P_{ACU}	-0.09129	-0.01972	-0.07201	0.00654	-0.00094	0.35382	-0.00264	-0.00037	-0.02483	-0.05155
P_{MMV}	-0.14159	0.00286	0.00063	0.01020	-0.00591	0.01950	-0.00774	-0.00004	-0.03837	-0.08175
P_P	-0.13232	0.00244	0.00034	0.07145	-0.00542	0.01923	-0.00730	-0.00079	-0.03613	-0.07695
P_{VB}	-0.58941	0.21111	0.65579	0.05012	-0.04544	-2.98292	-0.05478	1.98742	-0.15743	-0.35536
P_{nTC}	-0.00251	0.00414	0.00204	0.00021	0.00121	-0.00173	0.00011	0.00126	-0.00084	0.00107
P_{contr}	-0.14291	-0.00466	-0.00159	0.00961	-0.00655	0.01774	-0.00900	-0.00747	-0.03954	-0.09270
P_{PS}	-0.14376	-0.00359	-0.00518	0.00883	-0.00329	0.01710	-0.00660	-0.00603	-0.04022	-0.08938
P_{MPVFT}	0.00139	-0.00124	-0.00355	0.00315	-0.00007	0.00611	-0.00236	-0.00415	-0.04181	-0.09195
P_{CPV}	-0.00277	0.00400	0.00199	0.00025	0.00099	-0.00197	0.00012	0.00159	-0.00075	0.00122

To estimate the error in the linearization of the parameters of the working process, it is necessary to calculate the difference between the values of the deviations of the parameters of the working process Z_{li}, obtained using a linear mathematical model, and the deviations of the parameters of the working process Z_i, calculated using the initial nonlinear model for a given deviation of the state parameter Δj:

$$\delta_i = Z_{il} - Z_i, \tag{20}$$

where Z_{li} is calculated by the equation

$$Z_{li} = H_{ij} \cdot \Delta_j. \tag{21}$$

Linearization errors are defined for all state parameters used to describe the diagnosed faults. As indicated above, the task of the developed diagnostic algorithm is to localize the malfunction in the area where the deviation of the controlled parameter of the working process from the serviceable value is in the range from 1.5% to 10%. Therefore, it is important that the linearization error is as small as possible when calculating the deviations of the parameters of the Z_{li} working process in the specified range. Using a non-linear mathematical model, four values of the deviations of the state parameter $\Delta_{1.5\% j}$, $\Delta_{-1.5\% j}$, $\Delta_{10\% j}$, and $\Delta_{-10\% j}$, were determined for each defect, corresponding to the deviations of the controlled parameter of the working process from the serviceable value by 1.5% and 10% in the positive and negative directions.

It is necessary to estimate the dependence of the linearization errors on the coefficient of variation α to estimate the diagnostic errors caused by the linearization of the model. To do this, formula (20) was used to determine the linearization errors when calculating the parameters of the working process using the MIC from Table 2 and the deviations of the state parameters $\Delta_{1.5\% j}$, $\Delta_{-1.5\% j}$, $\Delta_{10\% j}$, and $\Delta_{-10\% j}$. These errors were calculated as a percentage (multiplied by 100).

When calculating the relative deviations of the working process parameters Z_{li} using the MIC from Table 2 in cases of deviation of the state parameter by $\Delta_{1.5\% j}$ and $\Delta_{-1.5\% j}$, the linearization error for most cases does not exceed 2%. However, to calculate the relative deviation P_{VB} with deviations $\Delta_{-1.5\% 5}$, $\Delta_{-1.5\% 7}$ and $\Delta_{-1.5\% 8}$, the linearization errors δ_7 were 5%, 4% and 45%, respectively, and for calculating the deviation P_{2c} with the deviations of the state parameters $\Delta_{-1.5\% 5}$, $\Delta_{-1.5\% 7}$ and $\Delta_{-1.5\% 8}$, errors δ_3 were 7.2%, 5.4% and 2.5%, respectively. Such significant errors are associated with the nonlinearity of the mathematical model. Figures 7 and 8 show the dependence of the change in the relative deviation of the working process parameter on the change in the state parameters k_{PV1}, k_{CV2} and F_{VB} when calculating using a nonlinear mathematical model.

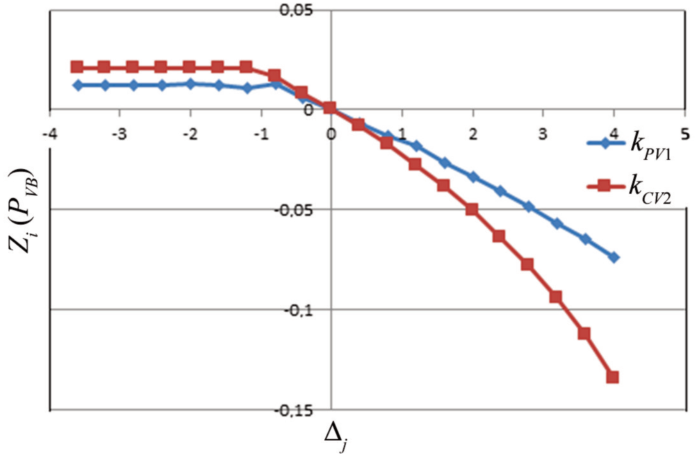

Figure 7. Dependence of the change in the relative deviation P_{VB} on the change in the state parameters k_{PV1}, k_{CV2}.

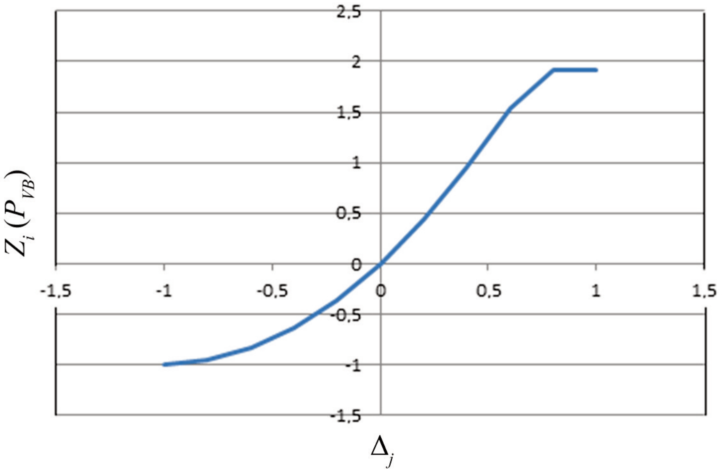

Figure 8. Dependence of the change in the relative deviation P_{VB} on the change in the state parameter F_{VB}.

Figure 7 shows that the parameter of the working process P_{VB} does not change with a negative deviation of the state parameters k_{PV1} and k_{CV2}. There is also a change in the slope of this dependence (line of state) with an increase in Δ_j, which negatively affects the accuracy of calculations using a linear mathematical model. After analyzing Fig. 8, it can be concluded that the parameter of the working process P_{VB} changes only within certain limits $-1 \leq \Delta_8 \leq 0.9$, while the change in the state parameter P_{VB} by $\Delta_8 = -1$ is still insufficient to obtain a deviation of the controlled parameter of the working process Q_{eq} by 1.5%. The fact that the fuel governor is insensitive to negative deviations of certain state parameters compels us to conclude that a linear mathematical model with a MIC, which is formed using a linearization algorithm with two-sided variation α, when applied to calculations, will give incorrect results.

When calculating Z_{li} with increased deviations of the state parameters $\Delta_{10\%\ j}$ and $\Delta_{-10\%\ j}$, the linearization errors increased for all working process parameters. This is because a change in the slope of the state lines with an increase in the deviation Δ_j is characteristic for all state parameters. Therefore, when calculating Z_{li} with large deviations Δ_j, there are increased discrepancies between the values of Z_{li} calculated using a linear mathematical model and Z_i calculated using a nonlinear mathematical model. Taking into account all of the above, we can conclude that for nonlinear systems such as a fuel governor, instead of (19), it is necessary to apply the one-sided linearization algorithm with a variable coefficient of variation α:

$$H_{ij} = \frac{Y_i^+ - Y_{0i}}{Y_{0i}\alpha_j}. \qquad (22)$$

The coefficient of variation α_j has different values for each state parameter and changes with the change in the deviation of the controlled parameter of the work process. The coefficient of variation α_j should be equal to the deviation of the state parameter Δ_j, which theoretically leads to the observed deviation of the controlled parameter of the working process Z_i. For example, if, as a result of measuring the controlled parameter of the working process, its deviation was 1.5%, then to linearize the mathematical model of this working process with some deviation of the state parameter Δ_j, it is necessary to select the appropriate coefficient of variation: for the case of deviation of the controlled parameter of the working process by 1.5% $\alpha_j = \Delta_{1.5\%\,j}$, and for deviation -1.5% $\alpha_j = \Delta_{-1.5\%\,j}$. We have calculated the MIC with different values of the coefficient of variation. Analysis of the results obtained showed that the maximum linearization error does not exceed 0.005%.

Thus, to minimize the linearization error, it is necessary to compile a table of coefficients of variation. The coefficient of variation should change depending on the deviation of the controlled parameter of the working process Y_i in the range from 1.5% to 10% with a gradation of not less than 0.05%.

7. CONCLUSIONS

Based on the calculation results, it can be concluded that the developed mathematical model makes it possible to simulate both stationary and non-stationary processes in the fuel governor pump of a helicopter engine. The model includes the key elements necessary to simulate the operation of a real fuel governor pump at the stage of adjustment during production, repair and operation, making it possible to diagnose the technical state of the fuel governor pump at these stages of its life cycle. Analysis of linearization errors revealed, at the initial stage of designing a diagnostic algorithm, that the common linearization method with a two-sided variation of state parameters with a constant coefficient of variation is not suitable for linearization of nonlinear systems such as a fuel governor pump. The proposed method of one-sided linearization with a variable coefficient of variation significantly reduces the linearization error, making it possible for this linear mathematical model to be used in the diagnostic algorithm in the future.

REFERENCES

[1] Birger, I.A., 1978, *Tekhnicheskaya diagnostika* [*Technical diagnostics*], Mechanical Engineering Publishing, Moscow, pp. 97–105.

[2] Bouamama, B.O., Biswas, G., Loureiro, R., Merzouki, R., 2014, "Graphical methods for diagnosis of dynamic systems: Review," *Annual Reviews in Control*, 38, pp. 199–219.

[3] Fentaye, A., Baheta, A., Gilani, S., Kyprianidis, K., 2019, "A Review of Gas-Path Diagnostics: State-of-the-Art Methods, Challenges and Opportunities," *Aerospace*, 6 (7), p. 83.

[4] Yepifanov, S.V., Kuznetsov, B.I., Bogaenko, I.N., Grabovskij, G.G., Djukov, V.A., Kuz'menko, S.A., Rjumshin, N.A., Sameckij, A.A., 1998, *Sintez sistem upravleniya i diagnostirovaniya gazoturbinnykh dvigateley* [Synthesis of automatic control and diagnostics systems of gas turbine engines], Engineering Publishing, Kyiv, pp. 43–72.

[5] Glikman, B.F., 1986, *Matematicheskiye modeli pnevmogidravlicheskikh sistem* [Mathematical models of pneumohydraulic systems], Science Publishing, Moscow, pp. 66–106.

[6] Köster, M.A., 2017, "On Modeling, Analysis and Nonlinear Control of Hydraulic Systems," Ph.D. thesis, Karlsruhe Institute of Technology.

[7] Merritt, H.E., 1967, *Hydraulic Control Systems*, John Wiley & Sons, Inc., New York, London, Sydney, pp. 35–41.

[8] Ascher, U.M., Petzold, L.R., 1998, *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*, SIAM. pp. 233–244.

[9] Shampine, L. F., Reichelt, M.W., Kierzenka, J.A., 1999, "Solving Index-1 DAEs in MATLAB and Simulink," SIAM Review, 41, pp. 538–552. means that airplanes powered by turbofan engines will be in operation well beyond 2050.