

# Analysis of Methods of Determining the Safe Ship Trajectory

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**ABSTRACT:** The paper describes six methods of optimal and game theory and artificial neural network for synthesis of safe control in collision situations at sea. The application of optimal and game control algorithms to determine the own ship safe trajectory during the passing of other encountered ships in good and restricted visibility at sea is presented. The comparison of the safe ship control in collision situation: multi-step matrix non-cooperative and cooperative games, multi-stage positional non-cooperative and cooperative games have been introduced. The considerations have been illustrated with examples of computer simulation of the algorithms to determine safe of own ship trajectories in a navigational situation during passing of eight met ships.

## 1 INTRODUCTION

For safety of navigation, the ships are obliged to comply with the International Regulations for Preventing Collisions at Sea (COLREG). However, these Rules refer only to two ships and under the conditions of good visibility.

In the situation of a restricted visibility the Regulations only specify recommendations of a general nature and are not able to consider all the necessary conditions which determine the passing course [1,2,3,4,6,7].

Consequently, the actual process of a ship passing other ships very often occurs in conditions of uncertainty and conflict accompanied by an inadequate co-operation of the ships within COLREG Rules.

It is, therefore, reasonable to investigate the methods of a ship safety handling using principles of

the theory based on optimal control and differential games [5,11,12,14,16,26,27].

## 2 CONTROL PROCESS

The process of handling a ship as a multidimensional dynamic object depends both on the accuracy of the details concerning the current navigational situation obtained from the Automatic Radar Plotting Aids ARPA anti-collision system and on the form of the process model used for the control synthesis [15,20].

The ARPA system ensures monitoring of at least 20  $j$  encountered ships, determining their movement parameters (speed  $V_j$ , course  $\psi_j$ ) and elements of approaching to own ship moving with speed  $V$  and course  $\psi$  to satisfy  $D_{j,\min} = DCPA_j$  - Distance of the Closest Point of Approach, and  $T_{j,\min} = TCPA_j$  - Time to the Closest Point of Approach and also assess the risk of collision  $r_j$  (Fig. 1).

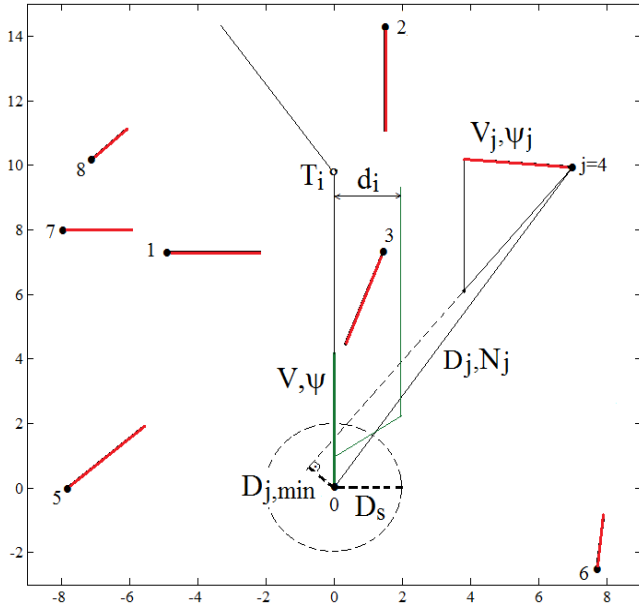


Figure 1. The situation of own ship passing eight encountered ships at sea.

The model of the process consists both of the kinematics and the dynamics of the ship's movement, the disturbances, the strategies of the encountered ships and the quality control index of the own ship [10,24].

The diversity of possible models directly affects the synthesis of the ship's control algorithms which are afterwards affected by the ship's control device, directly linked to the ARPA system and consequently determines the effects of safe and optimal control [3,28].

### 3 COMPUTER PROGRAMS FOR DETERMINING THE SAFE OWN SHIP TRAJECTORY

#### 3.1 Computer program STATOPT of static optimization

Goal control function has form:

$$I_0^* = \min_{u_0 \in U_0 = \bigcap_{j=1}^m U_{0,j}} \{L[x_0(t_k)]\} = L^*, \quad j=1,2,\dots,m \quad (1)$$

$L$  refers to the continuous function of the manoeuvring goal of the own ship, describing the distance of the ship at the initial moment  $t_0$  to the nearest turning point  $T_i$  on the reference route of the voyage and  $d_i$  is the final deviation safe trajectory of the reference trajectory (Fig. 1).

#### 3.2 Computer program DYNOPT of dynamic optimization with neural state constraints

Determination of the optimal control of the ship in terms of an adopted index of the control quality may be performed by applying Bellman's principle of optimization. The constraints for the state variables and the control values generate the neural constraints procedure in the computer program [17,23].

The optimal time for the ship to go through  $k$  stages is as follows:

$$t_k^* = \min_{u_{1,k-2}, u_{2,k-2}} (t_{k-1}^* + \Delta t_k), \quad k=3,4,\dots,K \quad (2)$$

#### 3.3 Computer program MATGAM\_C of multi-step cooperative matrix game

The matrix game  $R = [r_j(s_j, s_0)]$  includes the value a collision risk  $r_j$  with regard to the determined strategies  $s_0$  of the own ship and those  $s_j$  of the  $j$ -th encountered ship [8,13,18,19,21,22].

The value of the risk of the collision  $r_j$  is defined as the reference of the current situation of the approach described by the parameters  $D_{j,\min}$  and  $T_{j,\min}$  to the assumed assessment of the situation defined as safe and determined by the safe distance of approach  $D_s$  and the safe time  $T_s$  – which are necessary to execute a manoeuvring to avoid collision with consideration actual distance  $D_j$  between own ship and  $j$ -th encountered ship [25,29].

As a result control goal function has form:

$$I_0^* = \min_{s_0} \min_{s_j} r_j \quad (3)$$

#### 3.4 Computer program MATGAM\_NC of multi-step non-cooperative matrix game

Goal function (3) for non-cooperative matrix game has the form:

$$I_0^* = \min_{s_0} \max_{s_j} r_j \quad (4)$$

#### 3.5 Computer program POSGAM\_C of multi-stage cooperative positional game

The optimal control of the own ship  $u_0^*(t)$  is determined from the condition:

$$I_0^* = \min_{u_0 \in U_0 = \bigcap_{j=1}^m U_{0,j}} \left\{ \min_{u_j \in U_j} \min_{u_{0,j} \in U_{0,j}(u_j)} L[x_0(t_k)] \right\} = L^* \quad (5)$$

$$j=1,2,\dots,m$$

The value of control is calculated at each discrete stage of the ship's movement by applying the Simplex method to solve the problem of the triple linear programming, assuming the relationship (5) as the goal function and the control constraints.

#### 3.6 Computer program POSGAM\_NC of multi-stage non-cooperative positional game

Goal function (5) for non-cooperative positional game has the form:

$$I_0^* = \min_{\substack{u_0 \in U_0 = \bigcap_{j=1}^m U_{0,j} \\ j=1,2,\dots,m}} \left\{ \max_{u_j \in U_j} \min_{u_{0,j} \in U_{0,j}(u_j)} L[x_0(t_k)] \right\} = L^* \quad (6)$$

#### 4 COMPUTER SIMULATION

Computer simulation of six programs was carried out in Matlab/Simulink software on an example of the real navigational situation of passing  $m=8$  encountered ships in good visibility  $D_s=0,5$  nm and restricted visibility  $D_s=2,0$  nm (nautical miles) (Figures 2-13).

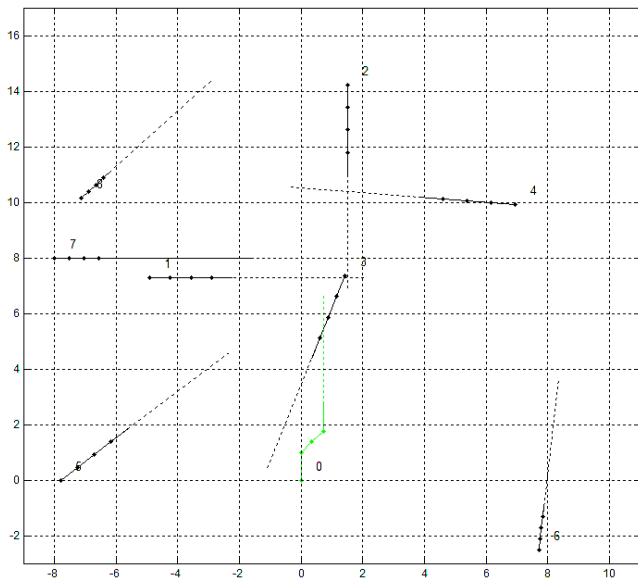


Figure 2. Computer simulation of static optimization STATOPT program determining of the safe own ship trajectory in situation of passing eight encountered ships in good visibility at sea,  $D_s=0,5$  nm,  $d=0,72$  nm (nautical mile).

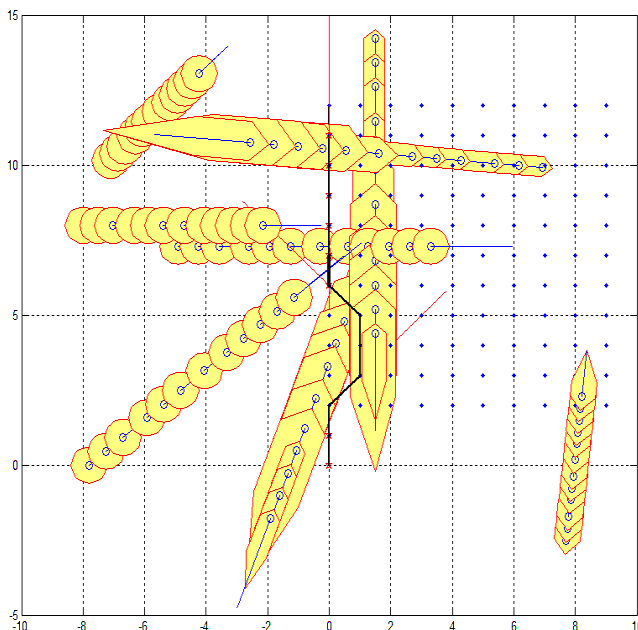


Figure 4. Computer simulation of dynamic optimization DYNOPT program determining of the safe own ship trajectory in situation of passing eight encountered ships in good visibility at sea,  $D_s=0,5$  nm,  $d=0,72$  nm (nautical mile).

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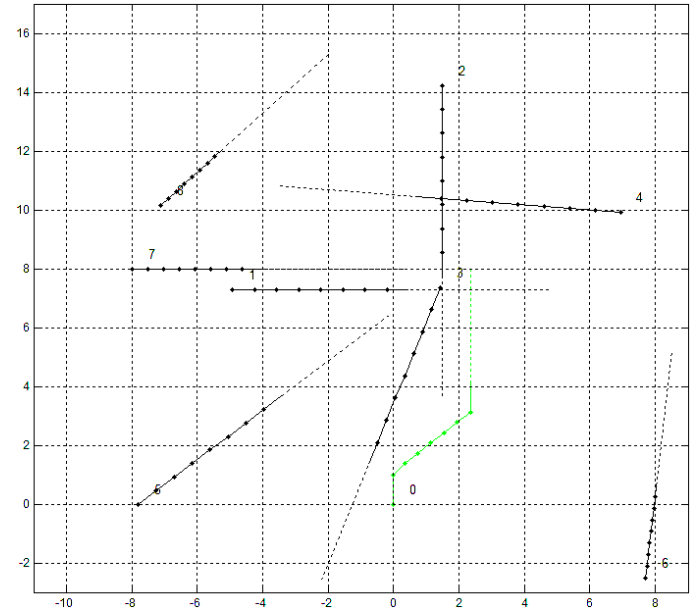


Figure 3. Computer simulation of STATOPT program determining of the safe own ship trajectory in situation of passing eight encountered ships in restricted visibility at sea,  $D_s=2,0$  nm,  $d=2,35$  nm (nautical mile).

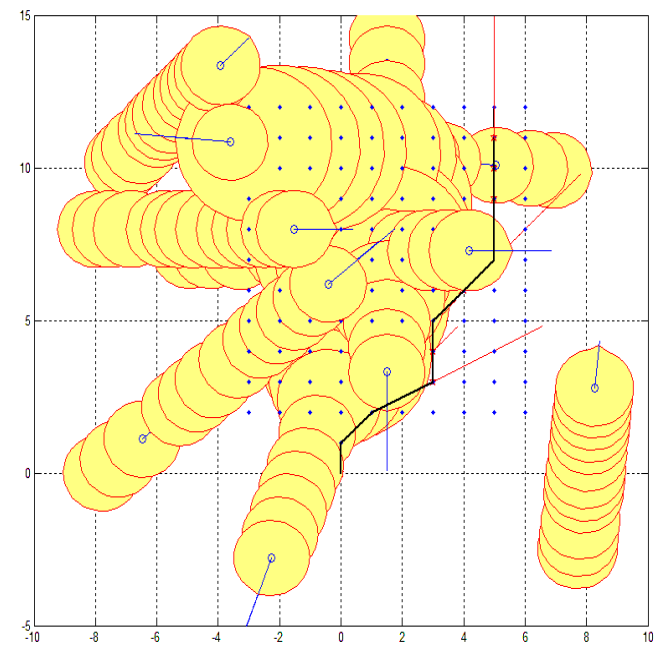


Figure 5. Computer simulation of dynamic optimization DYNOPT program determining of the safe own ship trajectory in situation of passing eight encountered ships in restricted visibility at sea,  $D_s=2,0$  nm,  $d=5,0$  nm (nautical mile).

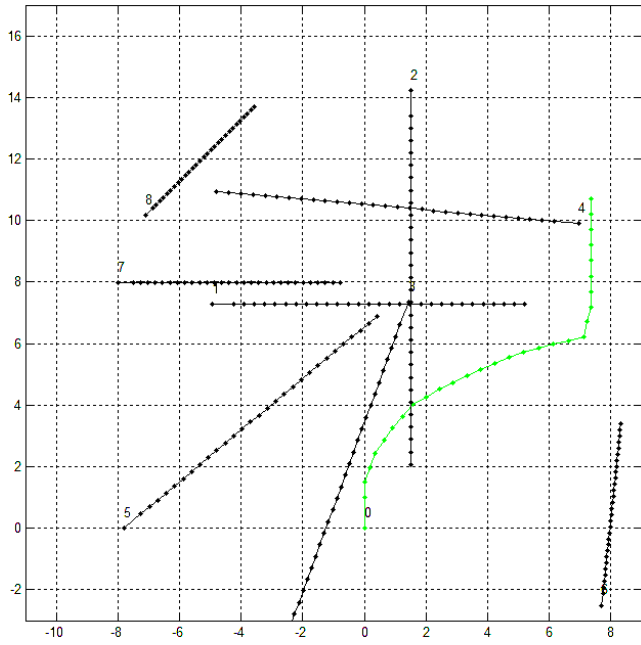


Figure 6. Computer simulation of multi-step cooperative matrix game MATGAM\_C program determining of the safe own ship trajectory in situation of passing eight encountered ships in good visibility at sea,  $D_s=0,5$  nm,  $d_i=7,2$  nm (nautical mile).

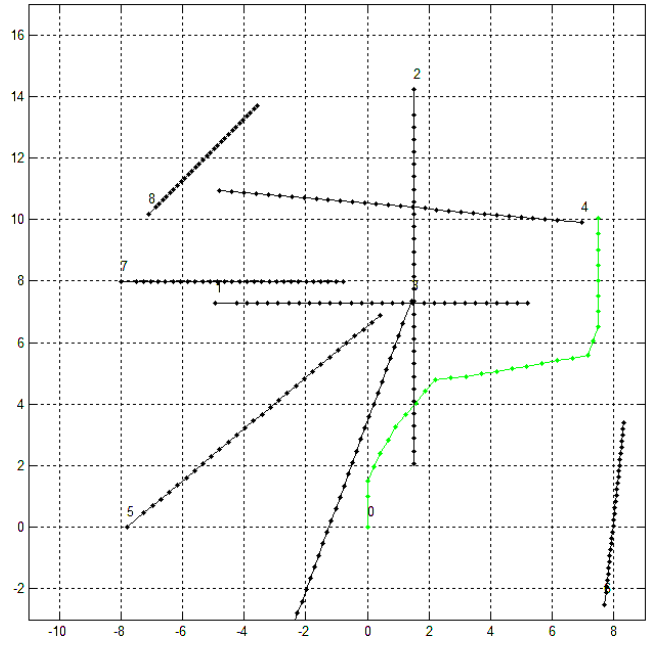


Figure 7. Computer simulation of multi-step cooperative matrix game MATGAM\_C program determining of the safe own ship trajectory in situation of passing eight encountered ships in restricted visibility at sea,  $D_s=2,0$  nm,  $d_i=7,8$  nm (nautical mile).

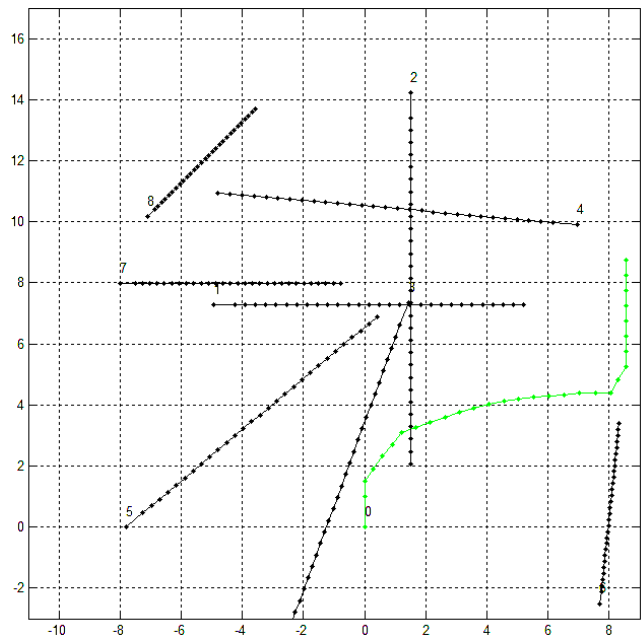


Figure 8. Computer simulation of multi-step non-cooperative matrix game MATGAM\_NC program determining of the safe own ship trajectory in situation of passing eight encountered ships in good visibility at sea,  $D_s=0,5$  nm,  $d_i=8,8$  nm (nautical mile).

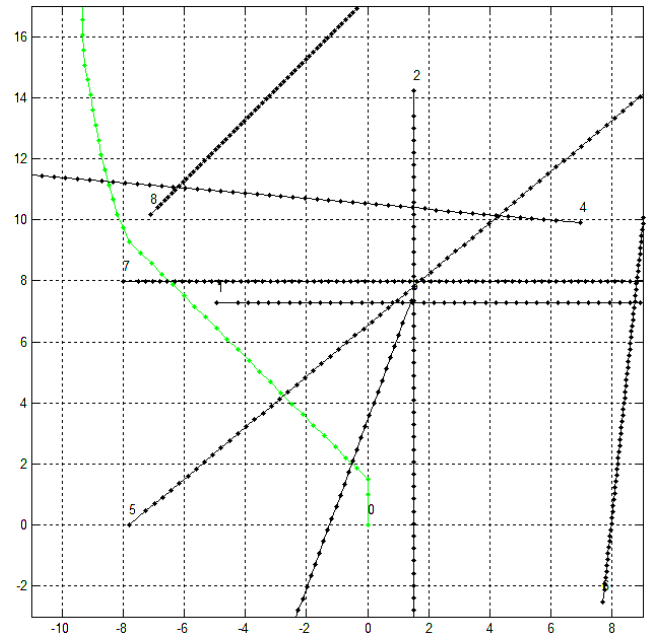


Figure 9. Computer simulation of multi-step non-cooperative matrix game MATGAM\_NC program determining of the safe own ship trajectory in situation of passing eight encountered ships in restricted visibility at sea,  $D_s=2,0$  nm,  $d_i=9,2$  nm (nautical mile).

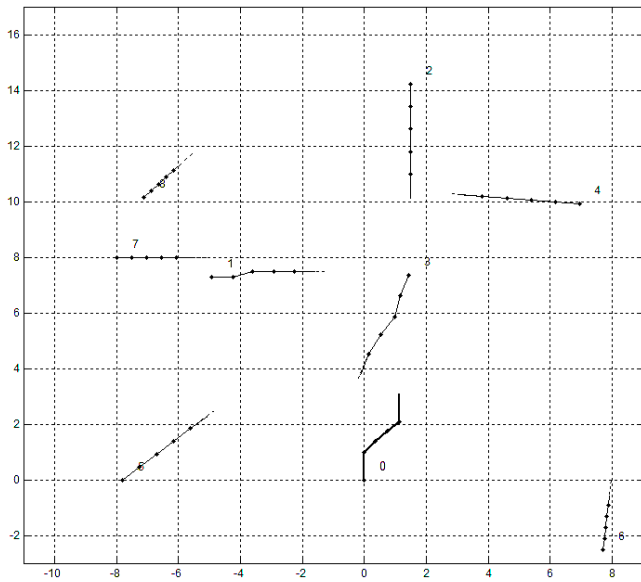


Figure 10. Computer simulation of multi-stage cooperative positional game POSGAM\_C program determining of the safe own ship trajectory in situation of passing eight encountered ships in good visibility at sea,  $D_s=0,5$  nm,  $d=1,14$  nm (nautical mile).

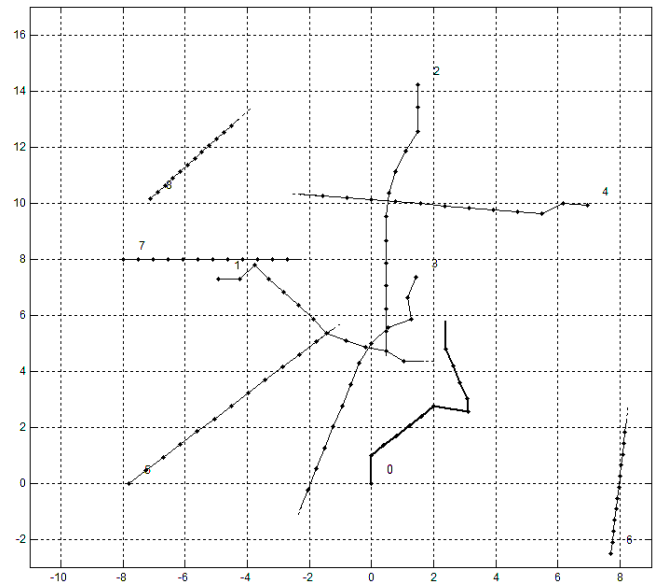


Figure 11. Computer simulation of multi-stage cooperative positional game POSGAM\_C program determining of the safe own ship trajectory in situation of passing eight encountered ships in restricted visibility at sea,  $D_s=2,0$  nm,  $d=2,40$  nm (nautical mile).

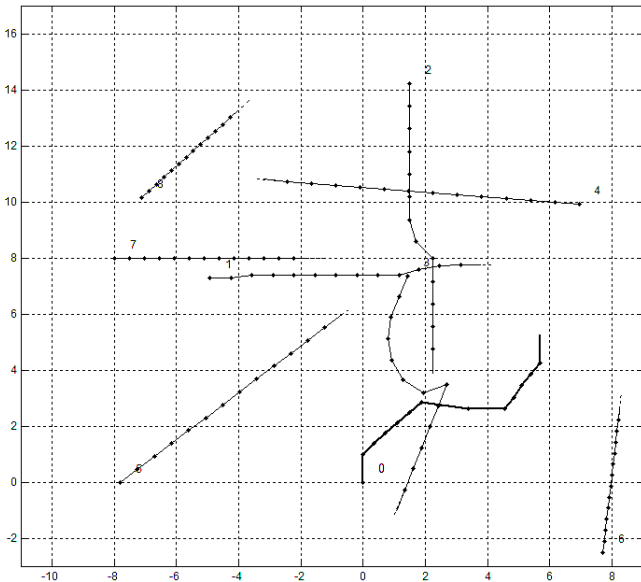


Figure 12. Computer simulation of multi-stage non-cooperative positional game POSGAM\_NC program determining of the safe own ship trajectory in situation of passing eight encountered ships in good visibility at sea,  $D_s=0,5$  nm,  $d=5,68$  nm (nautical mile).

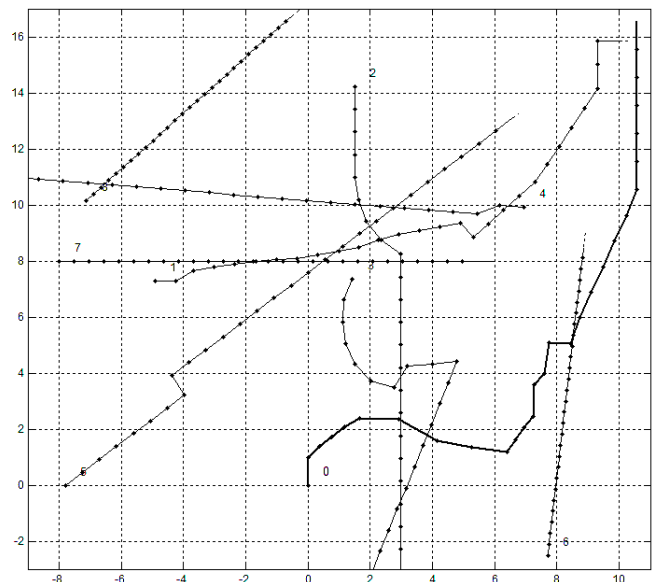


Figure 13. Computer simulation of multi-stage non-cooperative positional game POSGAM\_NC program determining of the safe own ship trajectory in situation of passing eight encountered ships in restricted visibility at sea,  $D_s=2,0$  nm,  $d=10,57$  nm (nautical mile).

## 5 CONCLUSIONS

The synthesis of an optimal and game control on the models of: static, dynamic with neural network, multi-step matrix game and multi-stage positional game makes it possible to determine the safe game trajectory of the own ship in situations when she passes a greater  $j$  number of the encountered objects.

The trajectory has been described as a certain sequence of manoeuvres with the course and speed.

The computer programs designed in the Matlab also takes into consideration the following: COLREG

Rules, advance time for a manoeuvre calculated with regard to the ship's dynamic features and the assessment of the final deviation between the real and reference trajectories.

The essential influence to form of safe and optimal trajectory and value of deviation between real and reference trajectories has a degree of cooperation between own and encountered ships.

The computer programs provides a formal model of the actual decision-making process leading ship navigator and can be used with the system of computer-aided navigator when deciding to manoeuvre in case of collision at sea.

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