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EFFECTS OF NONLINEARITY ON BOLT FORCES FOR THE OPERATIONAL STATE OF A MULTI-BOLTED CONNECTION

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A b s t r a c t

In the paper modelling and calculations of an asymmetrical multi-bolted connection at the operational stage are presented. The physical model of the joint is based on a flexible flange element that is connected with a rigid support by means of linear spring elements, which substitute bolts. Between the flange element and the support, the linear Winkler model of a contact layer is taken into consideration. The multi-bolted system is preloaded and then subjected to an eccentric normal load. Influence of nonlinearity of the contact layer between joined elements on computational values of bolt forces has been investigated. Results of calculations for several different values of the joined element's thickness are pointed out.

Introduction

Multi-bolted connections are systems of many contacting bodies. Characteristics of contact joints are usually nonlinear. Accordingly, multi-bolted connections should be treated as the nonlinear systems. The source of this nonlinearity are also gaskets and washers, which are often used in such joints (BOUZID, CHAMPLIAUD 2003, SAWA et al. 2003). However, the extent of this nonlinearity depends on loads acting on the connection. It can be made an argument that for analyses of a multi-bolted connection at the operational

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stage, the assumption of nonlinearity of the contact layer between joined elements is not recommended. This is possible under the following conditions:
– the multi-bolted connection is preloaded,
– characteristics of the nonlinear contact layer are described as a power function (MISRA, HUANG 2012, WANG et al. 2013) or a polynomial function (KONOWALSKI 2009, YASTREBOV et al. 2011).

In the previous paper (WITEK, GRZEJDA 2005) some results of theoretical investigations of an asymmetrical preloaded multi-bolted connection of a flange and a rigid support, subjected to an external normal load, were released. In the model of the joint, bolts were treated as linear springs and between the flange element and the support, the nonlinear model of the contact layer was taken into consideration. In the current paper some new results of investigations of an analogical model of the joint are presented. In the new model, the linear model of the contact layer between joined elements is taken into account. For modelling and calculations of the multi-bolted connection the finite element method – FEM (ZIENKIEWICZ, TAYLOR 2005) is used.

Physical model of the multi-bolted connection

A general structure of the multi-bolted connection model results from an idea presented in article (WITEK, GRZEJDA 2005). The model of the joint is based on a flexible flange element that is fastened to a rigid support by means of k one-sided linear spring elements (GRZEJDA 2009), which substitute bolts, preloaded by forces F_{mi} (Fig. 1*b*). Spring properties of the i-th bolt's model (for $i = 1, 2, ..., k$) are determined from the formula

$$c_{yi} = \frac{1}{\sum_{n} \frac{1}{c_n}} \tag{1}$$

where:
c_n – denotes the linear stiffness coefficient of the n-th bolt's fragment.

A contact layer between the flange element and the support is modeled as the Winkler model (WANG et al. 2005). This type of the contact layer may be applied in this case, because the considered joint is loaded only by normal forces. The Winkler model of the contact layer is described by means of l one-sided linear spring elements, which are discretely distributed in the contact plane and characterized by the following relationship

$$R_j = A_j \cdot \mathrm{f}(u_j) \tag{2}$$

where:
R_j – the force in the centre of the j-th elementary contact area,
A_j – the j-th elementary contact area,
u_j – deformation of the j-th linear spring element (for $j = 1, 2, ..., l$).

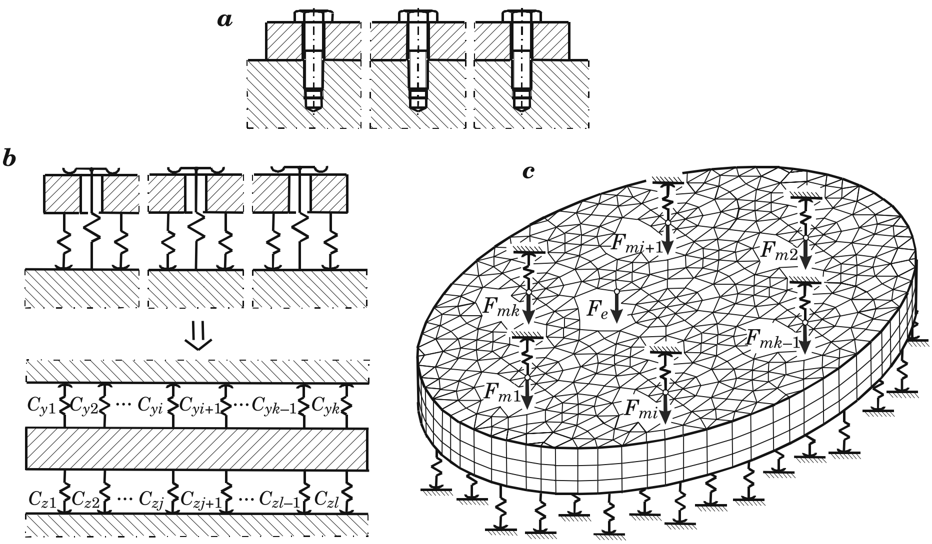

Fig. 1. Multi-bolted connection: *a* – diagram of the joint, *b* – description of system spring properties, *c* – FEM-model of the joint (composed of linear springs, which substitute bolts; the discrete flange element and linear springs, which are elements of the contact layer)

Creation of the finite element mesh at the contact surface of joined elements consists in performing the following steps:

– division of the contact area between joined elements into elementary contact areas,

– determination of the centres of gravity of individual elementary contact areas,

– insertion nodes at the centres of gravity of elementary contact areas,

– addition linear spring elements at the nodes defined in the previous phase.

The equation of system equilibrium (Fig. 1) can be written as

$$\mathbf{K} \cdot \boldsymbol{q} = \mathbf{p} \tag{3}$$

where:
$\mathbf{K}$ – the stiffness matrix,
$\boldsymbol{q}$ – the displacements vector,
$\boldsymbol{p}$ – the loads vector.

In the operational state, the loads vector $\boldsymbol{p}$ is composed of external normal loads F_e (Fig. 1*c*).

The generating procedure of the stiffness matrix **K** is presented in works (GRZEJDA 2009, WITEK, GRZEJDA 2005). Adopting the division of the joint into three subsystems (*B* – the set of bolts, *F* – the flange element model, *C* – the linear Winkler model of the contact layer), (3) can be rewritten in the form

$$\begin{bmatrix} \mathbf{K}_{BB} & \mathbf{K}_{BF} & \mathbf{0} \\ \mathbf{K}_{FB} & \mathbf{K}_{FF} & \mathbf{K}_{FC} \\ \mathbf{0} & \mathbf{K}_{CF} & \mathbf{K}_{CC} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{q}_B \\ \boldsymbol{q}_F \\ \boldsymbol{q}_C \end{bmatrix} = \boldsymbol{p} \tag{4}$$

where:
$\mathbf{K}_{BB}$, $\mathbf{K}_{FF}$ $\mathbf{K}_{CC}$ – are the stiffness matrices of subsystems *B*, *F*, *C*,
$\mathbf{K}_{BF}$, $\mathbf{K}_{FB}$, $\mathbf{K}_{FC}$, $\mathbf{K}_{CF}$ – are the matrices of elastic couplings among subsystems *B*, *F*, *C*.

On the grounds of so defined model of the multi-bolted connection, displacements of bolts and bolt forces after the operational state has been completed can be evaluated.

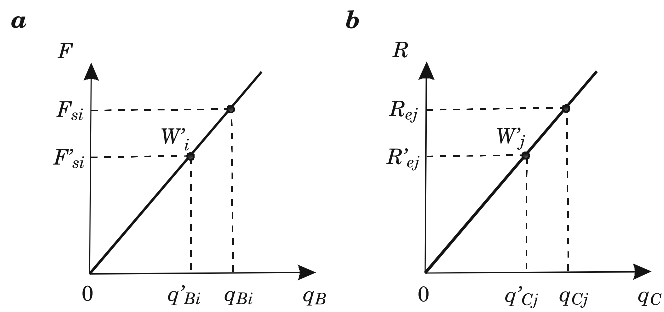

Fig. 2. Determining of the working load: *a* – in the case of linear springs, which substitute bolts, *b* – in the case of linear springs, which are elements of the contact layer

A starting point for calculations of the multi-bolted connection at the operational stage are the data obtained at the end of its assembly operation (GRZEJDA 2013, WITEK, GRZEJDA 2006). The model of the system proposed for the assembly condition makes it possible to analyze how the tightening sequence affects the preload distribution both during the multi-bolted connection's assembly and after it has been completed.

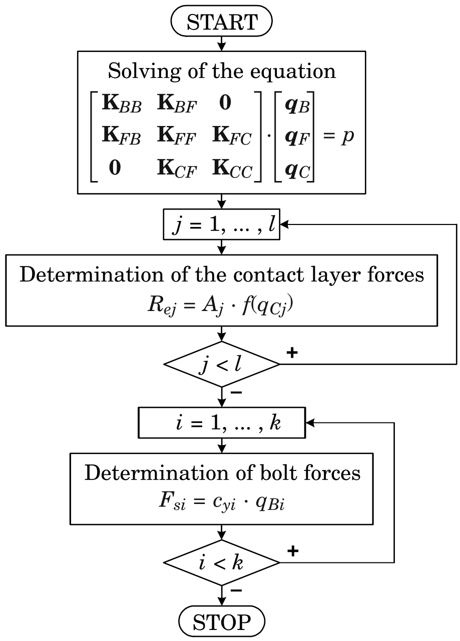

Fig. 3. Block diagram of iterative calculations of the multi-bolted connection

As a result of solving the equation (4) one obtains the displacements vector of bolts $\boldsymbol{q}_B$

$$\boldsymbol{q}_B = \mathrm{col}(q_{B1}, q_{B2}, \dots, q_{Bi}, \dots, q_{Bk}) \tag{5}$$

Final displacements of bolts q_{Bi} are measured from the working points $W_i^{'}$, which determine tension of bolts in the previous step of calculations (Fig. 2*a*). On the basis of so defined displacements q_{Bi}, forces in bolts F_{si} can be computed using the formula

$$F_{si} = c_{yi} \cdot q_{Bi} \tag{6}$$

As a result of solving the equation (4) one obtains the displacements vector of linear springs $\boldsymbol{q}_C$ too, which can be evaluated from the relation

$$\boldsymbol{q}_C = \mathrm{col}(q_{C1}, q_{C2}, \dots, q_{Cj}, \dots, q_{Cl}) \tag{7}$$

Final displacements of elements of the contact layer q_{Cj} are measured from the working points $W_j^{'}$, which determine their tension in the previous step of calculations (Fig. 2*b*). On the basis of so defined displacements q_{Cj}, forces in the contact layer R_{ej} can be computed from the relation (2) for u_j equal to q_{Cj}.

The diagram of iterative calculations of the multi-bolted connection is shown in Figure 3.

Calculations of the multi-bolted connection at the operational stage

According to the presented method, computations of an asymmetrical multi-bolted connection were performed. A simplified FEM-based model of the joined element is shown in Figure 4*a*. A contact surface between joined elements as well as the bolt's arrangement and their numeration are shown in Figure 4*b*.

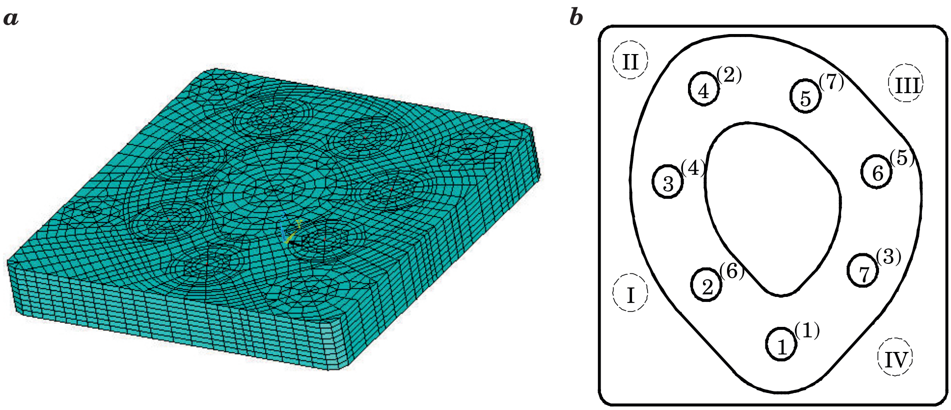

Fig. 4. Considered multi-bolted connection: *a* – simplified FEM-based model, *b* – joint characteristic

Calculations were carried out for three values of the joined element's thickness h (for $h \in$ {20 mm, 40 mm, 80 mm}). Characteristics of linear contact springs are described as the following function (GRZEJDA 2009)

$$R_j = A_j \cdot (26.873 \cdot u_j) \tag{8}$$

To fastening of the joint, the bolts M10×1.25 were used. The standard preload of bolts F_{mi} is equal to 20 kN. The tightening sequence taken here on (GRZEJDA 2009), is parenthesized in Figure 4*b*. After the preloading process, the multi-bolted connection is subjected to an eccentric normal load F_e equal to 50 kN acting consecutively at four points (I, II, III, IV) shown in Figure 4*b*.

Calculations were realized in ANSYS software. To build the model of the multi-bolted connection, the following finite elements were applied:
– link elements, which substitute bolts,
– solid elements, to model the flange element,
– link elements with characteristics consistent with the formula (8), to model the contact layer.

The number of elements used for modelling of the multi-bolted connection as a function of the joined element's thickness is set up in Table 1.

Table 1

Size of the FEM-models as a function of the joined element's thickness

Subsystem	Number of elements		
	h = 20 mm	h = 40 mm	h = 80 mm
B	7	7	7
F	19,256	39,396	39,396
C	664	664	664

Results of calculations were put together in graphs illustrated in Figure 5 and Figure 6 according as both the individual thickness of the flange element h and four points of the load F_e acting. In the respective figures, values of bolt forces F_{si} related to preloads F_{mi} (WITEK, GRZEJDA 2006) are presented. For describing the results of calculations a following nomenclature is introduced:
– the L-model – the model of the joint with the linear Winkler model of the contact layer,
– the NL-model – the model of the joint with the nonlinear Winkler model of the contact layer (WITEK, GRZEJDA 2005).

The shape of the figures is nearly symmetrical around the points of the load F_e acting. This symmetry is determined by the nature of the adopted type of the contact layer, which takes into account only properties of the contact in the normal direction. Significant increases in the bolt forces in bolts lying closest to the points of external loading are related to high values of local contact pressure in this part of the joint.

In most of the cases, values of forces in individual bolts computed according to the L-model of the joint are smaller than their values obtained according to the NL-model of the joint. Analysis of relative difference between obtained bolts forces is done on the basis of the W index

$$W = \left| \frac{F^{L}_{s\,\max} - F^{NL}_{s\,\max}}{F^{NL}_{s\,\max}} \right| \tag{9}$$

where:
$F^{L}_{s\,\max}$ – the maximal force in the bolt according to the L-model of the joint,
$F^{NL}_{s\,\max}$ – the maximal force in the bolt according to the NL-model of the joint.

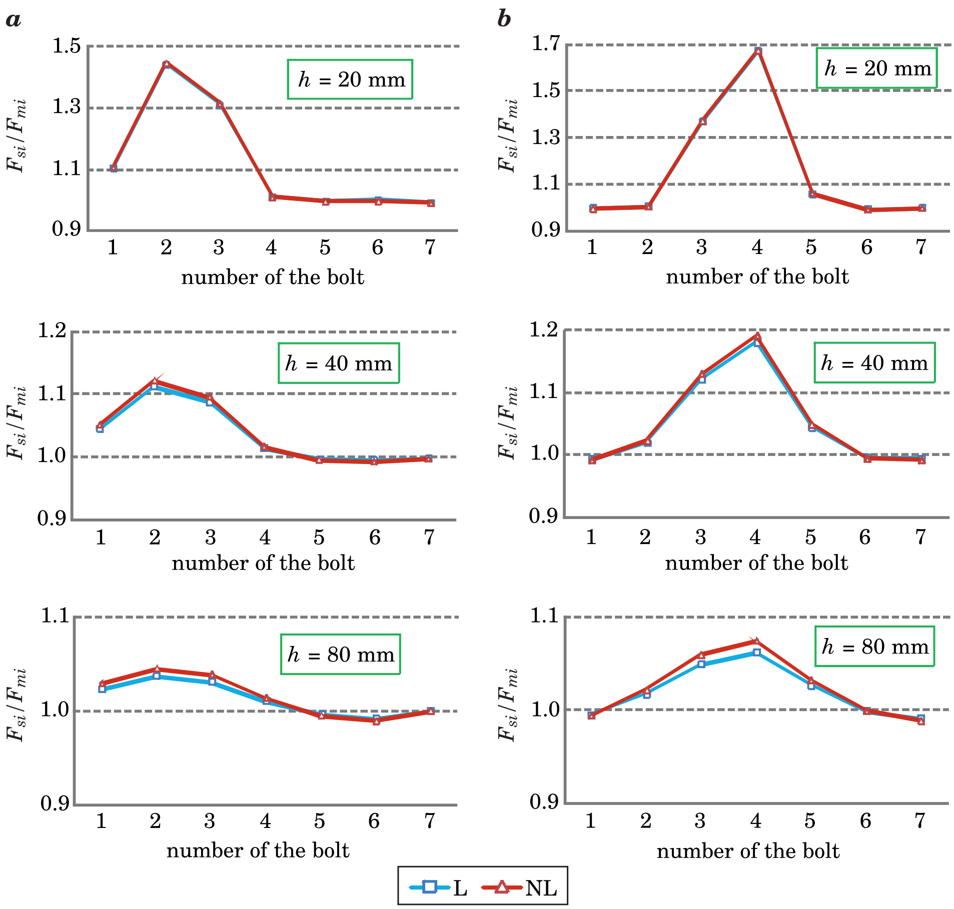

Fig. 5. Bolt load values in the joint externally loaded at the point: *a* – I, *b* – II

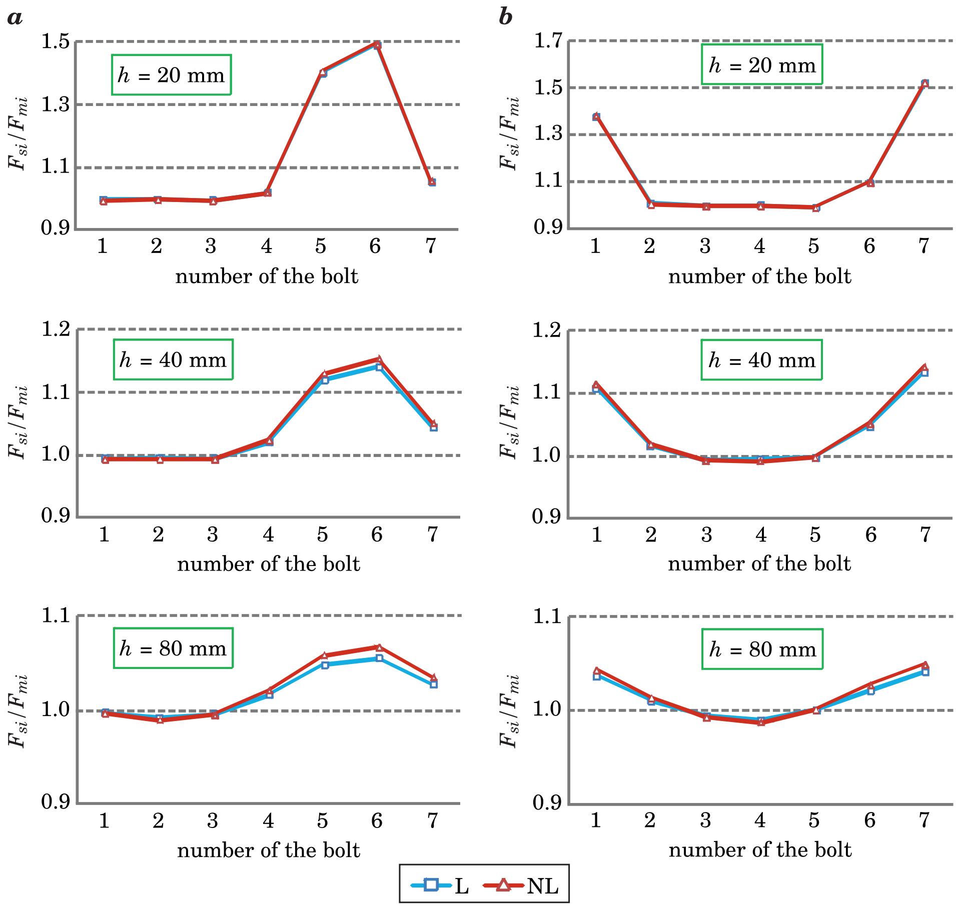

Fig. 6. Bolt load values in the joint externally loaded at the point: *a* – III, *b* – IV

Table 2

W index values as a function of the joined element's thickness

	h = 20 mm	h = 40 mm	h = 80 mm
	Point No. I		
	0.41	0.81	0.78
	Point No. II		
W [%]	0.28	0.87	1.10
	Point No. III		
	0.36	1.05	1.12
	Point No. IV		
	0.34	0.68	0.75

W index values as a function of the thickness of the joined element are set up in Table 2. On the grounds of comparisons, it can be noted that the accuracy of calculations results by means of the linear multi-bolted connection model decreases with reduction of joined element's flexibility. Depending on the joined element's thickness, the error of estimation may range from 0.28 to 1.12%.

Conclusions

In the case of multi-bolted connections preloaded pursuant to technical standards and then subjected to an eccentric normal load, nonlinearity of the contact layer between joined elements has negligible influence on computational values of bolt forces. Wherefore for analyses of such joints, the linear Winkler model of the contact layer can be applied. Owing to this fact, one achieves significantly higher efficiency of the modelling, which is brought on both the smaller complexity of the problem and considerably shorter process time.

The block construction of the multi-bolted connection model allows to expand its applicability. This is possible by modifying individual subsystems B, F or C. In the case of using this model to analyses of the joint loaded by an arbitrary force, it would be necessary to apply a different type of the contact layer model.

The presented model of the multi-bolted connection can be successfully used also in load capacity analysis of any joint in which a flexible flange element is connected with a rigid support (for a review, see DOMINIKOWSKI, BOGACZ 2009).

References

BOUZID A.H., CHAMPLIAUD H. 2003. *On the use of dual kriging interpolation for the evaluation of the gasket stress distribution in bolted joints*. Analysis of bolted joints, Proc. of the 2003 ASME Pressure Vessels and Piping Conference, Cleveland, Ohio, p. 77–83.

DOMINIKOWSKI S., BOGACZ P. 2009. *Determination of internal forces in end plates of simple end plate joints*. Technical Sciences, 12: 83–94.

GRZEJDA R. 2009. *Modelling and analysis of a multi-bolted joint of a flange element fastened to a nonlinear spring layer*. Doctor's thesis, West Pomeranian University of Technology, Szczecin.

GRZEJDA R. 2013. *Effects of nonlinearity of a contact layer between elements joined in a multi-bolted connection on preload values in bolts*. Bulletin of the Military University of Technology, article in press.

KONOWALSKI K. 2009. *Experimental research and modeling of normal contact stiffness and contact damping of machined joint surfaces*. Advances in Manufacturing Science and Technology, 33(3): 53–68.

MISRA A., HUANG S. 2012. *Micromechanical stress-displacement model for rough interfaces: Effect of asperity contact orientation on closure and shear behavior*. International Journal of Solids and Structures, 49(1): 111–120.

SAWA T., IWAMOTO T., FUNADA K., OMIYA Y. 2003. *FEM stress analysis and sealing performance in pipe flange connections with gaskets subjected to external bending moment (case where internal fluid is liquid)*. Analysis of bolted joints, Proc. of the 2003 ASME Pressure Vessels and Piping Conference, Cleveland, Ohio, p. 85–95.

WANG L., LIU H., ZHANG J., ZHAO W. 2013. *Analysis and modeling for flexible joint interfaces under micro and macro scale*. Precision Engineering, 37(4): 817–824.

WANG Y.H., THAM L.G., CHEUNG Y.K. 2005. *Beams and plates on elastic foundations: a review*. Progress in Structural Engineering and Materials, 7(4): 174–182.

WITEK A., GRZEJDA R. 2005. *Analysis of a nonlinear multi-bolted joint loaded by a normal force*. Archives of Mechanical Technology and Automation, 25(2): 211–219.

WITEK A., GRZEJDA R. 2006. *An analysis of the preload of a nonlinear multi-bolted joint*. Archives of Mechanical Technology and Automation, 26(2): 271–280.

YASTREBOV V.A., DURAND J., PROUDHON H., CAILLETAUD G. 2011. *Rough surface contact analysis by means of the Finite Element Method and of a new reduced model*. Comptes Rendus Mécanique, 339(7–8): 473–490.

ZIENKIEWICZ O.C., TAYLOR R.L. 2005. *The finite element method for solid and structural mechanics*. Elsevier Butterworth-Heinemann, Oxford.