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INSIGHT INTO VIBRATION SOURCES IN TURBINES

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Abstract

Despite of nearly 100 years of turbine engine development and design, blade vibrations remain a great engineering challenge. The rotating turbine blades' vibrations lead to cyclic oscillations, which result in alternating stress and strain in harsh environments of high temperature and pressure. In modern aeroengines, high hot flow velocities might generate erosion and corrosion pitting on the metal surfaces, that leverage remarkably mean stresses. The combination of both mean and alternating stresses can lead to unexpected engine failures, especially under resonance conditions. Then, alternating stress amplitudes can exceed the safety endurance limit, what accelerates the high cyclic fatigue leading quickly to catastrophic failure of the blade. Concerning the existing state-of-the-art and new market demands, this paper revises forced vibrations with respect to excitation mechanisms related to three design levels: (i) a component like the blade design, (ii) turbine stage design consisting of vanes and blades and (iii) a system design of a combustor and turbine. This work reviews the best practices for preventing the crotating turbine and compressor blades from High Cyclic Fatigue in the design process. Finally, an engine commissioning is briefly weighed up all the pros and cons to the experimental validations and needed measuring equipment.

Keywords: vibrations, blade, gas turbine, turbine engine, strain gauge, tip timing, Additive Manufacturing
Article Category: Research Article

INTRODUCTION

Since the beginning of turbojet engines, the fatigue failure of blades caused by vibrations was a major challenge in engine operations [12]. Until today this problem has not been fully solved as the High Cycle Fatigue (HCF) accounts for most of turbojets' failures. Nearly 90% of HCF problems are solved during the engine development phase but the remaining 10% of HCF damages generates about 30% of all maintenance costs [4]. A typical jet engine consists of an air intake, compressor, combustor, turbine, and an exhaust diffuser (Fig. 1). To leverage the HCF damage avoidance above 90%, the paper revises the excitations mechanisms in terms of (i) a component design of a single bladed disc, (ii) sub-system design of a turbine stage consisting of vanes and a bladed disc and (iii) system design level of the entire turbine including a combustor and exhaust diffusor, as illustrated in Fig. 1. In addition, this work considers the impact of manufacturing tolerances of the traditional processes like forging and casting, as well as the 3D additive manufacturing on vibrations of the bladed disc.

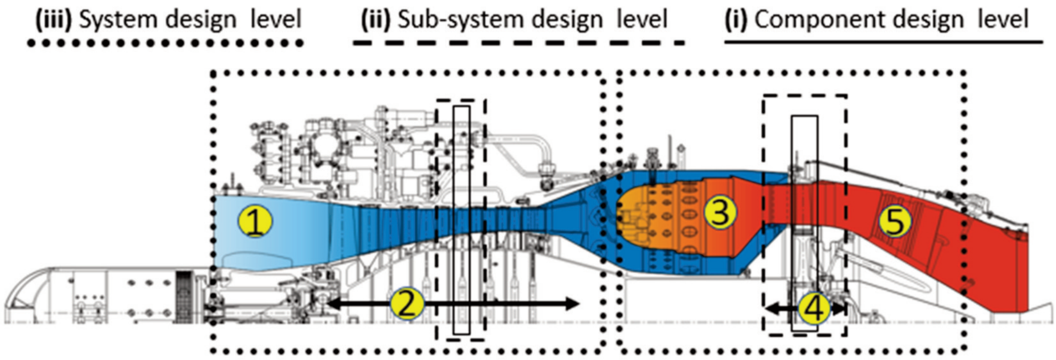


Figure 1. The cross-section view of K-15 jet engine consisting of (1) air intake, (2) compressor, (3) combustor, (4) turbine, and (5) exhaust diffuser.

At the idle, cruise, and take-off speeds and their thermal conditions, each rotating component of the engine must be safely designed regarding the fatigue endurance of alloys of the blade and disc (see the component design level in Fig. 1). While the mechanical loading can be predicted with a high reliability, metal temperatures are usually determined with some numerical uncertainties depending on flow turbulences characteristic of different operation conditions. The thermal conditions change blade temperatures directly impacting alloy properties, e.g. Young Modulus. Therefore, the blade frequencies vary and might be predicted wrongly. Thus, the prediction of proper thermal boundary conditions upon the blade and disc is a crucial task in the design process aimed at achieving the failure-free HCF regime. These temperature distributions on the blade and disc are determined in a traditional way with the body and fluid analyses performed separately. Nowadays, for a highly cooled blade, the Conjugated Heat Transfer (CHT) method, in which the body and fluid analysis are combined on their interface as a coupled solution, has become a required engineering process for predicting reliable thermal boundaries acting upon the rotating blade.

BLADED DISC VIBRATIONS AND RESONANCE CONDITIONS

In the design process of a rotating turbine, the blade is modelled with an elastic disc (see the component design level in Fig. 1) through frictional connections on the fir tree or other root types. For the most reliable metal temperatures obtained from the CHT analysis, the overall static deformations of the blade and disc are calculated for take-off conditions. This FE computation delivers the most critical mean stresses in the blade and disc.

For the computed overall static deformations and mean stresses, the frequencies of the rotating, freestanding blade are calculated. Until the late 1990s, the rigidly clamped blade was considered and there are different elastic oscillations along the radial direction of the freestanding airfoil, which are defined with number $i = 1, 2, 3, 4, \ldots, \infty$ (Fig. 2) as the airfoil mode number. In the case of a thin forged disc such as in Fig. 1, besides the radial airfoil modes i, circumferential oscillations of the bladed disc must be taken into account, as a result of nodal diameter n of the disc elasticity, where $n = 0, 1, 2, \ldots, n^*$ for even $n^* = N/2$ or odd $n^* = (N-1)/2$ number of N blades in the turbine row.

Thus, instead of one frequency f_i of the rigidly clamped blade, the bladed disc vibrations of mode i consists of n eigenfrequencies $f_{i,n}$ of the rotating disc assembly as shown in the Nodal Diameter diagram in Fig. 2. By computing the Nodal Diameter diagrams for different rotational speed Ω, the Campbell diagram is numerically created to recognize potential harmonic excitations (see k harmonic in Fig. 2). Although the bladed disc resonances can be identified in the Nodal Diameter diagram, the Campbell diagram is often used for direct comparison with the measured resonances with a tip timing system or/and strain gauges and the telemetry system.

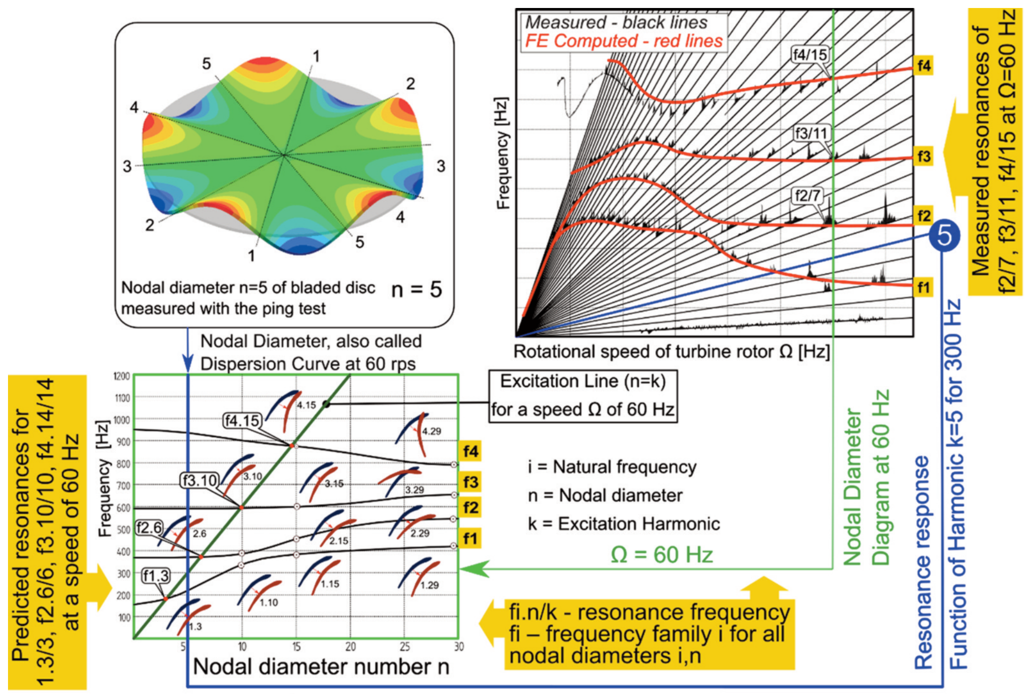


Figure 2. Fundamental tools in prediction of resonances of the rotating blade disc for the component design level.

For the resonance case of the disc assembly, there are two fundamental conditions. The first one says that the excitation frequency is equal to disc frequency $f_{i,n}$ (see Fig. 2). And the other one requires that the excitation form of oscillating pressure $p(\varphi,\upsilon)$ along the circumferential direction equals the nodal diameter number n of vibrating disc assembly (e.g. $k= n= 5$ as shown in Fig. 2). Then, the excitation energy is fed into the rotating bladed disc and the engine experiences the resonance condition. Thus, the rotating blade must be designed as the resonance free or the resonance proof component under circumstances of the variable rotational speed and varying thermal conditions.

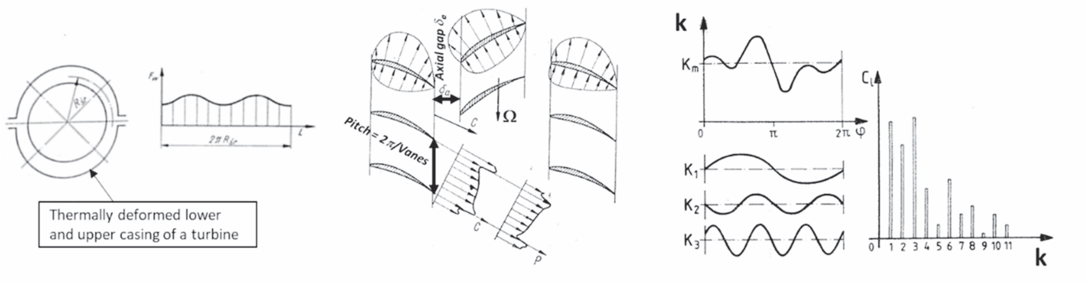
EXCITATION RANGES AND MECHANISMS

In the correctly designed and manufactured engine, only resonance vibrations of a rotating blade may be critical for HCF failure-free operation. Due to a wide spectrum of excitation forces, total avoidance of the resonances is impossible to achieve, especially for variable rotating speeds of the jet engine. Thus, two design concepts of either frequency tuning or resonance proof must be taken into account concerning the known sources of excitations divided into three major groups:

a) Low Frequency Range (LFR) of excitation sources from the 1st up to 5th–7th harmonics (depending on the design philosophy of a specific company) due to thermal ovalization of a turbine casing, asymmetrical gas flow, differences between upstream and downstream flow, number of structs of the air-intake and exhaust diffuser, and rotor-blade interaction,
b) Middle Frequency Range (MFR) depending on a number of burners, clocking effects by using the same number of vanes in a compressor or/and turbine, number of cans, a number of struts above 7,
c) High Frequency Range (HFR) as wake interaction and non-uniform pressure distribution on the rotating blade with respect to numbers of upstream and downstream Nozzle Guide Vanes (NGV).

Figure 2 illustrates the LFR of interest, where the computation of the eigenfrequency f_4 of the bladed disc overestimates the measured resonances (see black curves) because of too coarse of FE mesh representing the blade.

During the engine operation, the flow is non-uniform along the circumferential direction due to medium temperature, density, and pressure differences. In addition, the casing of the turbine is thermally deformed, which generates the non-uniform pressure distribution of engine harmonic $k = 2$ such as illustrated in Fig. 3a, which corresponds to LFR. Another strong excitation mechanism of HFR is the upstream and downstream flow acting on the rotating blade (Fig. 3b). In this case, the ratio of the axial distance da to the vane pitch $2\pi/N_V$ (where N_V means a number of vanes) shall be large enough for reducing excitation amplitudes on the rotating blade that are determined with Fast Fourier Transformation (Fig. 3c). Indeed, modern engines are designed with this ratio being decreased to meet market demands of the high energy density.



a) Excitation LRF of k = 2

b) Excitation HRF of upstream and downstream, where c & P mean flow velocity and pressure

c) FFT (Fast Fourier Transformation) applied to non-uniform pressure fordetermining LRF excitation amplitude C for harmonic k = 1, 2, …, 11

Figure 3. The fundamental excitation mechanisms caused by (a) thermal ovalization and other geometrical offsets between the upper and lower housings of the turbine, (b) upstream and downstream flow, and (c) analytical scheme for determining excitation harmonic *k* with FFT applied to the measured or computed circumferential flow distribution [15].

COMPUTATIONAL HIGH CYCLIC FATIGUE

Back in the 1990s, the resonance amplitude of the fundamental first mode of a rotating blade was determined with [17]:

$$\sigma_A = \sigma_{BN} \chi \frac{sP}{2\xi}, \tag{1}$$

where σ_A and σ_{BN} denote the resonant and bending stresses of the blade respectively, while the coefficient χ determines the degree of mechanical coupling between the blades through the elastic disc or/and additional elements such as a shroud or a winglet. The damping properties of the blade are determined by means of the empirical constant damping ratio ξ, whereas the empirical coefficient s defines the blade excitation as the percentage of the constant pressure P of gas or air acting on the rotating blades of the turbine or the compressor, respectively.

Besides the intensity (level) "$s\,P$" of the excitation, the resonance amplitude depends on damping and the system excitability χ defined as a phase relation of the excitation pressure and the vibration forms. Based on the Finite Element Method (FEM) and unsteady Computational Fluid Dynamics (CFD) the resonance stress amplitude $\sigma_{A,i,n}$ is calculated with a more precise manner as [15]:

Excitability of the bladed disc

$$\sigma_{A,i,n} = \left(\sigma_{real,i,n} + j\sigma_{imag,i,n}\right) \frac{\left\{\phi_{i,n,complex}\right\}^{*T} \left\{P_{v,complex,k=n}\right\}}{\left[\left(f_{i,n}^2\right)^2 + \left(2\left(\xi_{i,n,u} + \xi_{i,n,a}\right) f_{i,n}\,\upsilon\right)^2\right]^{1/2}}. \tag{2}$$

Frequency tuning in the Campbell diagram (Fig. 2)

Energy dissipation for the bladed disc

where j is the unit imaginary number, $\xi_{i,n,}$ and $\xi_{i,n,a}$ mean the overall frictional damping generated e.g. by the under-platform damper and aerodynamic damping of mode shapes i,n of the bladed disc. The excitation frequency υ is equal to $k\Omega$ where Ω is the rotational speed for harmonic excitations, $\phi_{i,n}$ denotes the vibration of the bladed disc representing with two travelling waves along the circumferential direction of the frequency i oscillating with a nodal diameter n, as illustrated for n = 5 in Fig. 2 as the standing wave.
In general, the alternating stresses depend on the three key drivers:

a) excitability between the mode shape of the blade disc and excitation form of the oscillating pressure as predicted in the unsteady or Harmonic Balance CFD analyses,
b) frequency turning between the eigenfrequency $f_{i,n}$ of the blade disc and excitation frequency υ, that are realized with the Finite Element Analysis,
c) the overall damping, especially obtained from frictional under-platform dampers [16], while the material damping does not exceed 0.02% of the Ni-based alloy and aero-dynamic damping is below 0.3% of the modal damping ratio [17]. Otherwise, too high aerodynamic damping values denote about not efficient stage performance for the design airfoil profile.

EXCITATION SOURCES

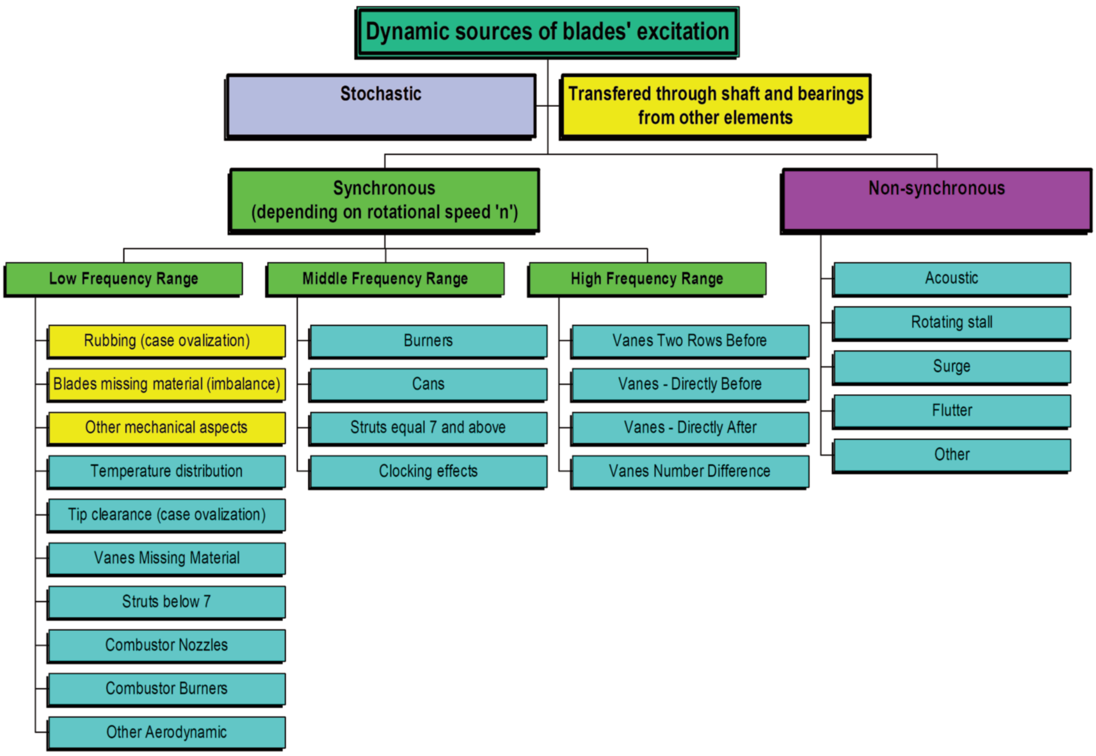


Figure 4. Systematic classification of excitation sources [9].

In relation to frequency, excitations of the rotating bladed disc can be divided into:

- synchronous,
- asynchronous,

- coming from other engine elements,
- stochastic vibration of the whole engine and flow excitation.

Predominant are the synchronous vibrations, in which the frequencies are multiples of the engine rotational speed. In Figure 4, the excitation sources are summarized with respect to the component, sub-system and system design levels. Another classification could be made taking into account the nature of the phenomenon:

- flow (blue-marked) – sourcing from changes in the flow of the working fluid – aerodynamic footprint, turbulence, temperature fields, differences in pressure and velocity along the circumference, change in the blade tip gap around the circumference, acoustic effects, instability of the flow,
- mechanical (yellow-marked) – transferred through the disc among other engine components, frictional rubbing of the rotating blade tip against the engine case, rotor and blade structure interaction, dynamic coupling between the generator and the turbine, rotor imbalance.

Excitation Sources at Component Design Level

With respect to the component design of a single bladed disc, mistuning effects must be considered in detail. In reality, the bladed disc is a system of N blades, whose geometries slightly differ from each other due to manufacturing tolerances, resulting in the mistuning effect. In the design process, the blade mistuning is not a deterministic quantity. In practice, the Whitehead factor as an empirical measure of amplification of the alternating stress can be used for the design safety.

The Root Cause Analysis was performed for the cracked compressor blade [6] with respect to erosion effects and mistuning. The mean and alternating stresses at the cracked location are enlarged with the fatigue notch factor N_f from the specimen testing and empirical mistuning factors of Whitehead $\theta_W = \left(1+\sqrt{n^*}\right)/2$ [18], of Kenyon-Griffin $\theta_{K-G} = \left(1+\sqrt{n^*/2}\right)/2$ [7], and $\theta_H = \sqrt{N}$ [5] to the alternating stress in the Haigh diagram (Fig. 5.). The alternating stresses amplified with Whitehead factor correspond well the experimental data measured with tip timing.

In the well-established design practice, mistuning of the blades is assessed through the frequency scatter of all blades in the turbine row measured with the ping test [10, 11]. For the fundamental mode 1, the nominal frequency f_1 of the rigidly clamped blade at the ping tests might vary ±1.5% or even ±2.5% in the casting and forging process, respectively. In the 3D Additive Manufacturing (AM), the repeatability of the blade geometry is almost perfect as demonstrated in Figure 6 for two examples of gas turbine blades obtained from AM production. Therefore, the AM-ed blades weakly coupled circumferentially with a light and elastic disc, mistuning impact on the alternating stresses of the bladed disc shall be analyzed in detail. In addition, the forced responses of these blades should be measured with the Tip Timing system to measure resonance amplitudes of each blade regarding the amplification of alternating stresses in the Haigh diagram (Fig. 5).

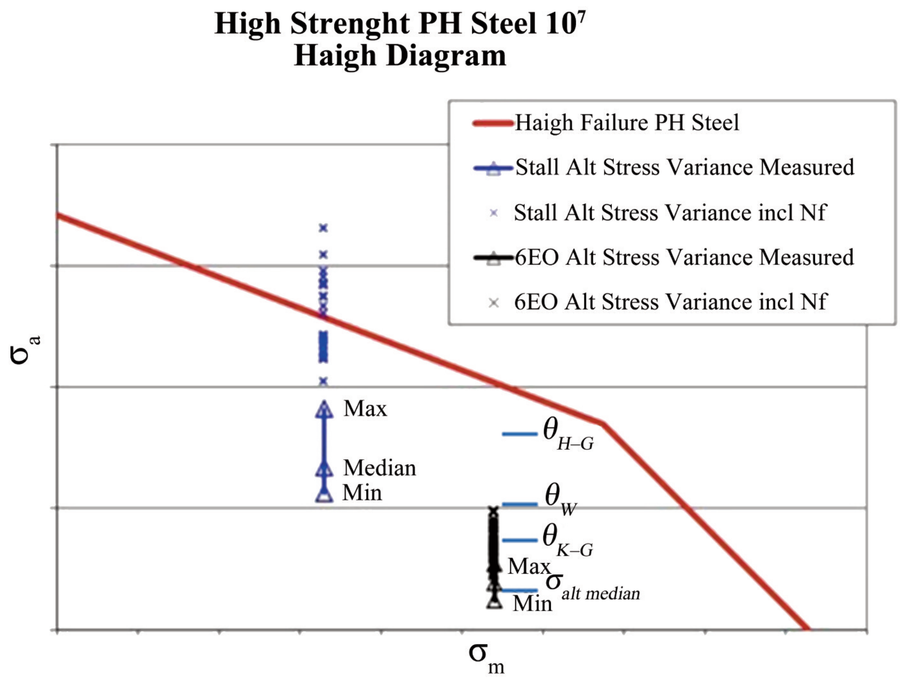


Figure 5. The measured Compressor Blade Haigh Diagram for smooth contour with erosion effects included, where the empirical mistuning amplification factors are added for reference [6].

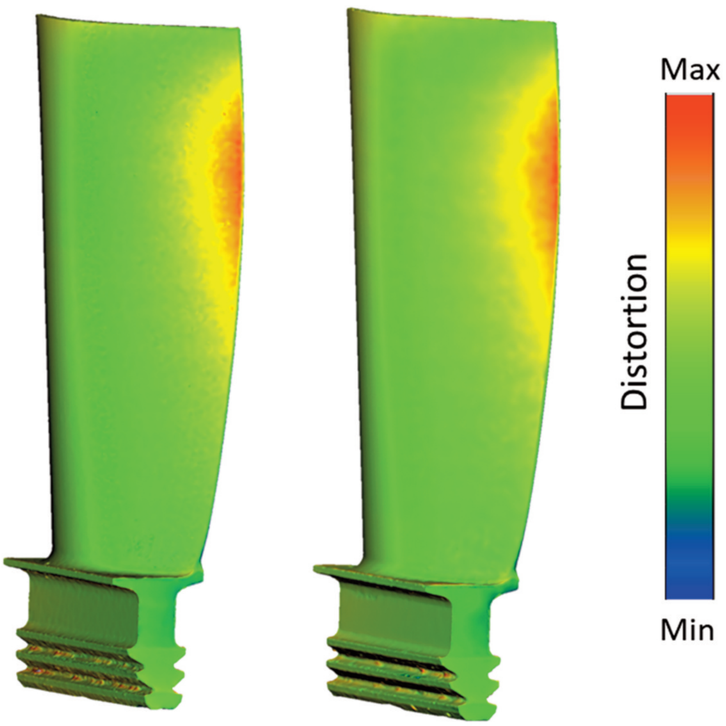


Figure 6. Optical scanned geometries of two, uncooled gas turbine blades produced with 3D Additive Manufacturing showing almost identical geometries.

Non-synchronous vibrations are related to the component safety design and do not directly depend on the rotational speed of the engine. The blades are excited by the coherent instability of the flow occurring at certain engine operation conditions. There is also a risk of HCF failure if the non-synchronous excitation frequency is close to a natural frequency of the blade and a resonance occurs. In the case of a turbine, two non-synchronous vibration sources, flutter can be dangerous.

- Flutter – self-excited vibrations of low frequency occurring within the flow of the fluid close to the so-called critical flutter speed [8]. A reduced frequency is the parameter used for flutter risk assessment; it is defined as the Strouhal number S_T as:

$$S_T = \frac{c_{prof} \cdot \omega}{2 \cdot \dot{u}_{g-p}}. \tag{3}$$

This parameter can be interpreted as a ratio of time required for a fluid particle to pass a distance of blade chord half c_{prof} with velocity $\dot{u}_{g-p}$ and time equal to one vibration period (Fig. 7).

The above formula may be presented in a modified way, providing the frequency in Hertz rather than the angular frequency ω [rad/s] and a total profile chord instead of its half, changing the parameter values and attention must be paid at data analysis.

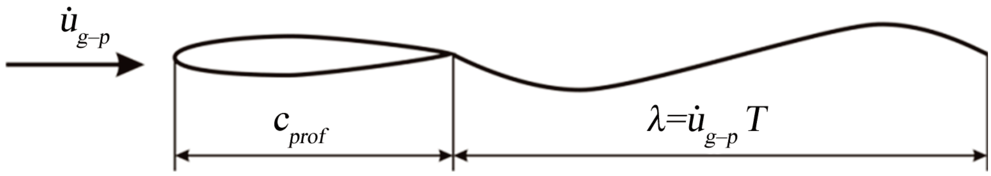


Figure 7. Interpretation of the reduced frequency.

In practice, blades flutter occurs with the first bending mode shape if reduced frequency is below 0.4, and for the first torsional mode shape for values in range of 0.4 and 0.7 [14]. In the design process, if the initial criterion is not met, more sophisticated analyses are performed, e.g., numerical FEA with fluid-structure coupling.

In the turbine, flutter affects mainly the blades of last stages, due to their low natural frequencies caused by dimensional properties [2]. The risk of flutter can be diminished by means of increasing the natural frequencies through applying a shroud, snubbers or welded joints.

In the compressor, additional asynchronous excitations may occur. Their roots are in a flow:

- Surge – occurs in axial and radial compressors during operation above the maximum pressure. Surge is an adverse phenomenon as it generates powerful pulsations of pressure within the fluid, resulting in vibrations of the engine structure, and of the blades in particular. It can lead to permanent blades damages and serious engine outages.
- Rotating stall – local separation of the flow from the blade airfoils can accumulate into local rotating stall regions. They can move around the gas-path circumference with a rotating speed lower than the engine rotating speed causing excitations of the blades (Fig. 8.)

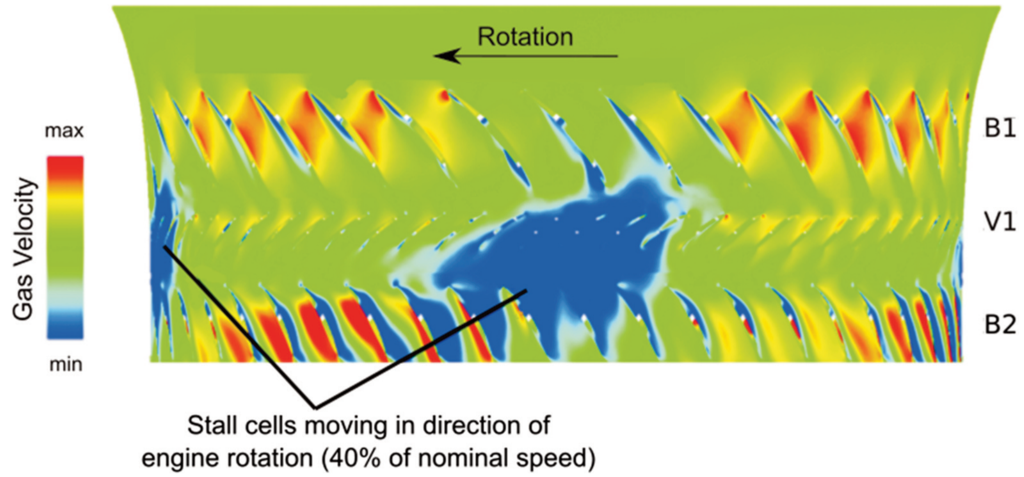


Figure 8. Flow velocity in compressor during rotating stall [3].

The rotating stall can evolve into a surge. Both the phenomena can occur within the compressor but, indirectly, they also affect the turbine blades. Flow instability of the air during compression passes into the combustor, and the turbine, subsequently. Additionally, the compressor vibrating motion is transferred via the engine structure and through the rotor shaft and the bearings and this way affects the turbine blades.

Excitation Sources at Sub-System Design Level

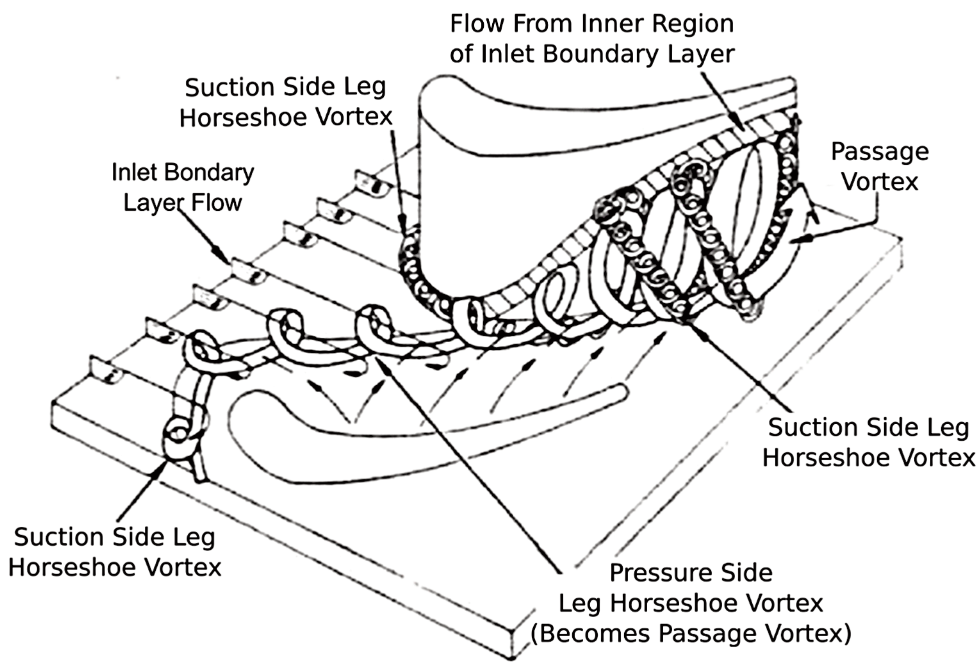


Figure 9. Vortex generation on the blade profile [13].

For the sub-system design such as an interaction between the rotating blades and vanes (Fig. 3b), mainly upstream and downstream effects must be considered. The number of vanes has a remarkable impact on the excitation level of the rotating blade due to the gap/pitch ratio. In addition, a proper three-dimensional curved vane and the optimization of the end-wall secondary flow (Fig. 9) lead to a remarkable reduction of the dynamic loading on the rotating blade.

Excitation Sources at System Design Level

For the system design level, the combustion and blade interaction shall be analyzed in a multi-disciplinary way. Flame instabilities within the combustor (Fig. 10) generate waves in the working fluid and acoustic resonances within the combustor can be excited. The exit conditions as interface to the turbine can vary remarkably form can to can along the circumferential direction. Thus, the non-uniform pressure distribution oscillates with an acoustic frequency ε, which might have an impact on the rotating blade.

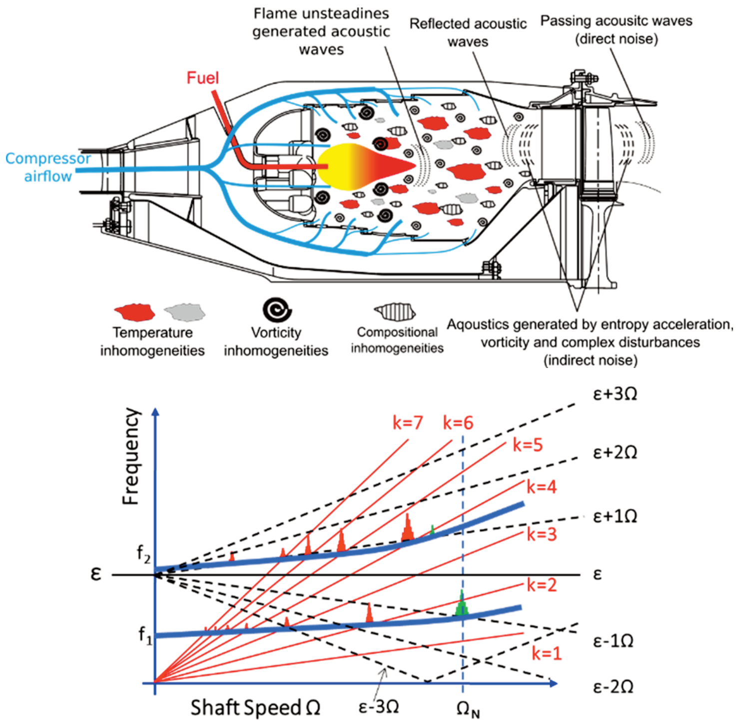


Figure 10. Flame instabilities in the combustor and Campbell diagram of rotating turbine blade with a rotational speed **Ω** extended on acoustic interaction at the acoustic frequency ε.

If these pulsations at frequency are strong, such as for instance in turbochargers, the blade frequencies should avoid these resonances. To predict these resonances, the acoustic analysis must be performed to predict the cavities frequencies ε_1, ε_2 and higher. By measuring pulsation in the can with the pressure transducers, the acoustic mode shapes can be calibrated to assess the real excitation amplitudes acting on the rotating blade [1].

Another example of the engine design level against excitations is the blade and rotor interaction for which the eigenfrequencies $f_{i,n}$ of the blade disc shall be above an excitation harmonic k of 1. For lower harmonic 0 and 1, the vibrating shaft will feed the energy to the torsional and weak bending resonance vibrations of the bladed disc.

The blade and exhaust diffusor (Fig. 1) interaction is another example of the engine design level which prevents the dynamic interaction between these two components. In this case, the axial distance between the two components must be properly assessed to ensure a safe engine operation, especially for low load operations that usually generate turbulent flow in the turbine region.

The computing power of today's computers allows verification of the interaction between the various stages of guide vanes that produce the HFR. By mounting guide vanes with the same number of N_V, they can be aligned along the same line, e.g., along the compressor, or the next stage of N_V guide vanes can be shifted around the circumference direction by several degrees (see "pitch" in Fig. 2). Depending on the configuration, the excitation on the rotating blade between these guide vanes may increase or decrease by 20% depending on the geometry and other design features of the compressor. These phenomena should always be given experimentally due to the turbulence models used in CFD.

EXPERIMENTAL SYSTEMS FOR MEASURING BLADE VIBRATIONS

Although there is remarkable progress with CFD which allows predicting the excitation forces with an acceptable reliability especially for HRF, the measuring technique still relies on the strain gauge and tip timing methods. In Table 1, the qualitative comparison between the Tip Timing and Strain Gauge Systems is given in terms of its pros and cons.

Table 1. Qualitative comparison between the Tip Timing and Strain Gauge Systems.

Parameter of Classification	**Tip Timing System**	**Strain Gauges & Telemetry System**
Measured variable	velocity of blade oscillation	strain on the blade profile
Number of measured blades	all blades in the row	only a few blades w/ strain gauges
Sensors	contactless	strain gauge
Sensor location	in the casing of an engine	on the blade
Effort of application	low	very high
Preparation for measurement	short	very long
Effort measurement of further disc	slow	very high
Effort data transmission	low	moderate
Measurement resolution	moderate	very good
Measurement accuracy	good	very good
Influence of application on result	very low	moderate
Evaluation of the measurement	difficult	simple

Suitability for High Frequency Range	impossible	very good
Suitability for Middle Frequency Range	depending on oscillation amp.	very good
Suitability for Low Frequency Range	very good	very good
Suitability for coupled frequencies $f_{i.n}$	yes	yes
Suitability for mistuned bladed disc	yes	no
Cost of measurements	low	very high

CONCLUSIONS

To meet the market demand for higher performance, modern engines need: to operate with larger mass flow, to burn hydrogen at higher firing temperatures, to startup and shutdown twice or more frequently a day. The 3D Additive Manufacturing will deliver light, geometrically repeatable rotating blades. The required High-Power-Density of the modern turbine will reduce axial distances between the rotating bladed disc and the vanes what expose the entire machine to increased forced vibrations and higher risk of HCF damages. Thus, the engine outline must begin at the system design level beyond that traditionally accomplished which is mainly based on the interaction between the blade and vane of the sub-system design level.

New AM manufacturing technologies that deliver parts with a high degree of manufacturability and complex design features will be exposed to resonant vibrations at low damping values. The repetitive production of all parts can be a problem for blade flutter which is diminished well by mistuned blades.

In this context, the concept of instrumenting blades to measure vibrations as the only possible means of verifying and understanding forced vibrations in modern engines should be reviewed. The level of experimental mechanics must be related to modern computational methods to measure those phenomena that the designer expects by minimizing validation costs (which can be very high when using the traditional way of the blade instrumentation). Thus, new methods for sensor and strain gauge optimization on vibrating component are necessary at the level of system and sub-system design that consider manufacturing tolerances and mechanical interactions among components of the turbine stage.

REFERENCES

[1] Beretta-Müller, A. & Szwedowicz, J. (2015). Blade Resonant Forced Response Excited by Combustor Acoustic Eigenmodes. In: Proceedings of ASME Turbo Expo: Turbine Technical Conference and Exposition, June 15−19, 2015, Montreal, Quebec, Canada.

[2] Corral, R. (2012). Bladed Disks: Flutter, Structural Design of Aircraft Engines, RTO-AVT-207-06, 25 January, Belgium.

[3] Dodds, J., and Vahdati, M. (May 1, 2015). Rotating Stall Observations in a High Speed Compressor—Part II: Numerical Study. ASME. *J. Turbomach*. May 2015, Vol. 137, No. 5, 051003. DOI: 10.1115/1.4028558.
[4] El-Aini, Y., et al. (1997). High Cycle Fatigue of Turbomachinery Components-Industry Perspective, In: 33rd Joint Propulsion Conference. DOI: 10.2514/6.1997-3365.
[5] Han, Y., Xiao, B. & Mignolet, M. P. (2007). Expedient Estimation of the Maximum Amplification Factor in Damped Mistuned Bladed Disks, ASME Paper No. GT2007-27353, In: Proceedings of ASME Turbo Expo, May 14−15, 2007, Montreal, Canada.
[6] Hulme, J. C., Fiebiger, W., S. & Szwedowicz, J. (2015). Axial Compressor Blade Failure Design Mitigation and Its Validation, ASME Paper No. GT2015-43312, ISBN: 978-0-7918-5679-6, Proceedings of ASME Turbo Expo, June 15−19, 2015, Montreal, Canada.
[7] Kenyon, J.A, & Griffin, J.H. (2001). Forced Response of Turbine Engine Bladed Disks and Sensitivity to Harmonic Mistuning. In: Proceedings of the ASME Turbo Expo: Power for Land, Sea, and Air. Volume 4: Manufacturing Materials and Metallurgy; Ceramics; Structures and Dynamics; Controls, Diagnostics and Instrumentation; Education; IGTI Scholar Award. New Orleans, Louisiana, USA. June 4–7, 2001. V004T03A037. ASME. DOI: 10.1115/2001-GT-0274.
[8] Kurnik, W. (1997). *Bifurkacje dywergentne i oscylacyjne*, WNT, Warszawa, ISBN 83-204-2221-3.
[9] Moneta, G. (2019). *Damping Optimization of Turbine Blade Vibration*, Ph.D. Dissertation, Warsaw University of Technology.
[10] Moneta, G. & Jachimowicz, J. (2020). Impact of Manufacturing Tolerances on Stress in a Turbine Blade Fir-Tree Root, *Fatigue of Aircraft Structures*, Vol. 2020, No. 12, 92−101, DOI: 10.2478/fas-2020-0009.
[11] Moneta, G., Jachimowicz, J. & Osiński, J. (2014). Influence of Manufacturing Tolerances on Vibration Frequencies of Turbine Blade, *Machine Dynamics Research*, Vol. 38, No 1, 105−118.
[12] Peter, J. (1999). *The History of Aircraft Gas Turbine Engine Development in the United States: A Tradition of Excellence*, ASME IGTI publication.
[13] Schneider C.M., et. al. (2013). On the Unsteady Formation of Secondary Flow Inside a Rotating Turbine Blade Passage. ASME. *J. Turbomach.*, Vol. 136, No. 6, 061004-061004-10. DOI:10.1115/1.4025582.
[14] Srinivasan, A.V. (1997). Flutter and Resonant Vibration Characteristics of Engine Blades. In: Proceedings of the ASME International Gas Turbine and Aeroengine Congress and Exhibition. Volume 4: Manufacturing Materials and Metallurgy; Ceramics; Structures and Dynamics; Controls, Diagnostics and Instrumentation; Education; IGTI Scholar Award. Orlando, Florida, USA. June 2–5, 1997. V004T17A001. ASME. DOI: 10.1115/97-GT-533.
[15] Szwedowicz, J. (2008). *High Cyclic Fatigue, Structural Design of Aircraft Engines: Key Objectives & Techniques*. Lecture Series Monographs of von Karman Institute for Fluid Dynamics, B-1640 Rhode Saint Genèse, Belgium, ISSN: 0377-8312, ISBN: 978-2-930389-8-2-6, pp. 49.
[16] Szwedowicz, J. (2010, April). 30-year anniversary of friction damper technology in turbine blades. *Mechanical Engineering-CIME*, Vol. 132, No. 4, 54+. https://link.gale.com/apps/doc/A223657423/AONE?u=googlescholar&sid=bookmark-AONE&xid=d990d10d.
[17] Szwedowicz, J., et al. (April 3, 2008). Scaling Concept for Axial Turbine Stages With Loosely Assembled Friction Bolts: The Linear Dynamic Assessment. ASME. *J. Eng. Gas Turbines Power*. May 2008; Vol. 130, No. 3, 032504. DOI: 10.1115/1.2838995.
[18] Whitehead, D. S. (1988). The Maximum Factor by Which Forced Vibration of Blades Can Increase Due to Mistuning, ASME *J. Eng. Gas Turbines Power*, Vol. 120, No. 1, 115–119.