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Analysis of linear continuous-time systems by the use of the conformable fractional calculus and Caputo

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Abstract: The paper presents general solutions for fractional state-space equations. The analysis of the fractional electrical circuit in the transient state is described by the equation of the state and space equations. The results are presented for the voltage of a capacitor and current in a coil, for different alpha values. The Caputo and conformable fractional derivative definitions have been considered. At the end, the results have been obtained.

Key words: conformable fractional derivative, fractional electrical circuit, different fractional order system

1. Introduction

Fractional calculus has been used in the pure and applied branches of science and engineering in the last century. Mathematicians such as Liouville, Grünwald, Letnikov and Riemann developed the fundamentals of fractional calculus and they are mentioned in the papers [8, 10, 13–14].

This article uses fractional-order equations in application to modeling of electric circuits. One of the most interesting issues of fractional order calculus applications is an analysis of electrical circuits in the transient state [2–5, 7], where the Caputo fractional derivative definition is used.

The fractional circuit equations using the Caputo definition and a new definition of a conformable fractional derivative (CFD) are given in [1, 6].

The fractional order dynamical systems described by the state-space equations with fractional order derivatives are analyzed in the monographs [8]. For the state-space description of the fractional electrical circuit equipped with a supercapacitor we will consider the solutions in a general case. Finally, the results obtained for a model electric circuit using the Caputo and CFD definitions will be shown in graphs.

2. Fractional order state-space equations

Consider a fractional linear system described by equations [10]:

$$\begin{bmatrix} D_t^{\alpha_1} x_1(t) \\ \vdots \\ D_t^{\alpha_n} x_n(t) \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} u(t), \qquad \begin{matrix} 0 < \alpha_k \leq 1 \\ k = 1, \ldots, n \end{matrix}, \tag{1}$$

where $x_k(t) \in \Re^{n_k}$, $k = 1, \ldots, n$ represent the state, $u(t) \in \Re^m$ is the input vector (extortion) and $A_{kj} \in \Re^{n_k \times n_l}$, $B_k \in \Re^{n_k \times m}$, $j, k = 1, \ldots, n$ are the matrices with constant coefficients, ${}_0D_t^{\alpha_k} x_k(t)$ is the fractional order derivative of the vector $x_k(t)$ described by the Caputo and CFD definition.

Initial conditions for (1) have the form:

$$x_k(0) = x_{k0} \in \Re^{\bar{n}_k}, \qquad k = 1, \ldots, n. \tag{2}$$

3. The Caputo definition

The function defined by [10]

$${}_0^{\text{Cap}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{a+1-n}} \, \mathrm{d}\tau \tag{3}$$

is called the Caputo fractional derivative, where $n-1 < \alpha < n$, $n = 1, n, \ldots$, $\Gamma(x)$ is the Euler gamma function and $f^{(n)}(t) = \dfrac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}$.

The solution of state-space Equation (1), using the Caputo definition, has the form [10]:

$$x_{\text{Cap}}(t) = \Phi_0(t)\, x_0 + \int_0^t [\Phi_1(t-\tau)B_{10} + \cdots + \Phi_n(t-\tau)B_{n0}]\, u(\tau)\, \mathrm{d}\tau, \tag{4}$$

where

$$x_{\text{Cap}}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \in \Re^N, \qquad N = \bar{n}_1 + \cdots + \bar{n}_n, \qquad x_0 = \begin{bmatrix} x_{10} \\ \vdots \\ x_{n0} \end{bmatrix}, \tag{5a}$$

$$B_{10} = \begin{bmatrix} B_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad B_{n0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ B_n \end{bmatrix}, \tag{5b}$$

$$
\begin{aligned}
\Phi_0(t) &= \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty} T_{k_1\ldots k_n}\frac{t^{k_1\alpha_1+\cdots+k_n\alpha_n}}{\Gamma\left(k_1\alpha_1+\cdots+k_n\alpha_n+1\right)},\\
\Phi_1(t) &= \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty} T_{k_1\ldots k_n}\frac{t^{(k_1+1)\alpha_1+k_2\alpha_2+\cdots+k_n\alpha_n-1}}{\Gamma\left[(k_1+1)\alpha_1+k_2\alpha_2+\cdots+k_n\alpha_n\right]},\\
&\ \ \vdots\\
\Phi_n(t) &= \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty} T_{k_1\ldots k_n}\frac{t^{k_1\alpha_1+\cdots+k_{n-1}\alpha_{n-1}+(k_n+1)\alpha_n-1}}{\Gamma\left[k_1\alpha_1+\cdots+k_{n-1}\alpha_{n-1}+(k_n+1)\alpha_n\right]}
\end{aligned}
\tag{5c}
$$

and

$$
T_{k_1,\ldots,k_n} = \begin{cases}
I_N & \text{for } k_1=\cdots=k_n=0\\
\begin{bmatrix} A_{11} & \cdots & A_{1n}\\ 0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix} & \text{for } \begin{array}{l} k_1=1\\ k_2=\cdots=k_n=0\end{array}\\
\begin{bmatrix} 0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\\ A_{i1} & \cdots & A_{in}\\ 0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix} & \text{for } \begin{array}{l} k_1=\cdots=k_{i-1}=0\\ k_i=1\\ k_{i+1}=\cdots=k_n=0\end{array}\\
\begin{bmatrix} 0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\\ A_{n1} & \cdots & A_{nn}\end{bmatrix} & \text{for } \begin{array}{l} k_1=\cdots=k_{n-1}=0\\ k_n=1\end{array}\\
T_{10\ldots0}T_{01\ldots1}+\cdots+T_{0\ldots01}T_{1\ldots10} & \text{for } k_1=\cdots=k_n=1\\
\vdots & \vdots\\
T_{10\ldots0}T_{k_1-1,k_2,\ldots,k_n}+\cdots+T_{0\ldots01}T_{k_1,\ldots,k_{n-1},k_n-1} & \text{for } k_1+\cdots+k_n>0\\
0 & \text{if exsist } k_i<0,\ i=1,\ldots,n
\end{cases}
\ . \tag{5d}
$$

4. The CFD definition

If $n<\alpha\leq n+1$, $n\in N_0$, then the conformable fractional derivative (CFD) of n-differentiable at t function f (where $t>0$) is defined as [6]:

$$
{}^{CFD}D_t^{\alpha}(f)(t) = \lim_{\varepsilon\to 0}\frac{f^{([\alpha]-1)}\left(t+\varepsilon t^{[\alpha]-\alpha}\right)-f^{([\alpha]-1)}(t)}{\varepsilon}, \tag{6}
$$

where $[\alpha]$ is the ceiling of α – the smallest integer greater than or equal to α.

Using the definition (6) we get a simple rule [6]:

$$ {}^{CFD}D_t^{\alpha} f(t) = t^{[\alpha]-\alpha} f^{([\alpha])}(t), \tag{7}$$

where f is the $[\alpha]$ differentiable function for $t > 0$.

Considering the continuous linear system described by state-space Equation (1) and using (7) for ${}^{CFD}D_t^{\alpha_k} x_k(t) = t^{1-\alpha_k}\dot{x}_k(t)$, where $0 < \alpha_k \le 1$ we obtain:

$$\begin{bmatrix} t^{1-\alpha_1}\dot{x}_1(t) \\ \vdots \\ t^{1-\alpha_n}\dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} u(t), \qquad \begin{array}{c} 0 < \alpha_k \le 1 \\ k = 1, \ldots, n \end{array} . \tag{8}$$

Multiplying the k-th part of Equation (8) by $t^{\alpha_k - 1}$, where $k = 1, \ldots, n$, we have:

$$\underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}}_{x'(t)} = \underbrace{\begin{bmatrix} A_{11}(t) & \cdots & A_{1n}(t) \\ \vdots & \ddots & \vdots \\ A_{n1}(t) & \cdots & A_{nn}(t) \end{bmatrix}}_{A(t)} \underbrace{\begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} B_1(t) \\ \vdots \\ B_n(t) \end{bmatrix}}_{B(t)} u(t), \qquad \begin{array}{c} 0 < \alpha_k \le 1 \\ k = 1, \ldots, n \end{array} , \tag{9}$$

where

$$A_{ij}(t) = A_{ij} t^{\alpha_i - 1}, \quad i, j = 1, \ldots, n \ \text{ or } \ A(t) = \begin{bmatrix} A_{11} t^{\alpha_1 - 1} & \cdots & A_{1n} t^{\alpha_1 - 1} \\ \vdots & \ddots & \vdots \\ A_{n1} t^{\alpha_n - 1} & \cdots & A_{nn} t^{\alpha_n - 1} \end{bmatrix}, \tag{10a}$$

$$B_i(t) = B_i t^{\alpha_i - 1}, \quad i = 1, \ldots, n, \quad \text{or} \quad B(t) = \begin{bmatrix} B_1 t^{\alpha_1 - 1} \\ \vdots \\ B_n t^{\alpha_n - 1} \end{bmatrix}. \tag{10b}$$

The solution to the state-space equation

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t) \tag{11}$$

has the form [9]:

$$x(t) = e^{f(t)} x_0 + \int_0^t e^{f(t) - f(\tau)} B(\tau)\, u(\tau)\, \mathrm{d}\tau, \tag{12}$$

where

$$\begin{aligned} f(t) &= \int_0^t A(\zeta)\, \mathrm{d}\zeta = \int_0^t \left[A_{ij} \zeta^{\alpha_i - 1}\right]_{ij} \mathrm{d}\zeta = \\ &= \left[A_{ij} \int_0^t \zeta^{\alpha_i - 1}\, \mathrm{d}\zeta\right]_{ij} = \left[A_{ij} \frac{\zeta^{\alpha_i}}{\alpha_i}\bigg|_0^t\right]_{ij} = \left[A_{ij} \frac{t^{\alpha_i}}{\alpha_i}\right]_{ij}. \end{aligned} \tag{13}$$

Matrix (13) can be written in the following form:

$$f(t)=\begin{bmatrix} A_{11}\frac{t^{\alpha_1-1}}{\alpha_1} & \cdots & A_{1n}\frac{t^{\alpha_1-1}}{\alpha_1} \\ \vdots & \ddots & \vdots \\ A_{n1}\frac{t^{\alpha_n-1}}{\alpha_n} & \cdots & A_{nn}\frac{t^{\alpha_n-1}}{\alpha_n} \end{bmatrix}= \\ =\underbrace{\begin{bmatrix} A_{11} & \cdots & A_{1n} \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}}_{T_{10\ldots0}}\frac{t^{\alpha_1}}{\alpha_1}+\cdots+\underbrace{\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ A_{n1} & \cdots & A_{nn} \end{bmatrix}}_{T_{0\ldots01}}\frac{t^{\alpha_n}}{\alpha_n}. \tag{14}$$

Using the Maclaurin series of the exponential matrix function of (14) we have:

$$F_0(t)=e^{f(t)}=\sum_{k=0}^{\infty}\frac{[f(t)]^k}{k!}=\sum_{k=0}^{\infty}\frac{\left(T_{10\ldots0}\frac{t^{\alpha_1}}{\alpha_1}+\cdots+T_{0\ldots01}\frac{t^{\alpha_n}}{\alpha_n}\right)^k}{k!}= \\ =\sum_{k_1=0}^{\infty}\ldots\sum_{k_n=0}^{\infty}T_{k_1\ldots k_n}\frac{t^{k_1\alpha_1+\ldots+k_n\alpha_n}}{\alpha_1^{k_1}\ldots\alpha_n^{k_n}(k_1+\ldots+k_n)!}, \tag{15}$$

where $T_{k_1\ldots k_n}$ is given by the Formula (5d).

In a similar way we calculate the matrix $e^{f(t)-f(\tau)}$ being included in formula (12).

$$e^{f(t)-f(\tau)}=\sum_{k=0}^{\infty}\frac{[f(t)-f(\tau)]^k}{k!}=\sum_{k=0}^{\infty}\frac{\left(T_{10\ldots0}\frac{t^{\alpha_1}-\tau^{\alpha_1}}{\alpha_1}+\cdots+T_{0\ldots01}\frac{t^{\alpha_n}-\tau^{\alpha_n}}{\alpha_n}\right)^k}{k!}= \\ =\sum_{k_1=0}^{\infty}\ldots\sum_{k_n=0}^{\infty}T_{k_1\ldots k_n}\frac{(t^{\alpha_1}-\tau^{\alpha_1})^{k_1}\ldots(t^{\alpha_n}-\tau^{\alpha_n})^{k_n}}{\alpha_1^{k_1}\ldots\alpha_n^{k_n}(k_1+\ldots+k_n)!}. \tag{16}$$

Substituting Formula (10b) in (12), yields

$$\int_0^t e^{f(t)-f(\tau)}B(\tau)u(\tau)\,\mathrm{d}\tau=\int_0^t e^{f(t)-f(\tau)}\begin{bmatrix} B_1\tau^{\alpha_1-1} \\ \vdots \\ B_n\tau^{\alpha_n-1} \end{bmatrix}u(\tau)\,\mathrm{d}\tau= \\ =\int_0^t\left(\underbrace{e^{f(t)-f(\tau)}\tau^{\alpha_1-1}}_{F_1(t,\tau)}\underbrace{\begin{bmatrix} B_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B_{10}}+\cdots+\underbrace{e^{f(t)-f(\tau)}\tau^{\alpha_n-1}}_{F_n(t,\tau)}\underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ B_n \end{bmatrix}}_{B_{n0}}\right)u(\tau)\,\mathrm{d}\tau= \\ =\int_0^t\left(F_1(t,\tau)B_{10}+\cdots+F_n(t,\tau)B_{n0}\right)u(\tau)\,\mathrm{d}\tau, \tag{17}$$

where

$$
\begin{aligned}
F_1(t,\tau) &= e^{f(t)-f(\tau)}\tau^{\alpha_1-1} = \\
&= \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty} T_{k_1\ldots k_n}\frac{(t^{\alpha_1}-\tau^{\alpha_1})^{k_1}\ldots(t^{\alpha_n}-\tau^{\alpha_n})^{k_n}\,\tau^{\alpha_1-1}}{\alpha_1^{k_1}\ldots\alpha_n^{k_n}(k_1+\ldots+k_n)!}T_{k_1,k_2}\,, \\
&\ \vdots \\
F_n(t,\tau) &= e^{f(t)-f(\tau)}\tau^{\alpha_n-1} = \\
&= \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty} T_{k_1\ldots k_n}\frac{(t^{\alpha_1}-\tau^{\alpha_1})^{k_1}\ldots(t^{\alpha_n}-\tau^{\alpha_n})^{k_n}\,\tau^{\alpha_n-1}}{\alpha_1^{k_1}\ldots\alpha_n^{k_n}(k_1+\ldots+k_n)!}T_{k_1,k_2}\,.
\end{aligned}
\tag{18}
$$

Finally, the solution (12) takes the form:

$$
x_{CFD}(t) = F_0(t)x_0 + \int_0^t [F_1(t,\tau)B_{10} + \cdots + F_n(t,\tau)B_{n0}]\,u(\tau)\,\mathrm{d}\tau\,. \tag{19}
$$

5. Electrical circuit and general description of the problem

Consider the electrical circuit shown in Figure 1 with given resistors R_1, R_2 and, fractional coil L, supercapacitor C and source voltage e.

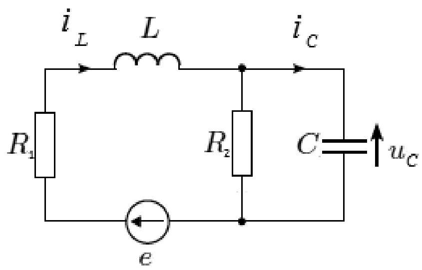

Fig. 1. Electrical circuit of R, L, C, e type [10]

The current $i_C(t)$ in the fractional capacitor is related to its voltage $u_C(t)$ by the formula:

$$
i_C(t) = C_{\alpha_1} D_t^{\alpha_1} u_C(t) = C_{\alpha_1}\frac{\mathrm{d}^{\alpha_1} u_C(t)}{\mathrm{d}t^{\alpha_1}} \qquad \text{for} \quad 0 < \alpha_1 < 1, \tag{20}
$$

where C_{α_1} is the pseudo-capacitance in units of F/sec$^{1-\alpha_1}$ of the fractional capacitor. Similarly, the voltage $u_L(t)$ on the coil (inductor) is related to its current $i_L(t)$ by the formula

$$
u_L(t) = L_{\alpha_2} D_t^{\alpha_2} i_L(t) = L_{\alpha_2}\frac{\mathrm{d}^{\alpha_2} i_L(t)}{\mathrm{d}t^{\alpha_2}} \qquad \text{for} \quad 0 < \alpha_2 < 1, \tag{21}
$$

where L_{α_2} is the pseudo-inductance in units H/sec$^{1-\alpha_2}$ of the coil.

Using Equations (20), (21) and Kirchhoff's laws may describe the transient states in the electrical circuit by the fractional differential equation:

$$\begin{bmatrix} D_t^{\alpha_1} x_1(t) \\ {}_0D_t^{\alpha_2} x_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t), \tag{22}$$

where $x_1 = [u_C]$ is the voltage across the supercapacitor, the component of $x_2 = [i_L]$ is the current in the coil, and the component of $u(t) = [e]$ and matrices are defined by

$$A_{11} = \left[-\frac{1}{R_2 C_{\alpha_1}} \right], \qquad A_{12} = \left[\frac{1}{C_{\alpha_1}} \right], \qquad A_{21} = \left[-\frac{1}{L_{\alpha_2}} \right], \qquad A_{22} = \left[-\frac{R_1}{L_{\alpha_2}} \right] \tag{23}$$

and

$$B_1 = [0], \qquad B_2 = \left[\frac{1}{L_{\alpha_2}} \right]. \tag{24}$$

For further analysis we refer to Equations (4), (19) with $\alpha_1 = 0.6$, $\alpha_2 = 0.9$. Let resistance $R_1 = 9.0\ \Omega$, $R_2 = 21.0\ \Omega$, fractional inductance $L_{\alpha_{21}} = 0.2$ H/sec$^{1-\alpha_2}$, fractional capacitance $C_{\alpha_1} = 11.0$ F/sec$^{1-\alpha_1}$ and the constant input $e = 1.0$ V. The initial conditions $i_L(0) = 0.0$ A, $u_C(0) = 0.0$ V and the state vector is $x(t) = [u_C(t)\ \ i_L(t)]^T$.

Solution 1 Using the Caputo definition of a fractional derivative we obtain the following solution to Equation (4):

$$\begin{aligned} x_{\text{Cap}}(t) &= \sum_{k=1}^{2} \int_0^t \Phi_k(t-\tau) B_{k0} u(\tau)\, \mathrm{d}\tau = \sum_{k=1}^{2} \int_0^t \Phi_k(t-\tau) B_{k0} e\, \mathrm{d}\tau = \\ &= \sum_{k=1}^{2} \left[\int_0^t \Phi_k(t-\tau)\, \mathrm{d}\tau \right] B_{k0} e = \sum_{k=1}^{2} \left[\int_0^t \Phi_k(\tau)\, \mathrm{d}\tau \right] B_{k0} e, \end{aligned} \tag{25}$$

where $\Phi_1(t)$ and $\Phi_2(t)$ are given by formulas (5c).

Solution 2 Using the CFD definition of fractional derivative (6) we obtain the following solution to Equation (19):

$$\begin{aligned} x_{CFD}(t) &= \sum_{k=1}^{2} \int_0^t F_k(t,\tau) B_{k0} u(\tau)\, \mathrm{d}\tau = \\ &= \sum_{k=1}^{2} \int_0^t F_k(t,\tau) B_{k0} e\, \mathrm{d}\tau = \sum_{k=1}^{2} \left[\int_0^t F_k(t,\tau)\, \mathrm{d}\tau \right] B_{k0} e, \end{aligned} \tag{26}$$

where $F_1(t)$ and $F_2(t)$ are given by formulas (18).

The comparison of the solutions for Caputo and CFD definitions are presented in Figures 2 and 3.

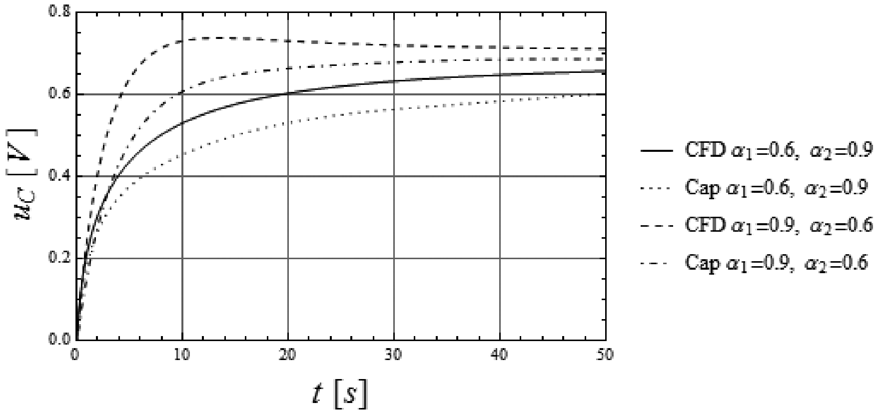

Fig. 2. Comparison of solutions using Caputo and CFD definitions for the capacitor for $\alpha_1 = 0.6$, $\alpha_2 = 0.9$, and for $\alpha_1 = 0.9$, $\alpha_2 = 0.6$

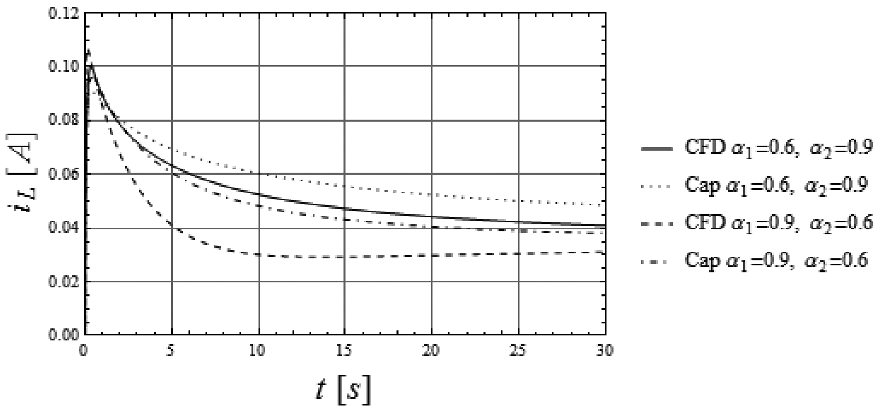

Fig. 3. Comparison of solutions using Caputo and CFD definitions for the coil for $\alpha_1 = 0.6$, $\alpha_2 = 0.9$, and for $\alpha_1 = 0.9$, $\alpha_2 = 0.6$

6. Conclusions

The paper presents the method of calculating the voltage and current on the elements of the fractional electrical circuit. The fractional order state-space description of the circuit gives a possibility for the analysis of different definitions of fractional order derivatives. In this paper, solutions for a model electrical circuit were derived using Caputo and conformable fractional derivative (CFD) definitions. The results obtained using the Caputo definition for the capacitor voltage calculation under the same alpha conditions have lower values than when using the CFD definition. The electric current with zero initial conditions in an electric circuit coil gets higher values when the Caputo definition is used, than when the CFD definition is applied.

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Comments on the paper "Analysis of linear continuous-time systems by the use of the Conformable Fractional Derivative and Caputo"

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Technical sciences aim, among others, to provide the simplest and most reliable solution to the investigated question. Such a solution should be delivered in the most efficient way. In the reviewed paper, quite complex theory has been used to solve a simple circuit theory task. The publication needs another condition to be fulfilled in order to assess it as a correct one. Specifically, other papers relevant to the discussed topic should be cited. Unfortunately, the reviewed paper does not meet the mentioned condition, because a number of relevant works have not been cited. No work of any theoretician has been cited, neither of famous ones as Prof. Prof. St. Fryze, T. Cholewicki, S. Wegrzyn, E. Philippow, W. Cauer, R. Dorf, L. Nejman nor of other not less prominent. No one from the younger generation, as to mention Prof. W. Mathis or Prof. S. Osowski has been cited, as well. All of the mentioned above scientists have dealt with circuit theory. My critical work titled "Fractional derivatives in electrical circuits theory – critical remarks" was published in AEE 1/2017. It is surprising that the work that describes a similar topic as the reviewed one and was published by the same journal, has not been cited. Additionally, my (or co-authored) other works on the same topic were published on PE 10/2016, 4/2017 and 1/2018. Also the works of Prof. P. Ostalczyk from PE 3/2017 and co-authored work of Prof. K.J. Latawiec "Fractional-order modeling of electric circuits (978-1-5386-1528-7/17$31.00©2017 IEEE", that is partially in line with the reviewed paper, are concurrent with the topic of the reviewed paper.

The work: Kaczorek T. "Positivity and Reachability of Fractional Electrical Circuits, Acta Mechanica et Automatica, Vol. 5, no. 2, pp. 42–51 (2011)", which has been cited by the author of the reviewed paper, names as "fractional electrical circuits" regular circuits with fractional derivatives, capacitances and inductivities in classic view, which do not meet the requirement of dimensional homogeneity. In the reviewed paper capacitance has been named "pseudo-capacitance" and inductance "pseudo-inductance". The author of the reviewed paper should elaborate on this. This gains on importance as the cited and reviewed papers refer to the same topic.

Following the work "Fractional-order modeling of electric circuits", the author of the reviewed paper applied the unit of farad divided by second to the power of one minus alpha. This way, the international system of units SI has been impacted, as it does not contain such a unit.

There is no reference to fractional derivatives in the sales folders of available super capacitors. The laws of physics should be followed.

The author of the reviewed paper does not notice that her work impacts the fundamentals of electrical engineering, specifically the Maxwellian equations, whitch has been pointed out by me in my publications mentioned earlier. This may lead to erroneous formulation of the Maxwellian equations, as it happened in the work of M.D. Ortigueira, M. Rivero, J.J. Trujillo, "From a generalized Helmholtz decomposition theorem to fractional Maxwell equations, Communications in Nonlinear Science and Numerical Simulation", 22 (2015), Issues 1–3, 1036–1049. I have shared my opinion with Prof. M. Ortigueira that Equations (58) and (59) in the mentioned work are formulated incorrectly.

My lecture from 30th November, 2016 at the Technical University of Warsaw (available at "Sikora Ryszard prof. Referat") touches similar questions as the reviewed paper.

I recommend to publish the reviewed paper simultaneously with the publication of all related reviews. I am of opinion that the reviews should be overt and therefore I ask to share my name together with my review. In the case I would wrong in my scientific views, I will admit it and apologize. I think that we shall be open and transparent in communicating our scientific views. I trust that the time of "one and the only one truth" in science is over. We all may both make mistakes and correct them.

I ask once more the Chairman of Committee on Electrical Engineering of the Polish Academy of Sciences to organize an open discussion on fractional derivatives' application in electrical engineering. Science does not profit from avoiding or limiting the discussion. Unfortunately, my strong opponent (PE 3/2017) (the specialist in fractional derivatives from the Technical University of Lodz) was not present during my lecture at ISEF'17, although this event was organized by His University.