AGH DRILLING, OIL, GAS • Vol. 31 • No. 3 • 2014

http://dx.doi.org/10.7494/drill.2014.31.3.431

Vasyl Moisyshyn*, Bogdan Borysevych*, Oleg Vytyaz*, Yuriy Gavryliv*

DEVELOPMENT OF THE MATHEMATICAL MODELS OF THE INTEGRAL DRILLING INDICES BASED ON THE DIMENSIONAL ANALYSIS

1. THEME TOPICALITY AND PROBLEM STATE

It is necessary to have mathematical models of the integral drilling indices, i.e. mechanical speed, value of torque, and energy consumption for the rock breaking process, to optimize the mode control of the drilling bit run. The controlled parameters of the greater part of the known models are the axial load on the drilling bit F_{st} and its rotation speed n_d. The circulation rate Q for most drilling rigs is considered to be a partially controlled parameter. It is defined discretely and believed to be sufficient if the further increase in the fluid effective injection rate does not lead to an increase in the drilling speed.

It was determined with the help of [1] that the utilization of the tools for stiffness change C and damping β in the bottom-hole assembly can both improve and worsen the integral drilling indices. At the present there are no studies of the influence of the drilling tool parameters C and β on the mechanical speed V_{mech}, value of torque $\overline{T}_d$ (average value on the bit), and energy consumption for rock breaking W_R on the well bottom-hole.

Taking into account the above mentioned, there arises urgency for developing mathematical models of the integral well drilling indices.

The objective of the study is to estimate the influence of the mode parameters F_{st} and n_d and drilling tool performance indices C and β on the integral drilling indices on the basis of the dimensional analysis, as well as to develop their mathematical models in accordance with the results of the bench tests.

2. DEVELOPMENT OF THE MATHEMATICAL MODELS OF THE MECHANICAL DRILLING SPEED

In accordance with [2] the dimensions of any physical magnitude may be presented as a product of the raised-to-power dimensions of the main physical magnitudes.

* Ivano-Frankivsk National Technical University of Oil And Gas, The Petroleum Engineering Faculty, Ukraine

The following formula is true for mechanical phenomena:

$$[X] = L^{\alpha}\, T^{\beta}\, M^{\gamma} \tag{1}$$

where:

L – length [m],
T – time [s],
M – mass [kg],
α, β, and γ – exponents.

The three similarity theorems are a theoretical basis for determining the conditions of transition from a model to a real process. The first and second theorem, which is called a π-theorem, determine the relations between the parameters of the drilling process modeling on the test stand and in the industrial conditions. The methods for finding out the similarity and ways of the similarity realization when developing the models are determined by the third theorem.

In accordance with the first theorem the drilling processes similarity on the test stand and in real conditions is characterized by the equality of all the similarity criteria that are dimensionless complexes of the physical magnitudes, composed of generalized coordinates and parameters, and they are described by the similar functional dependencies.

Let's consider the mechanical drilling speed to be a generalized coordinate and then let's describe the process of its chance with the help of the following dependence:

$$V_{mech} = f\,(F_{st} \cdot \omega_d \cdot C \cdot \beta \cdot D \cdot p_m \cdot Q) \tag{2}$$

where:

F_{st} – axial static drilling bit load [N],
ω_d – angular drilling bit rotation speed [s^{-1}],
C and β – stiffness [N/m] and drilling tool damping [N·s/m] respectively,
D – drill bit diameter [m],
p_m – rock hardness according to the mark [Pa],
Q – volumetric drilling mud flow rate [m^3/s].

Let's determine the dimensions of the shown above physical magnitudes with the help of the dimensions of the main measurement units of the SI System:

$$[V_{mech}] = \frac{\mathrm{m}}{\mathrm{s}} = L \cdot T^{-1};\ [F_{st}] \equiv [F_{res}] = \frac{\mathrm{kg \cdot m}}{\mathrm{s}^2} = M \cdot L \cdot T^{-2};\ [\omega] = \frac{\pi\, n_d}{30} = s^{-1} = T^{-1},$$

$$[C] = \frac{F_{st}}{1} = \frac{\mathrm{kg \cdot m/s^2}}{\mathrm{m}} = M \cdot T^{-2};\ [\beta] = \frac{F_{res}}{1} = \frac{\mathrm{kg \cdot m/s^2}}{\mathrm{m/s}} = M \cdot T^{-1},$$

$$[p_m] = \frac{F_{st}}{A} = \frac{\mathrm{kg \cdot m/s^2}}{\mathrm{m}^2} = M \cdot L^{-1} \cdot T^{-2};\ Q = \frac{F_{st}}{A} = \frac{\mathrm{m}^3}{\mathrm{s}} = L^3 \cdot T^{-1}.$$

In these formulas L – drilling tool deformation under the static load influence [m]; F_{res} – resistance force of the hydraulic vibration damper [N]; V – piston speed of the hydraulic vibration damper; A – mark basis area [m^2].

The exponents α, β, and γ of the physical magnitudes that determine the well drilling process are shown in Table 1.

Table 1

Exponents α, β, and γ of physical quantities that determine the process of drilling wells

Size	L	T	M
	α_i	β_i	γ_i
V_{mech}	1	−1	0
F_{st}	1	−2	1
ω	0	−1	0
C	0	−2	1
β	0	−1	1
D	1	0	0
p_m	−1	−2	1
Q	3	−1	0

Check the condition of independence of parameters D, p_m, Q, that is, convince ourselves that the determinant of the matrix of their dimensions is not equal to zero:

$$\begin{matrix} D \\ p_m \\ Q \end{matrix}\begin{vmatrix} 1 & 0 & 0 \\ -1 & -2 & 1 \\ 3 & -1 & 0 \end{vmatrix} =$$

$$= 1\cdot(-2)\cdot 0 + 0\cdot(-1)\cdot(-1) + 0\cdot 1\cdot 3 - 0\cdot(-2)\cdot 3 - (-1)\cdot 0\cdot 0 - (-1)\cdot 1\cdot 1 = 1 .$$

Make use of these independent parameters for finding the similarity criteria.

The first similarity criterion has the form:

$$\Pi_1 = \frac{V_{mech}}{D^{t_1}\cdot p_s^{n_1}\cdot Q^{p_1}} \tag{3}$$

We express this criterion through the dimensions of the relevant quantities (see Tab. 1).

$$\Pi_1 = \frac{L^1\cdot T^{-1}\cdot M^0}{(L^1)^{m_1}\cdot(L^{-1}\cdot T^{-2}\cdot M^1)^{n_1}\cdot(L^3\cdot T^{-1})^{p_1}} \tag{4}$$

From the condition of exponents equality of powers at L, T and M in the number at or and denominator we obtain the system of equations:

$$\begin{aligned} m_1 - n_1 + 3p_1 &= 1 \\ -2n_1 - p_1 &= -1 \\ n_1 &= 0 \end{aligned} \tag{5}$$

By substituting the value $m_1 = -2$, $n_1 = 0$, $p_1 = 1$ in (3), write down the first criterion of similarity:

$$\Pi_1 = \frac{V_{mech}}{\mathrm{D}^{-2} \cdot Q^1} \tag{6}$$

The second similarity criterion has the form:

$$\Pi_2 = \frac{F_{st}}{D^{m_2} \cdot p_S^{n_2} \cdot Q^{p_2}} \tag{7}$$

Expressing it through the dimensions of the relevant quantities (see Tab. 1), we obtain:

$$\Pi_2 = \frac{L^1 \cdot T^{-2} \cdot M^1}{(L^1)^{m_2} \cdot (L^{-1} \cdot T^{-2} \cdot M^1)^{n_2} \cdot (L^3 \cdot T^{-1})^{p_2}} \tag{8}$$

From the condition of equality of the exponent sat L, T and M in the numerator and denominator we obtain the system of equations:

$$\begin{aligned} m_2 - n_2 + 3p_2 &= 1 \\ -2n_2 - p_2 &= -2 \\ n_2 &= 1 \end{aligned} \tag{9}$$

By substituting the value $m_2 = 0$, $n_2 = 1$, $p_2 = 0$ in (7), we write the second criterion of similarity:

$$\Pi_2 = \frac{F_{st}}{p_m} \tag{10}$$

We write the third criterion of similarity:

$$\Pi_3 = \frac{\omega_d}{D^{m_3} \cdot p_s^{n_3} \cdot Q^{p_3}} \tag{11}$$

By submitting it through the dimension of the relevant quantities (see Tab. 1), we find:

$$\Pi_3 = \frac{L^0 \cdot T^{-1} \cdot M^0}{(L^1)^{m_3} \cdot (L^{-1} \cdot T^{-2} \cdot M^1)^{n_3} \cdot (L^3 \cdot T^{-1})^{p_3}} \tag{12}$$

$$\begin{aligned} m_3 - n_3 + 3p_3 &= 0 \\ -2n_3 - p_3 &= -1 \\ n_3 &= 0 \end{aligned} \tag{13}$$

By substituting the value $m_3 = -3$, $n_3 = 0$, $p_3 = 1$ in (11), we record:

$$\Pi_3 = \frac{\omega_d}{D^{-3} \cdot Q^1} \tag{14}$$

The fourth similarity criterion has the form:

$$\Pi_4 = \frac{C}{D^{m_4} \cdot p_s^{n_4} \cdot Q^{p_4}} \tag{15}$$

Expressing its dimensions through relevant quantities (see Tab. 1), we obtain:

$$\Pi_4 = \frac{L^0 \cdot T^{-2} \cdot M^1}{(L^1)^{m_4} \cdot (L^{-1} \cdot T^{-2} \cdot M^1)^{n_4} \cdot (L^3 \cdot T^{-1})^{p_4}} \tag{16}$$

$$\begin{aligned} m_4 - n_4 + 3p_4 &= 0 \\ -2n_4 - p_4 &= -2 \\ n_4 &= 1 \end{aligned} \tag{17}$$

By substituting $m_4 = 1$, $n_4 = 1$, $p_3 = 0$ in (15), we record:

$$\Pi_4 = \frac{C}{D^1 \cdot p^1_s} \tag{18}$$

Finally, the fifth similarity criterion has the form:

$$\Pi_5 = \frac{\beta}{D^{m_5} \cdot p_s^{n_5} \cdot Q^{p_5}} \tag{19}$$

Presenting through the dimension of the relevant quantities (see Tab. 1) we obtain:

$$\Pi_5 = \frac{L^0 \cdot T^{-1} \cdot M^1}{(L^1)^{m_5} \cdot (L^{-1} \cdot T^{-2} \cdot M^1)^{n_5} \cdot (L^3 \cdot T^{-1})^{p_5}} \tag{20}$$

$$\begin{aligned} m_5 - n_5 + 3p_5 &= 0 \\ -2n_5 - p_5 &= -1 \\ n_5 &= 1 \end{aligned} \tag{21}$$

By substituting $m_5 = 1$, $n_5 = 1$, $p_3 = -1$ in (19), we record the fifth criterion of similarity:

$$\Pi_5 = \frac{\beta}{D^4 \cdot p^1_m \cdot Q^{-1}} \tag{22}$$

Let's give the desired functional dependences:

$$\frac{V_{mech}}{D^{-2} \cdot Q} = f\left(\frac{F_{st}}{p_m} \cdot \frac{\omega_d}{D^{-3} \cdot Q} \cdot \frac{C}{D \cdot p_m} \cdot \frac{\beta}{D^4 \cdot p_m \cdot Q^{-1}}\right) \tag{23}$$

Based on the results of stand experimental studies [1] conducted according to a classic plan, the dependences $V_{mech} = f(F_{st})$, $V_{mech} = f(\omega_d)$, $V_{mech} = f(C)$ and $V_{mech} = f(\beta)$ are satisfactorily interpreted by a power function, which is an indicator for the dependence $V_{mech} = f(F_{st}) - \alpha_1$, for the dependence $V_{mech} = f(\omega_d)$, $- \alpha_2$, for $V_{mech} = f(C)$ and $V_{mech} = f(\beta)$ respectively by «$- \alpha_3$» and «$-\alpha_4$».

Let's give the desired functional dependences:

$$V_{mech} = D^{-2} \cdot Q \cdot k_1 \cdot \left(\frac{F_{st}}{p_m}\right)^{\alpha_1} \cdot \left(\frac{\omega_d}{D^{-3} \cdot Q}\right)^{\alpha_2} \cdot \left(\frac{C}{D \cdot p_m}\right)^{-\alpha_3} \cdot \left(\frac{\beta}{D^4 \cdot p_m \cdot Q^{-1}}\right)^{-\alpha_4} \quad (24)$$

Where k_1, α_1, α_2 α_3 α_4 – empirical coefficients is to be determined by the results of experimental studies conducted in accordance with the planned experiment. During these studies drilling a well was performed with bit 93 T at the fixed flow rate of drilling fluid (water) 7 – $Q = 7 \cdot 10^{-3}$ m^3/s, and hardness according to the mark of sandstone interlayers of Vorotyschenska strata – $p_m = 1440$ MPa $= 1440 \cdot 10^6$ Pa and $p_m = 2050$ MPa $= 2050 \cdot 10^6$ Pa.

The calculation for the interlayer with hardness $p_m = 1440$ MPa give:

$D^{-2} \cdot Q = (93 \cdot 10^{-3})^{-2} \cdot 7 \cdot 10^{-3} = 0{,}8093$; $D^{-3} \cdot Q = (93 \cdot 10^{-3})^{-3} \cdot 7 \cdot 10^{-3} = 8{,}7026$;

$D \cdot p_m = 93 \cdot 10^{-3} \cdot 1440 \cdot 10^6 = 1{,}3392 \cdot 10^8$;

$D^4 \cdot p_m \cdot Q^{-1} = (93 \cdot 10^{-3})^4 \cdot 1440 \cdot 10^6 \cdot (7 \cdot 10^{-3})^{-1} = 15388495$.

Granting this:

$$\begin{aligned} V_{mech} &= 0{,}8093 \cdot k_1 \cdot \left(\frac{F_{st}}{1440 \cdot 10^6}\right)^{\alpha_1} \cdot \left(\frac{\omega_d}{8{,}7026}\right)^{\alpha_2} \cdot \left(\frac{C}{1{,}3392 \cdot 10^8}\right)^{-\alpha_3} \cdot \left(\frac{\beta}{15388495}\right)^{-\alpha_4} = \\ &= 2{,}09 \cdot 10^{25} \cdot k_1 \cdot F_{st}^{\alpha_1} \cdot \omega_d^{\alpha_2} \cdot C^{-\alpha_3} \cdot \beta^{-\alpha_4} \end{aligned} \quad (25)$$

For interlayer hardness $p_s = 2050$ MPa we will have:

$D \cdot p_m = 93 \cdot 10^{-3} \cdot 2050 \cdot 10^6 = 1{,}9065 \cdot 10^8$;

$D^4 \cdot p_m \cdot Q^{-1} = (93 \cdot 10^{-3})^4 \cdot 2050 \cdot 10^6 \cdot (7 \cdot 10^{-3})^{-1} = 21907232$.

$$\begin{aligned} V_{mech} &= 0{,}8093 \cdot k_2 \cdot \left(\frac{F_{st}}{2050 \cdot 10^6}\right)^{\alpha_5} \cdot \left(\frac{\omega_d}{8{,}7026}\right)^{\alpha_6} \cdot \left(\frac{C}{1{,}9065 \cdot 10^8}\right)^{-\alpha_7} \cdot \left(\frac{\beta}{21907232}\right)^{-\alpha_8} = \\ &= 6{,}03 \cdot 10^{25} \cdot k_2 \cdot F_{st}^{\alpha_5} \cdot \omega_d^{\alpha_6} \cdot C^{-\alpha_7} \cdot \beta^{-\alpha_8} \end{aligned} \quad (26)$$

3. MATHEMATICAL MODELS DEVELOPMENT OF MOMENT CAPACITANCE FOR DRILLING PROCESS

Taking for the generalized coordinate the average value of torque on the bit, let's describe the process of its change as the functional dependence:

$$\overline{T}_d = f(F_{st} \cdot \omega_d \cdot C \cdot \beta \cdot D \cdot p_m \cdot Q) \quad (27)$$

The dimension of the moment capacitance – $\left[\overline{T}_d\right] = \mathrm{N} \cdot \mathrm{m} = \dfrac{\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{m}}{\mathrm{s}^2} = M \cdot L^2 \cdot T^{-2}$. Independent parameters, according to which we determine the similarity criteria are the same as in the definition of the functional dependence similarity criteria of the mechanical drilling speed. According to this similarity criteria 2–5 don't change.

The first similarity criterion has the form:

$$\Pi_1 = \frac{\overline{T}_d}{D^{m_1} \cdot p_m^{n_1} \cdot Q^{p_1}} \tag{28}$$

Expression through the criterion dimension corresponding values:

$$\Pi_1 = \frac{L^2 \cdot T^{-2} \cdot M^1}{(L^1)^{m_1} \cdot (L^{-1} \cdot T^{-2} \cdot M^1)^{n_1} \cdot (L^3 \cdot T^{-1})^{p_1}} \tag{29}$$

From the condition of exponents equality at L, T and M in the numerator and denominator we obtain the system of equations:

$$\begin{aligned} m_1 - n_1 + 3p_1 &= 2 \\ -2n_1 - p_1 &= -2 \\ n_1 &= 1 \end{aligned} \tag{30}$$

By substituting $m_1 = 3$, $n_1 = 1$, $p_1 = 0$, in (28), write down the first criterion of similarity:

$$\Pi_1 = \frac{\overline{T}_d}{D^3 \cdot p_m^1} \tag{31}$$

Let's give the desired functional dependence as:

$$\frac{\overline{T}_d}{D^3 \cdot p_\mathrm{m}^1} = f\left(\frac{F_{st}}{p_m} \cdot \frac{\omega_d}{D^{-3} \cdot Q} \cdot \frac{C}{D \cdot p_m} \cdot \frac{\beta}{D^4 \cdot p_m \cdot Q^{-1}}\right) \tag{32}$$

Based on the results of the stand experimental studies [1] conducted due to a classic plan, depending ces $\overline{T}_d = f(F_{st})$, $\overline{T}_d = f(\omega_d)$, $\overline{T}_d = f(C)$ are satisfactorily interpreted by a power degree function the exponent of which for the dependence $\overline{T}_d = f(F_{st})$ – «α_9», and for the dependence $\overline{T}_d = f(\omega_d)$ – «$-\alpha_{10}$», $\overline{T}_d = f(C)$ – «$-\alpha_{11}$». Dependences $\overline{T}_d = f(\beta)$ are satisfactorily interpreted by a linear function $\overline{T}_d = a_1 - b_1 \cdot \beta$.

Let's give the desired functional dependence as:

$$\begin{aligned} \overline{T}_d &= D^3 \cdot p_\mathrm{m} \cdot k_3 \cdot \left(\frac{F_{st}}{p_m}\right)^{\alpha_9} \cdot \left(\frac{\omega_d}{D^{-3} \cdot Q}\right)^{-\alpha_{10}} \cdot \left(\frac{C}{D \cdot p_m}\right)^{-\alpha_{11}} \cdot \left(\frac{a_1 - \mathrm{b}_1\beta}{D^4 \cdot p_m \cdot Q^{-1}}\right) = \\ &= \frac{Q}{D} k_3 \cdot \left(\frac{F_{st}}{p_m}\right)^{\alpha_9} \cdot \left(\frac{\omega_d}{D^{-3} \cdot Q}\right)^{-\alpha_{10}} \cdot \left(\frac{C}{D \cdot p_m}\right)^{-\alpha_{11}} \cdot (a_1 - b_1\beta) \end{aligned} \tag{33}$$

Where k_3, α_9, α_{10}, α_{11}, a_1, b_1 – empirical coefficients to be determined by the results of the experimental studies conducted for the planned experiment.

The calculation for the interlayer hardness $p_m = 1440$ MPa we have:

$Q/D = 7 \cdot 10^{-3}/93 \cdot 10^{-3} = 0{,}0753$; $D^{-3} \cdot Q = (93 \cdot 10^{-3})^{-3} \cdot 7 \cdot 10^{-3} = 8{,}7026$.

$D \cdot p_m = 93 \cdot 10^{-3} \cdot 1440 \cdot 10^{6} = 1{,}3392 \cdot 10^{8}$.

Given the above:

$$\begin{aligned}\bar{T}_d &= 0{,}0753 \cdot k_3 \cdot \left(\frac{F_{st}}{1440 \cdot 10^{6}}\right)^{\alpha_9} \cdot \left(\frac{\omega_d}{8{,}7026}\right)^{-\alpha_{10}} \cdot \left(\frac{C}{1{,}3392 \cdot 10^{8}}\right)^{-\alpha_{11}} \cdot (a_1 - b_1\beta) = \\ &= 1{,}26 \cdot 10^{17} \cdot k_3 \cdot F_{st}^{\alpha_9} \cdot \omega_d^{-\alpha_{10}} \cdot C^{-\alpha_{11}} \cdot (a_1 - b_1\beta)\end{aligned} \tag{34}$$

For interlayer hardness $p_m = 2050$ MPa we will have:

$D \cdot p_m = 93 \cdot 10^{-3} \cdot 2050 \cdot 10^{6} = 1{,}9065 \cdot 10^{8}$

$$\begin{aligned}\bar{T}_d &= 0{,}0753 \cdot k_4 \cdot \left(\frac{F_{st}}{2050 \cdot 10^{6}}\right)^{\alpha_{12}} \cdot \left(\frac{\omega_d}{8{,}7026}\right)^{-\alpha_{13}} \cdot \left(\frac{C}{1{,}9065 \cdot 10^{8}}\right)^{-\alpha_{14}} \cdot (a_2 - b_2\beta) = \\ &= 2{,}56 \cdot 10^{17} \cdot k_4 \cdot F_{st}^{\alpha_{12}} \cdot \omega_d^{-\alpha_{13}} \cdot C^{-\alpha_{14}} \cdot (a_2 - b_2\beta)\end{aligned} \tag{35}$$

4. DEVELOPMENT OF MATHEMATICAL MODELS OF POWER CONSUMPTION OF ROCK DESTRUCTION PROCESS IN THE BOTTOMHOLE

Taking for the generalized coordinate the average torque value on the bit, the process of its change will be described as functional dependence:

$$W_P = f\,(F_{st} \cdot \omega_d \cdot C \cdot \beta \cdot D \cdot p_m \cdot Q) \tag{36}$$

The dimension of the moment capacitance – $[W_P] = \text{N} \cdot \text{m/m}^3 = \frac{\text{kg} \cdot \text{m} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^3} = M \cdot L^{-1} \cdot T^{-2}$. Independent parameters, according to which we determine the similarity criteria are the same as in the definition of the functional dependence similarity criteria of the mechanical drilling speed. According to this similarity criteria 2–5 are not changed.

The first similarity criterion has the form:

$$\Pi_1 = \frac{W_P}{D^{m_1} \cdot p_m^{n_1} \cdot Q^{p_1}} \tag{37}$$

We express this criterion through the dimensions of corresponding values:

$$\Pi_1 = \frac{L^{-2} \cdot T^{-2} \cdot M^{1}}{(L^{1})^{m_1} \cdot (L^{-1} \cdot T^{-2} \cdot M^{1})^{n_1} \cdot (L^{3} \cdot T^{-1})^{p_1}} \tag{38}$$

From the condition of equality of exponents at L, T and M in the numerator and denominator we obtain the system of equations:

$$\begin{aligned} m_1 - n_1 + 3p_1 &= -1 \\ -2n_1 - p_1 &= -2 \\ n_1 &= 1 \end{aligned} \tag{39}$$

By substituting the value $m_1 = 0$, $n_1 = 1$, $p_1 = 0$ in (28), we write down the first criterion of similarity:

$$\Pi_1 = \frac{W_P}{p_m^1}. \tag{40}$$

Let's give the desired functional dependence as:

$$\frac{W_P}{p_{\mathrm{m}}^1} = f\left(\frac{F_{st}}{p_m} \cdot \frac{\omega_d}{D^{-3} \cdot Q} \cdot \frac{C}{D \cdot p_m} \cdot \frac{\beta}{D^4 \cdot p_m \cdot Q^{-1}}\right) \tag{41}$$

Based on the results of bench experimental studies [1] conducted due to a classic plan, the dependences $W_P = f\,(F_{st})$, $W_P = f\,(\omega_d)$ are satisfactorily interpreted by a power function, which is an indicator for the dependence $W_P = f\,(F_{st})$ – «$-\alpha_{15}$», and for the dependence $W_P = f\,(\omega_d)$ – «$-\alpha_{16}$». Dependence $W_P = f\,(C)$, which has a local minimum, is satisfactorily interpreted by the polynomial of the second degree $\overline{W}_P = A_1 - A_2 \cdot C + A_3 \cdot C^2$. Dependence $W_P = f\,(\beta)$ which has a local maximum is satisfactorily interpreted by a polynomial of the second degree $\overline{W}_P = A_4 + A_5 \cdot \beta - A_6 \cdot \beta^2$.

Let's present the required functional dependence as:

$$\begin{aligned} W_P &= p_m \cdot k_5 \cdot \left(\frac{F_{st}}{p_m}\right)^{-\alpha_{15}} \cdot \left(\frac{\omega_d}{D^{-3} \cdot Q}\right)^{-\alpha_{16}} \cdot \left(\frac{A_1 - A_2 \cdot C + A_3 \cdot C^2}{D \cdot p_m}\right) \cdot \left(\frac{A_4 + A_5 \cdot \beta - A_6 \cdot \beta^2}{D^4 \cdot p_m \cdot Q^{-1}}\right) = \\ &= \frac{Q}{D^5 \cdot p_m} \cdot k_6 \cdot \left(\frac{F_{st}}{p_m}\right)^{-\alpha_{15}} \cdot \left(\frac{\omega_d}{D^{-3} \cdot Q}\right)^{-\alpha_{16}} \cdot \left(A_1 - A_2 \cdot C + A_3 \cdot C^2\right) \cdot \\ &\cdot \left(A_4 + A_5 \cdot \beta - A_6 \cdot \beta^2\right) \end{aligned} \tag{42}$$

Where k_5, α_{15}, α_{16}, $A_1 \ldots A_6$ – empirical coefficients to be determined by the results of experimental studies conducted according to the planned factorial experiment.

The calculation for the interlayer hardness $p_m = 1440$ MPa give:

$$\frac{Q}{D^5 \cdot p_m} = \frac{7 \cdot 10^{-3}}{\left(93 \cdot 10^{-3}\right)^5 \cdot 1440 \cdot 10^6} = 6{,}9875 \cdot 10^{-7}$$

$D^{-3} \cdot Q = (93 \cdot 10^{-3})^{-3} \cdot 7 \cdot 10^{-3} = 8{,}7026.$

Inlight of outlined:

$$W_P = 6{,}9875 \cdot 10^{-7} \cdot k_5 \cdot \left(\frac{F_{st}}{1440 \cdot 10^6}\right)^{-\alpha_{15}} \cdot \left(\frac{\omega_d}{8{,}7026}\right)^{-\alpha_{16}} \cdot$$

$$\cdot \left(A_1 - A_2 \cdot C + A_3 \cdot C^2\right) \cdot \left(A_4 + A_5 \cdot \beta - A_6 \cdot \beta^2\right) = \quad (43)$$

$$= 8756{,}53 \cdot k_6 \cdot F_{st}^{-\alpha_{15}} \cdot \omega_d^{-\alpha_{16}} \cdot \left(A_1 - A_2 \cdot C + A_3 \cdot C^2\right) \cdot \left(A_4 + A_5 \cdot \beta - A_6 \cdot \beta^2\right).$$

For interlayer hardness $p_m = 2050$ MPa we have:

$$\frac{Q}{D^5 \cdot p_m} = \frac{7 \cdot 10^{-3}}{\left(93 \cdot 10^{-3}\right)^5 \cdot 2050 \cdot 10^6} = 4{,}9083 \cdot 10^{-7}.$$

$$W_P = 4{,}9083 \cdot 10^{-7} \cdot k_6 \cdot \left(\frac{F_{st}}{2050 \cdot 10^6}\right)^{-\alpha_{17}} \cdot \left(\frac{\omega_d}{8{,}7026}\right)^{-\alpha_{18}} \cdot$$

$$\cdot \left(A_7 - A_8 \cdot C + A_9 \cdot C^2\right) \cdot \left(A_{10} + A_{11} \cdot \beta - A_{12} \cdot \beta^2\right) = \quad (44)$$

$$= 8756{,}53 \cdot k_6 \cdot F_{st}^{-\alpha_{17}} \cdot \omega_d^{-\alpha_{18}} \cdot \left(A_7 - A_8 \cdot C + A_9 \cdot C^2\right) \cdot \left(A_{10} + A_{11} \cdot \beta - A_{12} \cdot \beta^2\right).$$

5. CONCLUSIONS

For the first time on the basis of dimensional analysis is using similar criteria we obtained a multifactorial mathematical model of the mechanical speed (25), (26), moment capacitance (33), (34) and energy intensity (43), (44) of drilling with roller-cone bits the sandstone interlayers of Vorotyschenska strata with hardness 1440 and 2050 MPa, respectively, taking into account the drive parameters and the stiffness and damping of the drilling tool.

REFERENCES

[1] Moysyshyn V.M., Borysevych B.D., Havryliv Yu.L., Zinchenko S.A.: *Stiykist' i kolyvannya buryl'noyi kolony*. Lileya-NB, Ivano-Frankivsk 2013.

[2] Myslyuk M.A., Zarubin Yu.O.: *Modelyuvannya yavyshch i protsesiv u naftohazopromysloviy spravi*. Ekor, Ivano-Frankivsk 1999.