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MODELLING AND MOTION ANALYSIS OF FIVE-BAR 5R MECHANISM

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This article presents the use of the potential of the present-day CAD/CAE systems in modelling and a kinematic analysis of a spatial five-membered lever mechanism 5R. A simulation model was developed. On the grounds of the methodology developed, numerical analysis was conducted. The results of the analysis are presented in diagrams.

Key words: CAD/CAE, numerical analysis, spatial dimension chain, collision, trajectory.

1. Introduction

The present study covers a kinematic analysis of a model of a spatial five-membered lever mechanism 5R. In order to develop the design, a designing methodology was used related to CAD/CAE computer technology (Bil and Budniak, 2012). The methodology proposed allows one to build multi-membered mechanisms that use among others spatial measurement chains.

The following are the primary elements of the method realized: 3D edge modelling of the members of the mechanism; modelling of spatial measurement chains, initial kinematic analysis of the 3D skeleton design; parametric modelling of the parts and assembly of the mechanism; a collision analysis of the elements of the mechanism in motion; an analysis of the kinematic quantities of the model for its work cycle (displacements, speeds, linear and angular accelerations); a visualization of the design, a photo-realistic presentation and animation of the work of the mechanism.

2. Modelling of five-bar 5r mechanism

The primary element in the process of designing or an analysis of mechanical systems that use computer aided processes is the creation of a virtual model of a spatial multi-membered mechanism including its appropriate elements. In the first stage of the creation of this model, one of the following concepts can be selected:

- 3D parametric edge modelling of the arms of the mechanism (Bil and Budniak, 2012), and further making an assembly in accordance with the data contained in the mathematical model (Bil, 2012a; 2012b; Bil and Budniak, 2014);
- modelling of a spatial measurement chain in CAD software, and further modelling of the arms and the assembly of the mechanism with the use of constructive geometry (constructional planes, axes and points, as well as local coordinate systems: related to the elements of the spatial measurement chain).

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In the present article, a concept was accepted where a generalized spatial five-membered lever mechanism 5R was constructed based on the spatial measurement chain created.

2.1. Spatial measurement chain

In the present study, an analysis was conducted of a spatial five-membered lever mechanism 5R that was constructed based on the mathematical model presented in the article (Bil and Budniak, 2014). Such a mechanism with five rotational pairs (Fig.1) will be formed when in the common point $S=S_2=R_4$ there will exist a kinematic pair with a common axis. In order to ensure a relative position of the points S_2 and R_4 with specific accuracy, one should above all determine the mutual position of the members of the mechanism. For a unique determination of the elements to be combined, the co-linearity bond of the collaborating rotational axes was used. The bond of the connection of edge ends was employed, as well. However, considering a "redefinition" of the closing members of the mechanism (connection of arms s_2 and s_4), which followed from errors in the definition of the linear and angular dimensions, a distance bond was introduced.

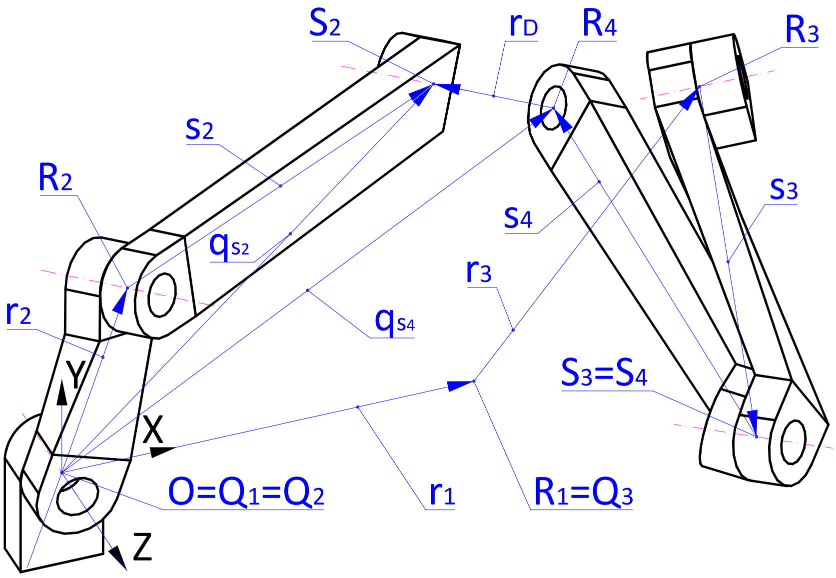

Fig.1. Location of the arms of the virtual model of 5R mechanism.

The r_Δ value, which at the same time is the value of the closing link of the spatial measurement chain, and which determines the error of the mutual location of the nodal points S_2 and S_4, depends on the accuracy of the mutual position of the following members: r_1, r_2, r_3, r_4, s_1, s_2, s_3 and s_2 and their workmanship accuracy.

In real conditions, as a result of inaccuracies in the workmanship and mutual position of the arms of the mechanism, the elements to be combined in the assembly position will have displacements and angular deviations in space that make their assembly difficult.

In a general case, the value of the closing link r_Δ is calculated from Formula (2.1)

$$\boldsymbol{r}_\Delta = \boldsymbol{q}_{s_2} - \boldsymbol{q}_{s_4} = \boldsymbol{s}_2 + \boldsymbol{s}_2 - \boldsymbol{s}_1 - \boldsymbol{s}_3 - \boldsymbol{s}_3 - \boldsymbol{s}_4 \tag{2.1}$$

where:

$\boldsymbol{q}_{s_2}, \boldsymbol{q}_{r_4}$ – vectors that determine the position of the points S_2 and R_4 in relation to the global system of coordinates *OXYZ*,

$\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3}, \boldsymbol{s_2}, \boldsymbol{s_3}, \boldsymbol{s_4}$ – vectors that determine the position of the nodal points R_1, R_2, R_3, S_2, S_3, S_4 in relation to their local systems of coordinates.

A description of the configuration of the 5R mechanism system can be considered as a description of the relative position of the local systems of coordinates connected with the individual links of the measurement chair: to the reference members r_1 and r_3 (the basis), the global system *OXYZ* is attributed. This approach to the description of this system arranges well and formalizes its modelling both in the area of the kinematics and dynamics of the mechanism.

At every moment, the movable elements of the 5R mechanism take a specific position in relation to the base and to one another. When analysing the position of this system, it is particularly important to determine the mutual position of the points S_2 and R_4 for the position of the remaining members of the mechanism given. Figure 2 presents the 5R mechanism, to which the following systems of rectangular coordinates were attributed:

- *OXYZ* – an absolute system of coordinates connected with the immovable arm r_1;
- $Q_2X_{R_2}Y_{R_2}Z_{R_2}$ – the local system of coordinates that constitutes the main assembly base in the determination of the position of the member r_2: it was accepted that the beginning of this system Q_2 and the axis Z_{R_2} coincide with the beginning of the absolute system of coordinates *OXYZ* and its axis *Z*;
- $Q_3X_{R_3}Y_{R_3}Z_{R_3}$, $R_2X_{S_2}Y_{S_2}Z_{S_2}$, $R_3X_{S_3}Y_{S_3}Z_{S_3}$, $R_4X_{S_4}Y_{S_4}Z_{S_4}$ – the local systems of coordinates that coincide with the main assembly bases that determine the position of the arms r_3, s_2, s_3 and s_4.

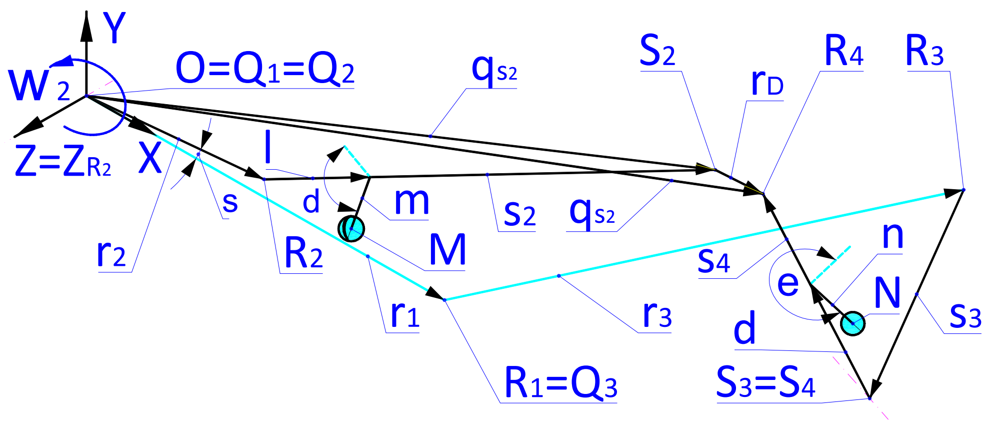

Fig.2. Description of the position of the points *M* and *N* of connecting members s_2 and s_4.

The vectors of the positions of the points S_2 and S_4 in their local systems of coordinates are as follows

$$S_{2R_2} = \begin{bmatrix} s_2 \\ 0 \\ 0 \end{bmatrix}, \qquad R_{4S_3} = \begin{bmatrix} s_4 \\ 0 \\ 0 \end{bmatrix} \tag{2.2}$$

The same points are described with the vectors S_2 and R_4 and determine their position in the *OXYZ* coordinate

$$S_2 = R_{R_2} \cdot S_{2R_2} + T_{R_2}, \tag{2.3}$$

$$R_4 = R_{Q_3} \cdot R_{4Q_3} + T_{Q_3} \tag{2.4}$$

where

$$R_{4Q_3} = R_{S_3} \cdot S_{4S_3} + T_{S_3} \tag{2.5}$$

R_{R_2} - rotation matrix that determines the rotation of the local system of coordinates $R_2X_{S_2}Y_{S_2}Z_{S_2}$ around axes X_{S_2}, Y_{S_2}, Z_{S_2}; T_{R_2} - vector that describes the position of the local system of coordinates $R_2X_{S_2}Y_{S_2}Z_{S_2}$ in the system of absolute coordinates *OXYZ*; R_{Q_3} - rotation matrix that determines the rotation of the local system of coordinates $Q_3X_{Q_3}Y_{Q_3}Z_{Q_3}$ around axes X_{R_3}, Y_{R_3}, Z_{R_3}; T_{Q_3} - vector that describes the position of the local system of coordinates $Q_3X_{R_3}Y_{R_3}Z_{R_3}$ in the system of absolute coordinates *OXYZ*; R_{S_3} - rotation matrix that determines the rotation of the local system of coordinates $R_3X_{S_4}Y_{S_4}Z_{S_4}$ around axes X_{S_4}, Y_{S_4}, Z_{S_4}; T_{S_3} - vector that describes the position of the local system of coordinates $Q_3X_{S_3}Y_{S_3}Z_{S_3}$ in the system of absolute coordinates *OXYZ*; s_2, s_4 – lengths of the arms of the 5R mechanism.

A complex movement of the arms r_2 and s_3 as well as the connectors s_2 and s_4 of the kinematic system of the mechanism under analysis constitutes the essence of the system. At any moment, the movable arms take a specific position in relation to the base: the combined arms r_1 and r_3, as well as to one another. When analyzing the kinematics of this system, it is of a particular importance to determine the relative position of the points *M* and *N* for a given position of the driving member r_2 (angle *s*), which is permanently connected with the movable connector s_2.

In the auxiliary (local) systems of coordinates $R_2X_{S_2}Y_{S_2}Z_{S_2}$ and $S_3X_{S_4}Y_{S_4}Z_{S_4}$ (Fig.2), one can describe the position of the points *M* and *N* with vectors of the lengths that are equal to *m* and *n* and are shifted along the axes X_{S_2} and X_{S_4} to the distances *l* and *d*, and are rotated around these axes by angles δ and ε

$$M_{S2} = \begin{bmatrix} l \\ m \cdot \cos\delta \\ m \cdot \sin\delta \end{bmatrix}, \qquad N_{S4} = \begin{bmatrix} d \\ n \cdot \cos\varepsilon \\ n \cdot \sin\varepsilon \end{bmatrix}. \tag{2.6}$$

In a particular case, the points *M* and *N* can for example be the gravity centre of the masses of the members s_2 and s_4. The points of the remaining movable members can be defined in a similar manner.

It is advantageous to have an analytical description of the position of the kinematic system in the form of an explicit function presented above, as this constitutes the point of departure for further analyses, not only kinematic ones. In the present article, an integrated CAD/CAE system was used to determine the kinematic parameters of the movement.

2.2. Solid model

The first stage of the construction of the virtual 5R mechanism was to create models that contained the constructive geometry of its individual parts. This geometry is formed by the structural planes, axes and structural points as well as the beginning of the local systems of coordinates. The parameters of constructive geometry, which determine the positions of the local systems of coordinates, their axes and points S_2 and R_4, were written in the form of modelling variables. The values of these variables correspond to the elements of the matrix of rotations and the vectors described in Eqs (2.2)-(2.6).

Based on the model of the base of the 5R mechanism, relations were defined that occur between the remaining elements of the assembly. For the purpose of an explicit determination of the positions of the individual elements of the mechanism, the assembly bases of the components to be combined were used that are adjacent to one another. Figure 3 presents the final view of the virtual model of the 5R mechanism including constructive geometry that constitutes an element of the spatial measurement chain.

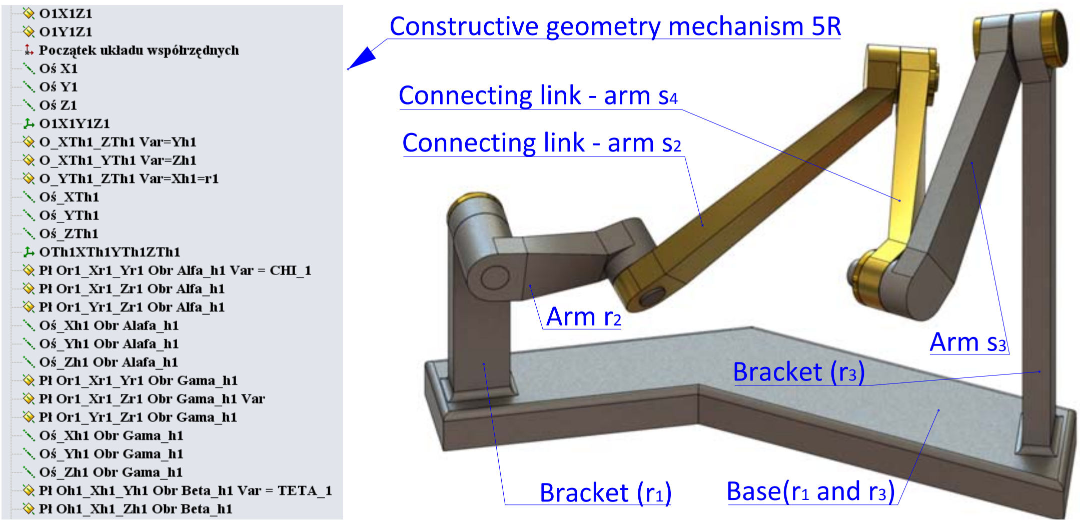

Fig.3. View of the virtual model of 5R mechanism.

These models are characterised by a high complexity level due to the complex constructive geometry. The rotational axes of the arms are located on different planes and are warped in relation to one another. At the same time, the semi-pairs of the neighbouring arms of the mechanism have a common point and common axes.

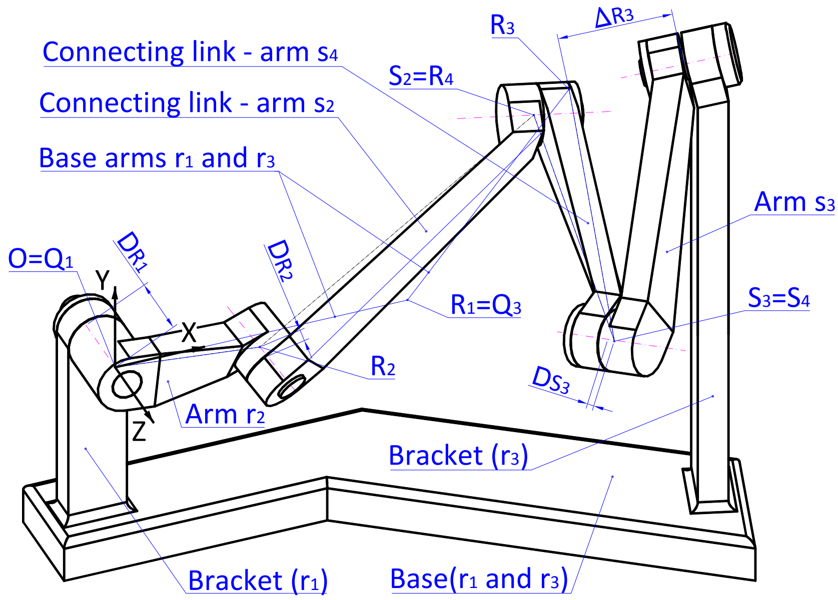

Fig.4. Graphical presentation of 5R mechanism that works with no collisions.

A creation of a model of a solid structure, where arm axes in the form of straight sections are connected with one another in the mechanism nodes, is not possible without avoiding collisions in motion. Therefore, a constructional solution was accepted where, in order to ensure the rotation of the arms, sliding bearings with roll journals were used. A sleeve is constituted by an opening in the housing of the arm of the mechanism. This housing is the sleeve. In order to avoid collisions, the fronts of the sleeves of the arms s_2 and s_4 are adjacent to one another in the plane that is perpendicular to the rotation axis in nodes $S_2=R_4$. At the same time, the remaining nodes, in accordance with Fig.4, were shifted along the individual rotation axes to the following distances: $\Delta_{R1}=6mm$, $\Delta_{R2}=2.5mm$, $\Delta_{R3}=15mm$, $\Delta_{S3}=1.2mm$.

3. Analysis of kinematic system

3.1. Analysis of position

A skeleton model of the 5R mechanism shown in Fig.2 was developed for the purpose of simulation tests. In order to describe the movement trajectory and the positions of the selected elements of the mechanism, an immovable system of coordinates $QXYZ$ was accepted. This system is connected with the immovable arm r_1, whose function is performed by the base of the mechanism with brackets. Point $O=Q_1$ which constitutes the beginning of the immovable system of coordinates is located in the node that connects the members r_1 and r_2. The X axis is connected with the member r_1, while the Z axis coincides with the rotation axis Z_{R_2} of the active arm r_2.

The parameters related to the measurements and masses of the arms were written as modelling variables. Due to the spatial situation of the structural elements of the mechanism, reference geometry was used in the form of structural planes and axes that determine the position of the essential parameters of the draft, such as the rotation axis of the arm r_2, the common rotation axis of the arms r_2 and s_4, r_3 and s_3, s_3 and s_4.

Owing to the use of the SolidWorks Motion software (Chan, 2011), it was possible to find the positions of the characteristic points of the members. The positions of these points are sought on their trajectories that are the result of those constraints (of lengths and angles) that are imposed by the individual members and kinematic pairs (Bil and Budniak, 2014).

Forcing of relative motion of the arms in order to perform a simulation of the movement was obtained by applying the rotation of the active arm r_2. This member performs rotational motion with a constant angular velocity $\omega_2 = 2\pi\ s^{-1}$. A numerical analysis of the relative position of the members of the 5R spatial mechanism was conducted for the points M and N: the movable connectors s_2 and s_4.

Figure 5 presents a five-membered mechanism 5R with trajectories τ_{R_2}, τ_M and τ_{S_2} marked by the following points: R_2, S_3, M, N and $S_2=S_4$.

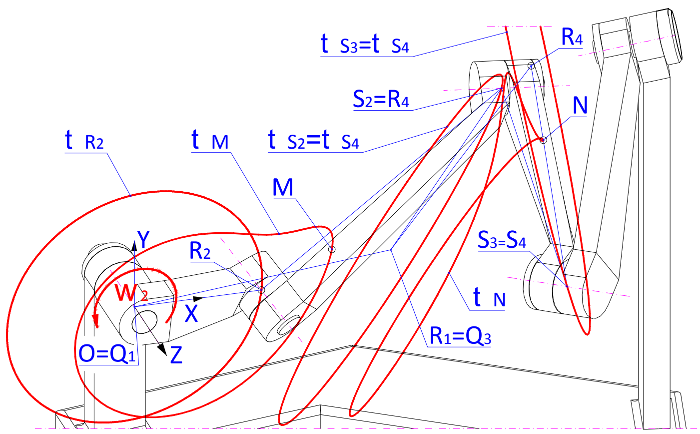

Fig.5. Trajectories of selected characteristic points of 5R mechanism.

If the active member is the movable arm r_2 that rotates with the constant rate of rotation ω_2, the configuration of the system is as follows:

- points R_2, S_2=R_4, S_3=S_4 move on the circle τ_{R_2}, $\tau_{S_2} = \tau_{R_4}$ and $\tau_{S_3} = \tau_{S_4}$,
- trajectories τ_M and τ_N of the points M and N (with the following parameters: $l=d=10mm$, $m=n=5mm$ and $\delta=\varepsilon=90°$) are complex curves in three-dimensional space. A change to the values of the coordinates of these points in the function of time t, during one work cycle, is presented in Fig.6.

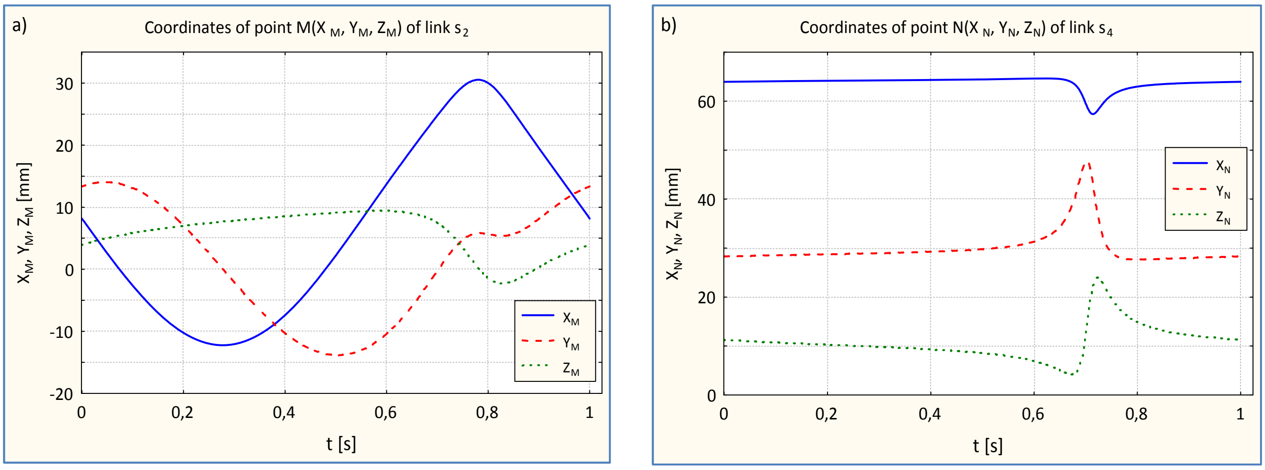

Fig.6. Coordinates of points, a) $M(X_M,Y_M,Z_M)$ of connector s_2, b) $N(X_N, Y_N, Z_N)$ of connector s_4.

The results presented in Figs 5 and 6 of simulation tests illustrate only selected factors that have an influence of the work of the 5R mechanism. The simulation model developed makes it possible to test the linear and angular displacements of the elements in the area of an analysis of the relative position of all of its members, the determination of the virtual space outline, where all the elements of the mechanism work in accordance with the function that follows from the mathematical model, etc.

3.2. Velocities and accelerations

Knowing those quantities that describe the configuration of the kinematic system of the 5R mechanism, one can describe the movement in the area of velocities and accelerations, which are defined as successive derivatives of linear and angular displacements in relation to time (Budniak and Bil, 2012). To determine these, vector coordinates, complex numbers, absolute coordinates etc. are used. In the present article, those parameters were determined on the grounds of numerical simulations in the SolidWorks Motion software (Chang, 2011).

An analysis of the linear velocity of the movement of the points of the 5R mechanism was conducted with the constant rate of rotation of the active member r_2, which rotates with angular velocity $\omega_2 = 2\pi\ s^{-1}$. Fig.7 presents a change to the velocities v_M and v_N for the points M and N.

The resultant linear velocity v_M (Fig.7a) is in the range from $80.9\ ms^{-1}$ to $158.2\ ms^{-1}$. The linear velocity is $v_M = 47.4 \div 917.9\ mms^{-1}$ (Fig.7b). The greatest changes to the linear velocities are observed in the time interval $t = 0.55 \div 0.9\ s$.

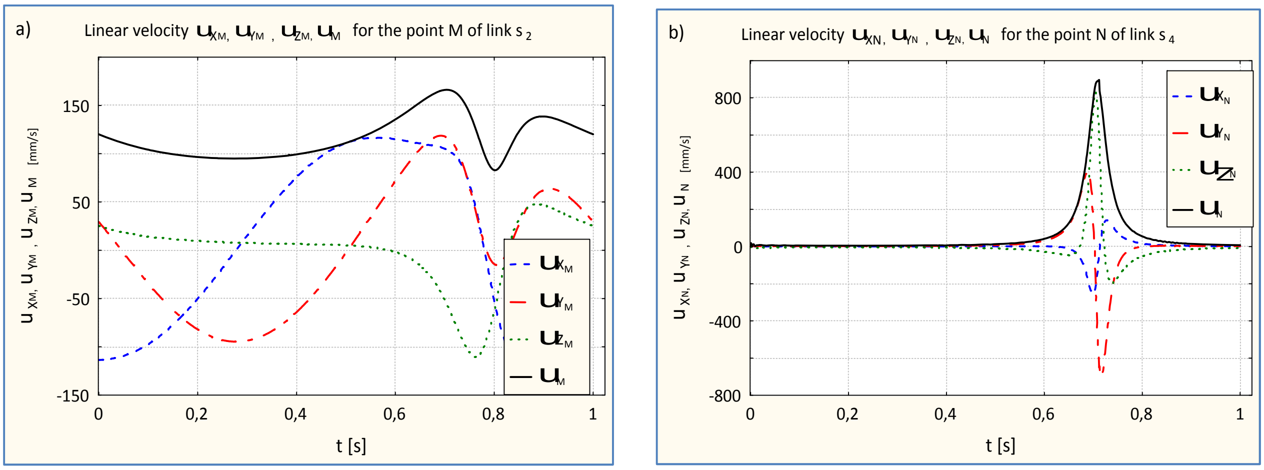

Fig.7. Linear velocities: a) v_M point M of connector s_2, b) v_N point N of connector s_4.

An analysis of the linear acceleration of the points M and N (Fig.8) was conducted with the same parameters as for the simulations of linear velocities.

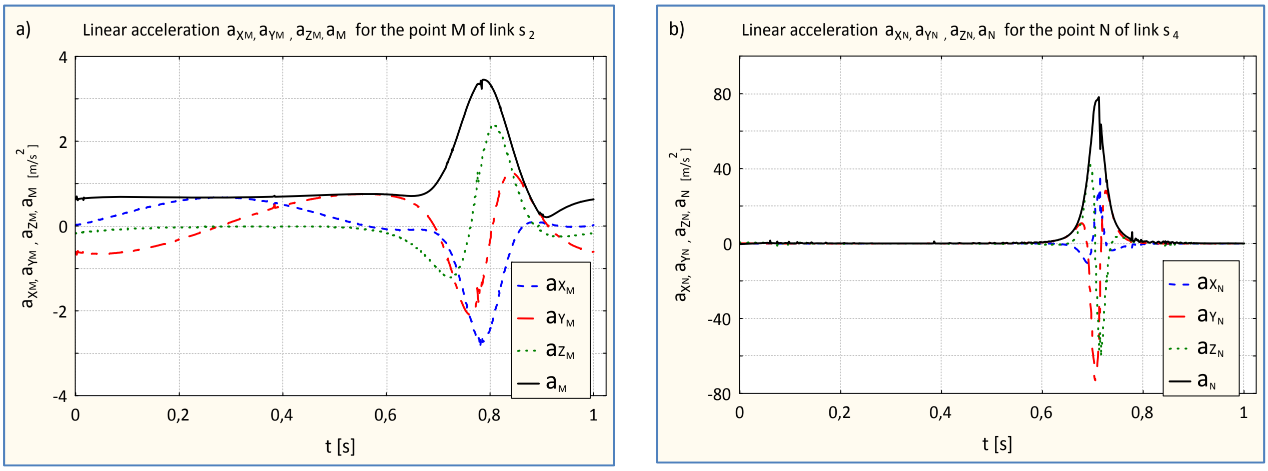

Fig.8. Linear accelerations: a) a_M point M of connector s_2, b) a_N point N of connector s_4.

The resultants of the acceleration of the points M and N in the time interval $t = 0 \div 0.65s$ are of a stable nature ($a_M \approx 0.74\ ms^{-2}$, $a_N \approx 0.5\ ms^{-2}$). However, in time $t=0.65 \div 0.85\ s$, an instability was observed of the acceleration value, which is in a wide range and reaches maximum values $a_{Mmax} \approx 3.4\ ms^{-2}$, $a_{Nmax} \approx 78.2\ ms^{-2}$. This is connected with large curvatures of the trajectories τ_M and τ_N.

Conclusions

The purpose of this article was to present the potential of the present-day CAD/CAE systems to design and analyse the designs of spatial multi-member mechanisms on the example of a five-membered spatial mechanism 5R. The simulation model presented allows kinematic analyses which enable the following: carrying out tests of collisions in motion, the determination of the relative positions of the characteristic points of the mechanism, the determination of the virtual space where all the elements of the mechanism work in compliance with the function that follows from the mathematical model. This system realizes the idea of virtual modelling of physical systems and it reduces the number of prototypes required to produce a new design.

The results presented in the study of sismulation tests illustrate only selected factors that have an impact on the operation of a spatial four-member mechanism 5R. The simulation model developed constitutes a point of departure for further analyses.

In the future, practical applications are foreseen of the results of the analyses presented in this article in an optimization of the spatial designs of spatial multi-member mechanisms.

Nomenclature

a_M, a_N – linear accelerations the points M and N [mm/s^2]
l, d – shift points M and N along the arms s_2 and s_4 [mm]
m, n – distance between the points M and N of arms $s2$ and $s4$ [mm]
$\boldsymbol{q}_{s_2}, \boldsymbol{q}_{r_4}$ – vectors that determine the position of the points S_2 and R_4
R_{R_2} – rotation matrix that determines the rotation of the local system of coordinates $R_2X_{S_2}Y_{S_2}Z_{S_2}$ around axes X_{S_2}, Y_{S_2}, Z_{S_2}
R_{Q_3} – rotation matrix that determines the rotation of the local system of coordinates $Q_3X_{Q_3}Y_{Q_3}Z_{Q_3}$ around axes X_{R_3}, Y_{R_3}, Z_{R_3}
R_{S_3} – rotation matrix that determines the rotation of the local system of coordinates $R_3X_{S_4}Y_{S_4}Z_{S_4}$ around axes X_{S_4}, Y_{S_4}, Z_{S_4}
$\boldsymbol{r}_\Delta$ – vectors the closing link
$\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{s}_2, \boldsymbol{s}_3, \boldsymbol{s}_4$ – vectors that determine the position of the nodal points R_1, R_2, R_3, S_2, S_3, S_4
S_2, S_4 – the vectors of the positions in the absolute system of coordinates $OXYZ$
S_{2R_2}, S_{4S_3} – the vectors of the positions of the points S_2 and S_4 in their local systems of coordinates
T_{R_2} – vector that describes the position of the local system of coordinates $R_2X_{S_2}Y_{S_2}Z_{S_2}$ in the system of absolute coordinates $OXYZ$
T_{Q_3} – vector that describes the position of the local system of coordinates $Q_3X_{R_3}Y_{R_3}Z_{R_3}$ in the system of absolute coordinates $OXYZ$
T_{S_3} – vector that describes the position of the local system of coordinates $Q_3X_{S_3}Y_{S_3}Z_{S_3}$ in the system of absolute coordinates $OXYZ$
v_M, v_N – linear velocities the points M and N [mm/s]
δ, ε – the angles of rotation of the arms m and n [grad]
τ_M, τ_N – trajectories of the points M and N
τ_{R_2}, τ_{R_4}, τ_{S_2}, τ_{S_3}, τ_{S_4}, – trajectories of the points R_2, R_4, S_2, S_3, S_4
ω_2 – angular velocity of the active arm r_2 [s^{-1}]

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