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On some wave problems in train-track dynamics

1 Introduction

Railway technology is a fast developing field, there are many new inventions, but also well-known issues that still are partially unsolved. Many of those are directly or indirectly related to oscillations and waves, which are induced by the moving loads exerted by wheels on the track. These result in noise emanation, damage of infrastructure, and of the vehicle itself.

The paper is devoted to the modelling and simulation of selected wave-related phenomena, this way trying to improve our understanding of undesirable effects, which are of great interest to researchers as well as to practitioners in railway engineering.

In particular in the case of high traveling speeds, the dynamic interaction of rail vehicles with the track becomes very complex. Some effects are still not fully understood even by authorities in the field. A wide palette of partial models is required to cover the most important aspects of the topic. Given the spatial scale of wave propagation in rails, far sometimes reaching simplifications are needed to obtain meaningful results.

So, for the study of certain phenomena, it is appropriate to substitute the contact forces exacted by a wheelset by concentrated forces. Descriptions based on combinations of Heaviside functions, can give important insight, because for the resulting differential equations analytical solutions are available. In this case, a much more crucial point of the analysis is the proper application of the radiation conditions, while a high resolution of the contact forces does not contribute to quality of the model. Such observations are valid both in the case of very long tracks, which can be treated as beams of length tending to infinity, and in the case of a finite length train with distributed or discrete interaction.

Frequently the contact load is described by a harmonically variation of a Dirac force, moving at constant speed. More advanced models of the rail vehicle are desirable. However, the simulation of coupled vehicle and track models is much more expensive and thus often too restrictive in time and space scales.

The dynamical behaviour, and particularly the stability of motion, may depend on the assumed modelling. The lower boundary of instability one can obtain by the use of a continuous model, like a set of densely distributed oscillators. It is obvious that in the case of determined mass and spring constants, together with geometrical parameters, the adequate choice of damping parameters is very important, [1]. A bad choice can destabilize the system and reduce the critical speed of the considered rail vehicle.

Some examples of high-speed trains made in France (Fig. 1) and a new train produced in Poland, DART (Fig. 2) may illustrate this. The PESA train is designed for operation at about two times lower speed than the TGV. The train shown in Fig, 1*a* reached the maximum speed of 318 km/h in December 1972, while the train shown in Fig. 1*b* reached the maximum speed of 574.8 km/h in April 2007, [2].

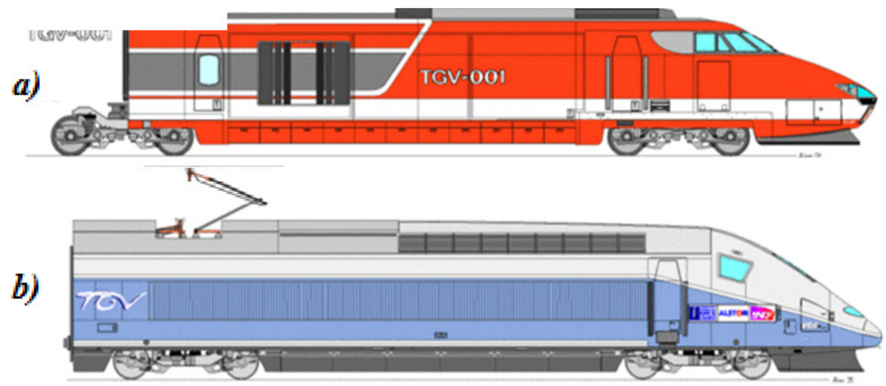

Fig. 1. The TGV trains which reached world records of speed a) in Dec. 1972 and b) April 2007, [2]

Fig. 2. The most recent train made by PESA, DART, which operates in Poland with maximum speed 160 km/h (optional 250 km/h), [3]

As follows from the investigation of general models of train-track interaction, their analysis is very complex and with higher accuracy more and more time consuming.

For this reason it is preferable to consider some phenomena by means of simplified models.

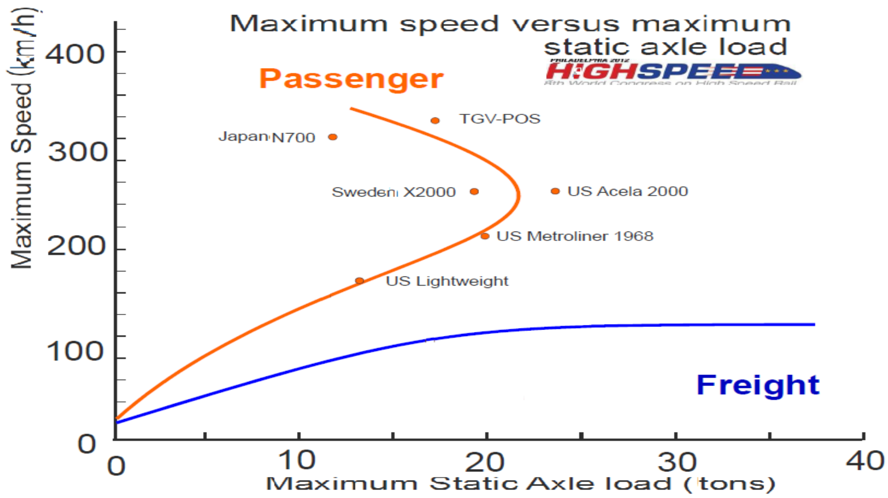

Fig. 3. World tendency of increasing maximum speed vs. static load for freight and passenger trains, [4]

Even today, many experts of railway infrastructure still believe – wrongly – that speed has no essential influence on the real load. That is why often the dynamic load is approximated by 1.15 – 1.30 of the static load. The scientific investigations [5], and the axial loads tendency shown in Fig. 3, indicate that the load of vehicle-infrastructure interaction is many times higher in the realm of high-speed rail traffic.

2 Dynamics and stability of interaction of continuous subsystems in relative motion

2.1 Physical and mathematical modelling

The problem of train-track interaction was considered theoretically in many papers. In the majority of early investigations, the track was assumed as rigid, and the train was modelled as a lumped system. Such modeling does not allow the consideration of various wave phenomena, which occur in reality. There analysis requires an approach by models as shown in Fig. 4. A study of some selected problems with such a setup, concerning the influence of particular system parameters on the stability of systems in relative motion, is given in [1] and [6]. Also some cases of nonlinearity of interaction were investigated in this framework, cf. [6].

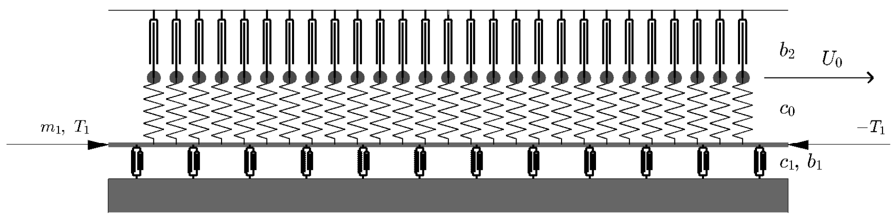

Fig. 4. Model of a moving set of oscillators interacting with a beam and viscoelastic surrounding

Let us consider the system shown in Fig. 4, consisting of two infinite continuous subsystems, which are in relative motion with the velocity U_0. For computational reasons, the continuous vehicle model is replaced by an infinite set of oscillators. Each of those has the mass m_0 which depends on the (mean) mass density of the (long) train and the length of the represented segment. The oscillators interact with a beam, based on a viscoelastic foundation. This submodel represents the track. Again, the continuous foundation is replaced by a sequence of spring-dashpot elements, characterized by their uniform elasticity modulus c_1 and the coefficient of damping b_1. The beam with the bending stiffness EI and the mass density m_1 is subjected to a compression force T_1caused e.g. by thermal effects. The equations of motion, written in the system of coordinates (x, y) connected with the track, take the form:

$$m_0\left(w_{tt} - 2U_0 w_{xt} + {U_0}^2 w_{xx}\right) + b_2 w_t + c_0(w - W) = 0\ , \qquad (1)$$

$$EIW_{xxxx} + T_1 W_{xx} + m_1 W_{tt} + b_1 W_t + c_1 W + c_0(W - w) = 0\ , \qquad (2)$$

where w and W are, respectively, the displacements of the oscillators and of the beam, both in direction of the coordinate y, i.e., perpendicular to the beam.

We are looking for the solution in the form of a travelling wave:

$$w(x,t) = Ae^{ik(x-Vt)}\ , W(x,t) = Be^{ik(x-Vt)}, \qquad (3)$$

where: A is the wave amplitude, k is the wave number and V is the travelling wave velocity. The speed of the moving vehicle is denoted by U_0.

After substituting the general form of the solution (3) into the set of equations (1), (2), one obtains a linear algebraic system for the unknown amplitudes A and B, from which follows the characteristic equation (6) of the problem:

$$m_0 A\left(-k^2V^2 + 2U_0k^2V - {U_0}^2k^2\right) - ib_2k(V - U_0)A + c_0(A - B) = 0\ , \qquad (4)$$

$$EIB(k^4 - T_1k^2) - m_1Bk^2V^2 + ib_1BkV + c_1B + c_0(B - A) = 0\ . \qquad (5)$$

The determinant of the system of equations (4), (5) has to vanish for nonzero solutions to exist, this condition is formulated in (6). Note that Φ takes complex values:

$$\Psi(V, U_0) = \Phi(V - U_0, V) = Re\ \Phi(V - U_0, V) + i\ Im\ \Phi(V - U_0, V) = 0\ . \qquad (6)$$

This equation allows the discussion of the stability problem.

2.2 Stability analysis

Graphical representations of the characteristic equation (6) are shown in Fig. 5*a* and Fig. 5*b* for the elastic and viscoelastic case, respectively. In the purely elastic case of the system, shown on the graph in Fig. 5*a*, the characteristic equation (6) has only real roots, it holds $Re\,\Phi(V-U_0,V)\equiv 0$. In the case of stable motion with constant speed, for any given wave number k, the line $U_0=const$ determines four different values of wave velocity V. When the line $U_0=U_{0_{critical}}$ becomes tangent to the ellipse (which represents the waves in the set of oscillators moving in phase with the waves in the beam), two of the wave velocities V are described by a double root, which constitutes the boundary of instability.

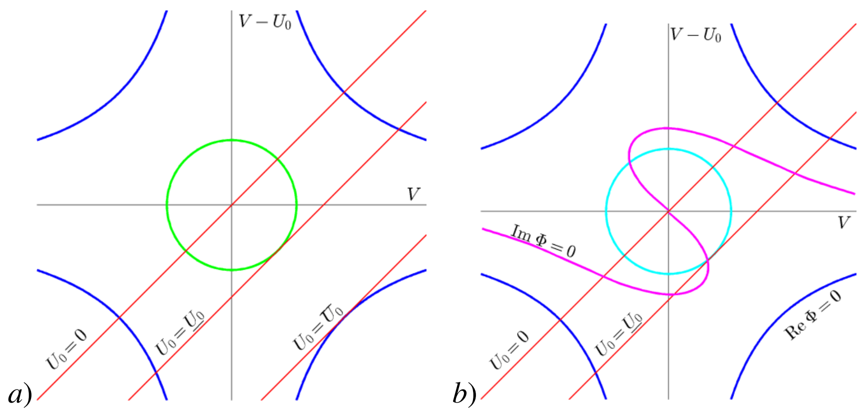

Fig. 5. Characteristic equation in the elastic case (a) and in the viscoelastic case (b)

The instability range is described as follows: $U_0\in\left[U_{0_{cr_1}},U_{0_{cr_2}}\right]=[\underline{U}_0,\overline{U}_0]$. In this range, two wave amplitudes are constant. From the remaining two, the first one is exponentially increasing and the second one decreasing. This is because in the case of real parameters of the system, roots are pairwise complex conjugate complex numbers:

$$V_1=V^*+\mathrm{i}\Upsilon,\qquad V_2=V^*-\mathrm{i}\Upsilon\,. \tag{7}$$

At least one of the discussed above roots fulfills the criterion of instability in the form $\mathrm{Re}\,(\mathrm{i}kV_\iota)>0\,,\ \iota\in\{1,2,3,4\}$.

In the case of the damped system, i.e. for viscoelastic elements, one can apply the Hurwitz [7] or Mikhajlov criterion of stability, or the generalized Mikhajlov criterion [1], which require the existence of four roots of the real part and three roots of the imaginary part of the characteristic equation, situated as shown in Fig. 5*b* for a speed $U_0<\underline{U}_0$. There are some visible qualitative differences between these cases. The critical speed of the viscoelastic system can be smaller than in the purely elastic case. Further, the range of instability in the elastic case is bounded by a finite value $\overline{U}_0$, while in the viscoelastic case the critical range has no upper bound.

3 Dynamics of interaction of a moving lumped system along a continuous one

The train, in general, is moving with a certain speed which can be assumed as constant. The forces by which the pantograph acts on the wire generate dynamic deflections in the traction. The disturbances propagating along the wire are reflected at supports and boundaries, transmitted to supporting wires and other contact wires, causing complicated interactions with the given and other pantographs, [8], [9]. A pantograph interacting at two points with the traction, as well as interaction between two pantographs may destabilize the system, [10], [11].

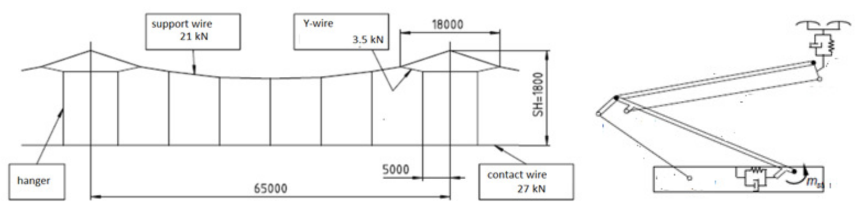

Fig. 6. Segment of traction and an exemplary model of the pantograph

Let us consider a hybrid viscoelastic system, which consists of a single oscillator interacting at two points with distance L with one of the simplest continuous systems – a string with mass density m_1 under tension T_1, see Fig. 6. Such a system can describe some phenomena occurring during the interaction of a pantograph with the contact wire.

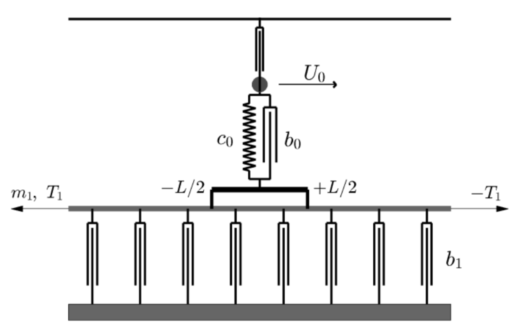

Fig. 7. Model of a single moving oscillator interacting at two points with a string in a viscous surrounding

Much more complicated is the interaction with a beam. This is because the number of waves is two times greater, and further the wave speed depends on the wave number. This means that there occurs dispersion of waves.

The equations of motion for both subsystems, written in the coordinate system connected with the discrete system, are:

$$m_0 w_{tt} + b_2 w_t + b_0(w_t - W_{0_t}) + c_0(w - W_0) = 0 \,, \tag{8}$$

$$\begin{gathered} m_1(W_{tt} - 2U_0 W_{xt} + {U_0}^2 W_{xx} + b_1(W_t - U_0 W_x) - T_1 W = \\ (P_0 + \cos(\omega t))\left(\delta\left(-\tfrac{L}{2}\right) + \delta\left(-\tfrac{L}{2}\right)\right). \end{gathered} \tag{9}$$

$$\begin{gathered} b_0(w_t - W_t) + c_0(w - W) = 2(P_0 + P\cos(\omega t)) \,, \\ 2W_0 = W\left(-\tfrac{l}{2}\right) + W\left(\tfrac{l}{2}\right). \end{gathered} \tag{10}$$

Instead of discussing equation (9) with the given excitation forces as inhomogeneity, it is also possible to study a homogeneous version of this equation together with the following interface conditions at $= \pm\frac{L}{2}$:

$$\lim_{\varepsilon\to 0}\left(T_1\left(W(x,t)_{|x=y-\varepsilon} - W(x,t)_{|x=y+\varepsilon}\right)\right) = P_0 + P\cos(\omega t) \,. \tag{11}$$

The purely elastic case of the system is easy to investigate, because the wave velocity in a string depends only on mass density m_1 and tension T_1. We expect solutions in the form of oscillatory motion with frequency ω of the mass m_0, which generates waves in the string travelling with speed U_0:

$$\begin{aligned} &W_0(t) = A\cos(\omega t), \\ &W(x,t) = B\cos\left(k_1\left(x + \tfrac{L}{2} - (v - U_0)t\right)\right) + B\cos\left(k_1\left(x - \tfrac{L}{2} - (v - U_0)t\right)\right). \end{aligned} \tag{12}$$

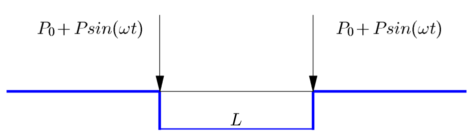

Fig. 8. Two concentrated interaction forces of discrete system acting on a string

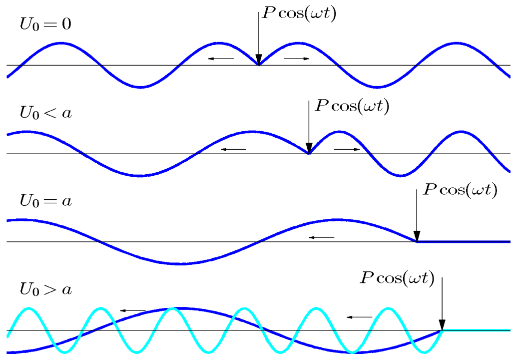

Fig. 9. Waves generated in the string by oscillating force for different speeds of the string, $a = \sqrt{\frac{T_1}{m_1}}$

By denoting $U = \frac{U_0}{a}$, $\Omega = \frac{\omega L}{c_0}$ and substituting the solution form (12) into the set of equations (8)–(10), we obtain again a characteristic equation, which can be written as:

$$\Phi = \Phi(U,\Omega) = Re\,\Phi(U,\Omega) + \mathrm{i}\,Im\,\Phi(U,\Omega) = 0\,. \tag{13}$$

Here, as previously in (6), $Re\,\Phi(U,\Omega)$ and $Im\,\Phi(U,\Omega)$ are the real and imaginary parts of the characteristic determinant Φ.

For selected physical parameters of string and oscillator, root curves of both parts of Φ in the (U,Ω)-plane are shown in Fig. 10.

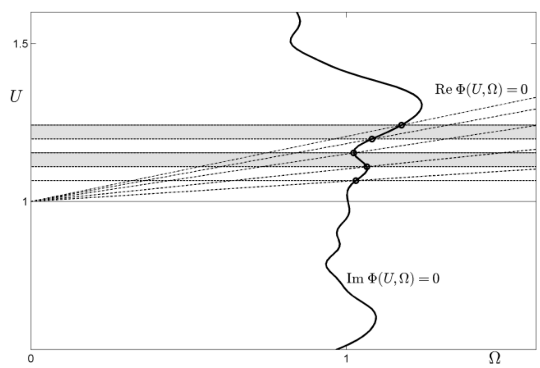

Fig. 10. Characteristic curves for selected parameters of the system

As can be easily seen, for all speeds $U < 1$ the motion is stable. In the range between 1 and 1.2, there is an infinite sequence of alternatively stable and unstable regions.

The crux is double points interaction. This brings about coupling between the source of wave generation and receiver generated disturbances. When the distance between interacting points $L > 0$ is finite, the system becomes stable in the case of an infinite length of string. The only exception is a travelling speed equal to the velocity of elastic waves, i.e. $U = 1$.

The presented above investigations are relatively complicated in comparison with the study of single lumped model of pantograph system dynamics. However, the analysis of real systems, when dumping and bending stiffness are taken into account, are much more complicated. Even in the case of a simple Bernoulli–Euler beam model, there can be already observed dispersion. Further, there is the possibility of waves travelling in the direction of the excitation source, which was discussed in [12]. Several ranges in the parameter plane of normalized speed and frequency have to be distinguished, where qualitatively different types of solution behavior occur.

4 Waves generated by continuous moving loads

Let us now consider a continuously distributed load described by a Heaviside function which was mentioned also in [13]. The front moves at constant speed. This case of dynamics driven by a moving load, can be seen as a model of a magnetically levitated vehicle acting on its track, In the linear case, due to the superposition principle, various shapes of loads can be approximated by linear combinations the discussed piecewise constant load.

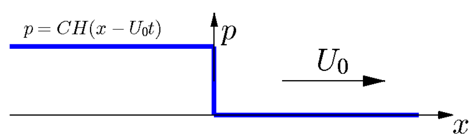

Fig. 11. Moving load with the velocity U_0 described by the Heaviside function $p(x,t) = CH(x - U_0 t)$

The equation of motion for the simple Bernoulli-Euler beam model subjected to load described by the Heaviside function moving with constant speed U_0 takes the form:

$$EIW_{XXXX} + mW_{tt} + hW_t + cW = p(X,t) = CH(X) \,, \tag{14}$$

where W is the displacements of the beam, EI – stiffness of the beam, m – mass density, h –foundation damping parameter, c – stiffness of foundation and C is the intensity of the load.

The solution of the problem can be obtained describing equation of motion in the moving system of coordinates $(X,t) = (x - U_0 t, t)$ with the speed U_0 and fulfilling following boundary conditions:

$$\lim_{X\to-\infty} W(X) = \frac{c}{C}\,, \qquad \lim_{X\to\infty} W(X) = 0\,, \tag{15}$$

$$\lim_{X\to 0-} W(X) = \lim_{X\to 0+} W(X)\,, \qquad \lim_{X\to 0-} W_X(X) = \lim_{X\to 0+} W_X(X)\,,$$
$$\lim_{X\to 0-} W_{XX}(X) = \lim_{X\to 0+} W_{XX}(X),\ \lim_{X\to 0-} W_{XXX}(X) \ = \lim_{X\to 0+} W_{XXX}(X). \tag{16}$$

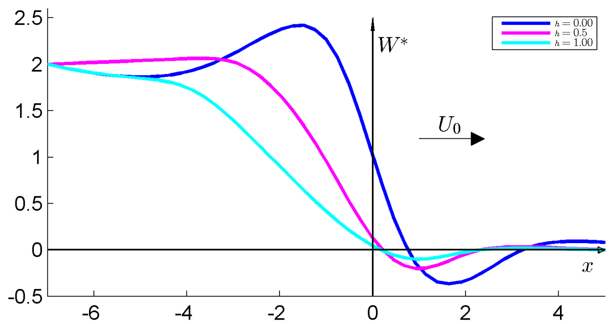

Fig. 12. Displacement of beam subjected to moving load described by the Heaviside function with the velocity $U = 0.8U_{cr}$ for various intensity of damping

We denote $U_{cr} = \sqrt[4]{\frac{4cEI}{m}}$, $U = \frac{U_0}{U_{cr}}$ and $W^* = \frac{W}{W_{st}}$, where W_{st} is the displacement under static load C.

After fulfilling the boundary condition (15) and (16) one can obtain the results shown in Fig. 12 and Fig. 13 for the subcritical case ($U = 0.8U_{cr}$) and supercritical case ($U = 1.2U_{cr}$), respectively.

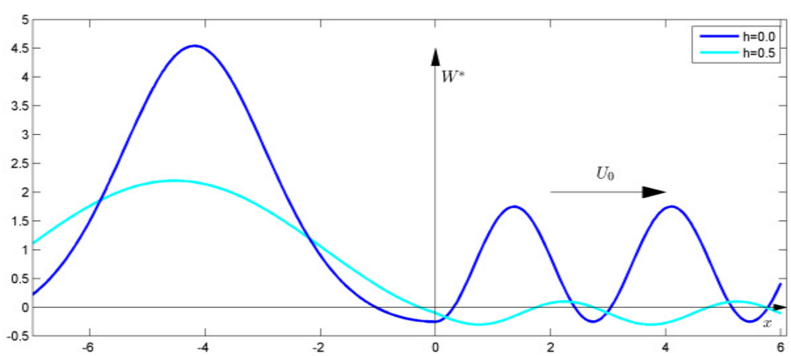

Fig. 13. Displacement of beam subjected to supercritical moving load $U = 1.2U_{cr}$ described by the Heaviside function in the elastic case, $h = 0$, and in the viscoelastic case with $h = 0.5$

The above analytical solution can be used for the approximation of different shapes of loads by means of superposition of any number of loads described by shifted Heaviside functions. In the case of other kinds of approximation, not based on the analytical techniques, special attention has to be paid to the correct definition of absorbing boundary condition. Additionally, the analytical approach allows investigating the consequences of speed variations of the contact point, especially with regard to the dynamic forces and the stability of solutions. This was recently shown in [14].

5 Conclusions

The dynamic interaction of rail vehicles with continuous systems describing the track and/or the catenary system – particularly in the case of high speeds – is relatively complex. Some effects are still not well recognized, and there is a need for further investigation.

In the presented paper, problems of stability were considered for the cases of a set of densely distributed oscillators and of a two-point oscillator, interacting with continuous systems like a beam on a viscoelastic foundation or a string under tension. Additionally to the above mentioned problems, the dynamics of a beam on viscoelastic foundation was considered.

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Summary

The paper is devoted to the modeling, analysis and simulation of selected travelling wave-related phenomena, excited by moving loads. The study improves our understanding of undesirable effects, which are of great interest to researchers as well as to practitioners in railway engineering. It is shown that the dynamic interaction of rail vehicles with continuous and discrete systems describing the track and/or the catenary system is particularly complex in the case of high-speed motion. Further, dynamics and stability of viscoelastic track models are considered.

Keywords: wave phenomena, high-speed motion, railway dynamics

Zagadnienia falowe w dynamice układu pociąg-tor

Streszczenie

Praca jest poświęcona modelowaniu, analizie i symulacji wybranych zjawisk związanych z falami bieżącymi, które wywołane są ruchomym wymuszeniem. Prezentowane badania mają na celu lepsze poznanie niepożądanych efektów, ważnych zarówno dla badaczy, jak i inżynierów pracujących w kolejnictwie. Wykazano, że dynamiczne oddziaływanie różnego typu pojazdów szynowych z ciągłymi i dyskretnymi układami opisującymi tor lub sieć zasilającą jest względnie złożone, w szczególności w przypadku dużych prędkości jazdy. Badania uzupełniono analizą dynamiki i stateczności lepko-sprężystych modeli toru.

Słowa kluczowe: zjawiska falowe, duże prędkości ruchu, dynamika kolejowa