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# SEMI-MARKOV MODEL OF QUALITY STATE CHANGES OF A SELECTED TRANSPORT SYSTEM

**Bogdan Landowski, Łukasz Muślewski**

*University of Science and Technology*
*Machine Maintenance Department*
*Prof. S. Kaliskiego Street 7, 85-789 Bydgoszcz, Poland*
*tel.: +48 52 3408208*
*e-mail: bogdan.landowski@utp.edu.pl*
*lukasz.muslewski@utp.edu.pl*

**Leszek Knopik**

*University of Science and Technology*
*Department of Management*
*Prof. S. Kaliskiego Street 7, 85-789 Bydgoszcz, Poland*
*tel.: +48 52 3408860*
*e-mail: leszek.knopik@utp.edu.pl*

**Piotr Bojar**

*PWSZ in Pila*
*Podchorążych 10, 64-920 Pila, Poland*
*tel.: +48 604 195 937*
*e-mail: pbbojar@gmail.com*

***Abstract***

*The study deals with the problems connected with evaluation and analysis of transport systems operational quality. The research object is a real, sociotechnical system (of the type H-M-E) whose functional quality is affected by: operators, equipment and technical state of he operated vehicles and the environmental impact. A scheme of an assessment model was built on the basis of a developed concept of a system operational quality and a random process was defined to be used for the analysis. The assessment process involves investigating the variability degree of properties included in the resultant model in a given time and determining whether and to what degree the obtained values meet the requirements of the system appropriate operational quality. On the basis of the above assumptions and in result of experimental tests, four qualitative states of a transport system functioning, which represent intervals of values of the obtained grades in given time moments, were distinguished. The study presents a model of system operation qualitative state changes developed with the use of Markov chain inserted into a certain semi-Markov process. A graph and a matrix for probability of transition between the states were created and boundary distributions of the process were determined on the basis of experimentally obtained data. A system of equations was determined for the distinguished Markov chain, on the basis of which a mathematical model for assessment and control of operational quality of selected types of transport systems will be built.*

***Keywords:*** *transport system, efficiency, operation and maintenance process, semi-Markov process, simulation tests*

## 1. Introduction

The article deals with the problems connected with evaluation of transport systems operation. These systems belong to sociotechnical systems of the type <H-M-E>, whose operational quality is affected by operators, equipment, and technical state of operated transport means as well as environmental factors. It is defined that: "the quality of a system is a set of properties expressed by means of numerical values, in given time t, determining to what degree the set requirements are satisfied" [9].

Therefore, in order to build an evaluation model, it is necessary to establish a set of assessment criteria to be used for identification of properties in order to describe a system in terms of its operational quality.

The process of evaluation involves investigating variability degree of the properties distinguished in the resultant model in given time moments and then defining whether and to what degree the obtained values meet the condition of the investigated system functioning desired quality [6].

It needs to be remembered that the properties included in the model should be characterized by actuality, verifiability or falsifiability and they need to meet the requirements such as significance, measurability, variability and lack of correlation with each other.

## 2. General characteristics of the research object

The research object is a public bus transport system which belongs to sociotechnical systems of the type <Human – Machine – Environment>, whose main task is to provide people with transport services in a given quantitative and territorial range, by means of transport means used in the system.

The investigated transport system provides transport services on the territory of a town and its suburbs. Transport services offered by the considered system need to be performed timely, reliably, efficiently and safely taking into consideration the impact of the natural environment. Thus, it is of key importance to ensure appropriate level of its operational quality.

The system was identified and divided into the following subsystems: decision making subsystem, information subsystem, traffic continuity assurance subsystem (operability assurance, diagnostic, fuel supply) and executive one consisting of elementary subsystems of the type <H – TO> (driver –bus) and the environment as a cooperating subsystem directly affecting functioning of the system and its subsystems.

## 3. Method for evaluation of a transport system operational quality

This section includes a description of the rules used for formulation of a method for a transport system operational quality evaluation [7, 8, 10].

It was assumed that the evaluator determines a set of assessment criteria for evaluation of a system operational quality K. Next, the research object is identified and on this basis, having in mind these criteria, they determine a set of properties – X describing the system in terms of its operational quality. It needs to be mentioned that the set of properties accepted for a description of the investigated system consists of two subsystems containing measurable properties and immeasurable ones. Immeasurable properties are those, which cannot be measured due to technical difficulties or the researcher's lack of knowledge. Accepted variability boundaries $X_{M,i}^{min}$, $X_{M,i}^{max}$, consistent with the system appropriate (desired) operational quality, need to be given for each measurable property describing system $X_{Mi}$ (i = 1, 2, ..., n) and similarly for each property assumed to be immeasurable, $X_{Nj}$ (j = 1, 2, ..., m), it is necessary to establish conditions of the desired quality so as to be able to state unequivocally statement whether a given property satisfies them. For this purpose, different values from 0 to m are assigned to immeasurable properties.

This means that in given time t, the system operates with the required quality only when the values of its measurable properties are contained within the established limits and immeasurable properties satisfy the conditions of the system required operational quality.

The evaluation process involves checking whether and to what degree particular properties from X set meet the required criteria K. Evaluation is performed on the basis of measured values of the properties in given time t (measurable properties), or states in which they are in given time t (immeasurable properties), through assigning to them suitable discriminant t. In connection with this, the level of a system operational quality in given time t, is determined by a set of significant properties $\{X_i\}$ i = 1, 2, ..., p accepted for its description from a preset point of view.

Let $X_i(t)$, i = 1, 2, ..., p, denote a property, which is a time dependent random variable, whose performance in given time t, describes the system operational quality:

$$X(t) = \langle X_1(t), X_2(t), ..., X_p(t)\rangle . \quad (1)$$

Component $X_i(t)$, i = 1, 2, …, p, of X(t) vector, is one-dimensional random process in R space, describing the i-th i – property of the system operational quality, whereas vector X(t) is a p – dimensional random process describing explicitly the system operational quality in space $R^p$, in given time t.

In order to assess the analysed system, it is necessary to determine such a set of significant properties of quality Z = $X_i$, i = 1, 2, ..., p, which is divided into n – separable subsets $Z_1$, $Z_2$, ..., $Z_n$, satisfying the below dependencies [6]:

$$\begin{gathered} Z_i \cap Z_j = \varnothing \quad \text{for } i \neq j, \\ Z(t) = Z_1(t) \cup Z_2(t) \cup ... \cup Z_n(t). \end{gathered} \quad (2)$$

Having in mind that the study deals with evaluation of <H-M-E> type transport system operational quality, whose elements are: a human (operator), machine (technical object) and the environment, the resultant model for evaluation of a system operational quality assumes a form described by the below dependence:

$$\begin{aligned} Z_1(t) &= \{X_1(t), ..., X_{k_1}(t)\}, \\ Z_2(t) &= \{X_{k_1+1}(t), ..., X_{k_2}(t)\}, \\ Z_3(t) &= \{X_{k_2+1}(t), ..., X_{k_3}(t)\}, \end{aligned} \quad (3)$$

where: $k_3 = p$.

It was assumed for the purpose of this study that evaluation of a transport system operational quality is modelling of the form [9]:

$$Y : T \times \Omega \to R , \quad (4)$$

which means that $Y(t,\omega)$, $t \in T$, $\omega \in \Omega$ is a measure of a system operational quality in time t, dependent on elementary event ω, where:

Y – measure of a system operational quality being a function of X(t) random variable vector, (reflecting length of $\overline{\Delta K}$ vector),

T = <0, +∞) – set of time moments,

**Ω** – set of elementary events ω,

R – set of real numbers.

The A random process reflecting the system operational quality is defined for the analysed system as:

$$Z_x(t) = \sum_{i=1}^{p} \alpha_i X_i(t) , \quad (5)$$

where:

$\alpha_i \geq 0, \sum_{i=1}^{p} \alpha_i = 1$, i = 1, 2, …, p, denote values of qualitative weights of particular properties, determining the analysed system operational quality,

$Z_x(t)$ – is a random process being a finite combination of processes $X_i(t)$, i = 1, 2, ..., p. The below inequality is obvious for $Z_x(t)$ process:

$$Z_x(t) \leq \sum_{i=1}^{p} \alpha_i q_i \,, \tag{6}$$

where:
$t \in T$, $q_i$ denotes the maximum value, desired for particular properties.

## 4. Description of the system qualitative operational states

Since the resultant model for evaluation of a system operational quality also includes properties foe which the smallest values reflect the desired state and their variability intervals so for the purpose of consistent interpretation of the obtained results they undergo recoding onto <0, 10> range, according to dependence 7.

$$Range = 10*(X_i - X_{min})/(X_{max} - X_{min}), \tag{7}$$

where:
$X_{min} = Min\ \{X_i\}$,
$X_{max} = Max\ \{X_i\}$.

Thus, knowing the system operation value from interval <0, 10>, in given time t, it is possible to define which qualitative state the system is in.

Four qualitative states were distinguished which reflect the values from the accepted range. Names of particular states were defined and adequate values have been assigned to them.

State I – 'desired' – reflects values from interval <8-10>. This state reflects a model desired quality of the system operation.

State II – 'acceptable' – <5.5-8), this is a state of the system correct operation which does not need undertaking any actions to reach state I, transition to which though is connected with undertaking a strategic decision, e.g. replacement of transport means or operators who have a negative influence on the system operational quality.

State III – 'limited' – <4-5.5). This state reflects a boundary quality level of a system operation where transport tasks are performed without compliance with the requirements and intervention is indispensable (change of operator, restoring full operability of a technical object, limiting or eliminating negative environmental factors), in order to reach state I or II of the system operational quality.

State IV – 'critical' – <0-4) – makes it impossible for the system to perform transport tasks. It requires renovation, exchange or modernization to guarantee a return to least state II of the system operational quality.

## 5. Model of semi-Markov qualitative state change process

Semi-Markov process is a generalization of Markov process where time intervals between transitions to particular states can be random variables, which depend on the current state and the possibility of next state occurrence. [1-5].

A natural approach to the semi Markov process is the renewal theory in which time intervals between occurrences of events do not need to have an exponential distribution.

For this reason, it is desirable to define Markov sequence of renewals as a sequence of a two-dimensional random variable.

Two elements of the two-dimensional random variable are observation times of $S_n$, n-th transition to next state and n-th observation $Y_n$, $n \geq 0$, $Y_n \in \{0, 1, 2, ..., n\}$.

Total probability of occurrence of $Y_{n+1} = j$ observation in the period between entrances $S_{n+1} - S_n \leq x$, which, depending on the observation history, satisfies the Markov property, is as follows:

$$P\{Y_{n+1}=j, S_{n+1} - S_n \leq x\ Y_n=i, S_n, Y_{n-1}, S_{n-1}, \ldots, Y_0, 0\} = P\{Y_{n+1}=j, S_{n+1} - S_n \leq x \mid Y_n=i\}, \tag{8}$$

let:

$$P\{Y_{n+1} = j, S_{n+1} - S_n \le x \mid Y_n = i\} = G_{i,j}(x). \tag{9}$$

Finally, a semi Markov process is a stochastic process, which records states of the renewal process in each moment of time.

To make it formal, let $\{(Y_n, S_n), n \ge 0\}$ be Markov sequence of renewals. Let state:

$$N(t) = \sup \{n \ge 0: Sn \le t\} \text{ and } X(t) = Y_{N(t)}. \tag{10}$$

Then stochastic process $\{X(t), t \ge 0\}$ is denoted as a semi-Markov process and $Y_n$ process is referred to as Markov chain X(t).

As defined in equation (9), matrix $G(x) = [G_{ij}(x)]$ was called the core of a semi Markov process. Next, selected properties of a semi-Markov process were discussed to be later classified.

A semi-Markov process is homogenous in terms of time if the probability is affected only by the time interval until transition to next state, not by the time when the interval starts.

$$P\{Y_{n+1} = j, S_{n+1} - S_n \le x \mid Y_n = i\} = P\{Y_1 = j, S_1 \le x \mid Y_0 = i\}. \tag{11}$$

A semi-Markov process is considered to be regular if in a finite time interval the number of state changes is finite.

A semi-Markov process cannot be reduced if it is possible to enter any other state from each state. In such a case, states are said to communicate with each other. This state is considered to be repeatable if the process returns to state j in a period shorter than infinity, otherwise referred to as transitory (if it never comes back). A state is considered positively repeatable if it comes back and the expected return to the state, assuming that the process has started in state i, is shorter than infinity. In a semi-Markov process, a repeatable state is called aperiodic if it is possible to enter it any moment.

The initial distribution of states $a = [a_j]$ is represented by a probability according to which, at the beginning $a_j = P\{X(0) = i\}$ and the system state is i. Eventually a semi Markov process is fully defined by the initial distribution of states $a = [a_j]$ and core $G(x) = [G_{ij}(x)]$.

For a positive, repeatable, irreducible and aperiodic semi Markov process boundary probabilities of being in state j during transition to state i, does not depend on i:

$$p_j = \lim P\{X(t) = j \mid X(0) = i\} = \Pi_i ET_i / \sum \Pi_i ET_i, \tag{12}$$

where:

$\Pi_i$, i = 1, 2, ..., n is a boundary probability inserted into Markov chain and $ET_i$ is a mean value of time of being in state i, i = 1, 2, ..., n.

## 6. Study of a system operational quality state changes

A graph and a matrix of system operational qualitative state transitions were developed for the considered research object. The graph reflecting possible transitions between the four distinguished states are shown in Fig. 1.

Matrix of transition probabilities was built on this basis:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{21} & 0 & p_{23} & p_{24} \\ p_{31} & p_{32} & 0 & p_{34} \\ p_{41} & p_{42} & 0 & 0 \end{bmatrix}. \tag{13}$$

Matrix P of state change probabilities for a Markov chain was built on the basis of a directed graph shown in figure was described by dependence (13), where:

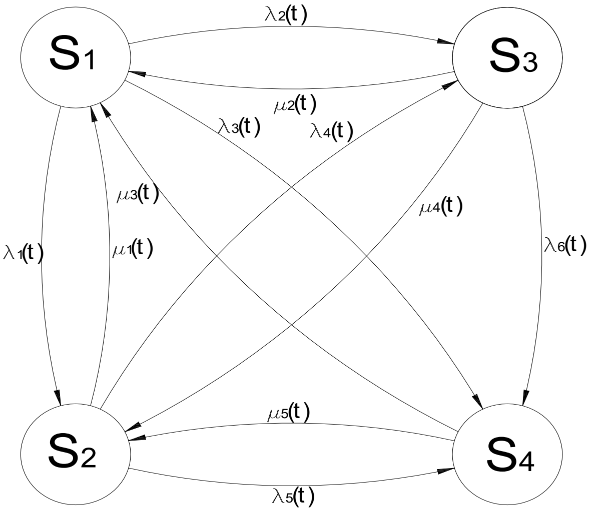


*Fig. 1. Graph of the process state changes*

$$P^T = \begin{bmatrix} 0 & p_{21} & p_{31} & p_{41} \\ p_{12} & 0 & p_{32} & p_{42} \\ p_{13} & p_{23} & 0 & 0 \\ p_{14} & p_{24} & p_{34} & 0 \end{bmatrix}. \tag{14}$$

where:

$p_{ij}$ – probability of transition from state $S_i$ to state $S_j$.

The first step in determination of boundary probabilities for a Markov chain is to build a system of matrix equations:

$$P^T \cdot \Pi = \Pi,$$

$$\begin{bmatrix} 0 & p_{21} & p_{31} & p_{41} \\ p_{12} & 0 & p_{32} & p_{42} \\ p_{13} & p_{23} & 0 & 0 \\ p_{14} & p_{24} & p_{34} & 0 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}. \tag{15}$$

A system of matrix equations can be written in the form of a linear equation system:

$$\sum_i p_{ij} \cdot \pi_i = \pi_j . \tag{16}$$

That is:

$$\begin{cases} p_{21} \cdot \pi_2 + p_{31} \cdot \pi_3 + p_{41} \cdot \pi_4 = \pi_1, \\ p_{12} \cdot \pi_1 + p_{32} \cdot \pi_3 + p_{42} \cdot \pi_4 = \pi_2, \\ p_{13} \cdot \pi_1 + p_{23} \cdot \pi_2 = \pi_3, \\ p_{14} \cdot \pi_1 + p_{24} \cdot \pi_2 + p_{34} \cdot \pi_3 = \pi_4. \end{cases} \tag{17}$$

A system of linear equations is a dependent system. In order to solve this system one of the equations was substituted by normalization condition of the following form:

$$\sum_i \pi_i = 1 . \tag{18}$$

Then, taking into consideration the normalization condition, the system of linear equations has the form:

$$\begin{cases} \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1, \\ p_{12} \cdot \pi_1 + p_{32} \cdot \pi_3 + p_{42} \cdot \pi_4 = \pi_2, \\ p_{13} \cdot \pi_1 + p_{23} \cdot \pi_2 = \pi_3, \\ p_{14} \cdot \pi_1 + p_{24} \cdot \pi_2 + p_{34} \cdot \pi_3 = \pi_4. \end{cases} \tag{19}$$

It was determined that for the analysed research object the matrix assumes the following form:

$$P = \begin{bmatrix} 0 & 0.6 & 0.3 & 0.1 \\ 0.1 & 0 & 0.65 & 0.25 \\ 0.2 & 0.7 & 0 & 0.1 \\ 0.15 & 0.85 & 0 & 0 \end{bmatrix}. \tag{20}$$

On the basis of matrix transition probabilities P, boundary probabilities were determined for the inserted Markov chain: $\Pi_1 = 0.126$, $\Pi_2 = 0.417$, $\Pi_3 = 0.309$, $\Pi_4 = 0.148$.

The following mean values of a technical object's being in states were determined on the basis of experimental tests: $ET_1 = 2$, $ET_2 = 6$, $ET_3 = 0.5$, $ET_4 = 1.5$.

For boundary probabilities of Markov process we obtain: $P_1 = 0.193$, $P_2 = 0.533$, $P_3 = 0.237$, $P_4 = 0.038$.

## Conclusions

The study deals with the problems connected with evaluation of operational quality of complex operation systems, on the example of a distinguished transport system, being the research object of this study. A general concept of evaluation method for sociotechnical systems operational quality has been described. Development of a model for transport system operation qualitative state changes based on a Markov chain inserted in a certain semi Markov-process has been considered. Boundary distribution of a semi-Markov process has been determined according to experimental based data. This distribution reflects behaviour of the system in a sufficiently long period of operation. Numerical simulation tests of the system can provide the basis for modification of the created model in order to raise the level of operational quality (reaching and maintaining the desired qualitative state) of the considered transport system which will provide the basis for further research.

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