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Probability of an Intermediate (Reduced Operational) State

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Abstract. This work examines with the form of the well-known sum: $p + q = 1$ – which is the sum of the probabilities of opposite events, in particular: the sum of the probabilities of the operational and non-operational (failure) states of a single element (a creation characterised by one output and any number of inputs). It was found that without significantly compromising the accuracy of the previous analyses, it was possible to introduce an additional component to the sum: $^{iii}pq3$, a component that embodies the probability of an intermediate state, or a reduced operational state. With a constant value of the sum of the components in question, their variation as a function of probability q was determined, following which in the function of the same variable the variation of the entropy of an element's i state was examined using Chapman–Kolmogorov equations; here the focus was on investigating the intensity of the transition from the operational state to the non-operational state or an intermediate state, and from an intermediate state to the non-operational state.

The meaning of intermediate probability was also referenced to the object: its diagnostic program, the entropy of structure, the full set of discriminable states, and the relevant transition intensities. It became indispensable in this respect to describe the object using the language of graph theory, in which the basic concepts are layers and an availability matrix. It should be noted that the subject object is an entity that comprises a set of individual elements, with a number and structure of connections that are consistent with the purpose of this entity.
Keywords: intermediate state, object element, element of the set of states, intensity, full set, entropy, credibility, diagnostic program

1. INTRODUCTION

Life constantly creates situations where it is necessary to make a decision with more than two options. Some of the most glaring examples include: health – illness – death; operational – intermediate – failure (non-operational); beauty – mediocrity – ugliness; very good – sufficient – insufficient; green – orange – red. In the case of technical diagnostics, a need for a decision arises when a component loses its properties gradually; where qualifying the result of a check as positive or negative poses serious problems. The imagination can then suggest a scale of the negative consequences of a false positive, or a false negative, all I and II type errors respectively.

With so much need to distinguish an intermediate state, or a reduced operational state, it is regrettable that there is no simple and intuitively understandable method to assess its significance, the probability of its occurrence. Why not change it? Why not, without any particular formalism or confusion, expand the well-known sum of probabilities of the operational and the non-operational states: $p + q = 1$, with an additional component, ${}^{iii}pq3$, which identifies the probability of an intermediate or reduced operational state in question?

It is hoped that the results of the analyses of the following equation:

$$ {}^{iii}p + {}^{iii}pq3 + q = 1 \tag{1} $$

will provide an answer to this question.

By deriving the following to the left of the equal sign:

$$ {}^{iii}p = \frac{1-q}{1+3q} \tag{2} $$

it can be expressed as a function of one variable:

$$ \frac{1-q}{1+3q} + 3q\frac{1-q}{1+3q} + q = 1 \tag{3} $$

The left-hand side superscript with a Roman numeral is to enhance the distinctness of the operational state probability, *p*, for the three- and five-value qualifiers of the state of an element. The probability of failure, *q*, is not subject to such a distinction.

In this paper, the term *intermediate state* is similar to the concepts known from reference literature, such as *reduced operational state*, *fuzzy strategy* and *uncertainty,* as preferred by, inter alia, the authors of the following works: [5, 8], [18] and [19] respectively. It could be stated that the *three-state division of an element* can correspond to any situation in which the model of research becomes a three-state graph, as found in [1, 9, 16, 20]. A *three-state division* may suggest the need to adapt *three-valued logic*. However, it has not been adapted here, due to the considered concept of object diagnostics. It has been believed that the object should undergo "strict" research programs in this respect, beginning with its input elements and ending with its output elements. This requires a full understanding of the constructional structure of the object. It is difficult to be indifferent to the contemporary challenges posed by Industry 4.0 and its concomitant concept, *prediction* [21, 22]. Therefore, can an understanding of the *probability of an intermediate state* be ignored by Industry 4.0?

2. PROBABILISTIC PROPERTIES OF AN ELEMENT

An element (entity), whether for technical or administrative, organisational or any other reasons, is understood here as an indivisible creation with any internal structure, characterised by any number of inputs and only one output. The components of the element are its physical components. Any single-input component can be an element. The component could be a wire or a radio receiver. Because of its components, an element can have any structure (serial, parallel bridged, etc.) to ensure reliability.

It is possible to determine the intermediate state or the non-operational state of a specific element if all its input signals are acceptable and its output signal[1] is either unacceptable or partially unacceptable. If present, any unacceptable or partially unacceptable input signal can make the output signal of the element unacceptable or partially unacceptable and obscures that element's diagnostic state[2] [14].

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[1] Any measurable quantity that allows one of the distinguishable diagnostic states of an element (entity) to be determined.

[2] The concept of "diagnostic state" is synonymous here with the term of "reliability state", which could equally well mean the "operational state", the "non-operational state" or an "intermediate state". The reference literature terms the concept of "reliability state" as a "malfunction state". It is thought that introducing the concept of "diagnostic state" will stop any lexical discussions.

2.1. Variability of probabilities of the particular states

Equation (1) expresses the natural consequence of introducing an intermediate state; firstly, by placing the value of its probability between the probabilities of the operational and the non-operational states, and secondly by indicating its significance as three times the product of probabilities of both states. Multiplying by three triplets is not incidental here – it leads to the intersection of all the components of equation (1) as a single point.

Equation (3) succinctly describes the variation of each of the components as a function of one variable, or the non-operational state probability, q. From the analysis of the variation of the components (Fig. 1), it can be concluded that the components reach a common point (and therefore an identical value equal to 1/3) for q = 1/3. It is significant and intuitively understandable that if q = 0 and q = 1, the value of ${}^{iii}p$ (Fig. 1, solid green line) gives the values of 1 and 0 respectively. This is compatible with a binary (zero-one) state assessment.

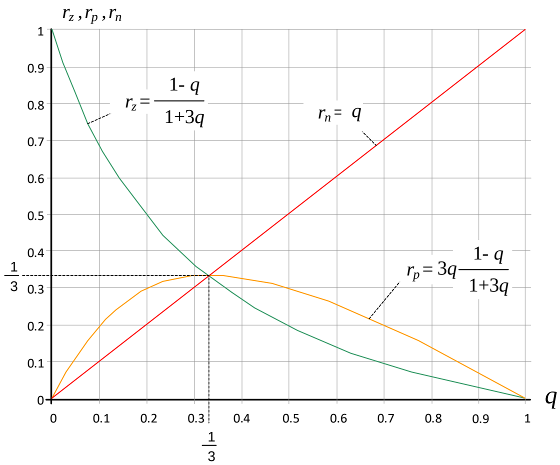


Fig. 1. Graphs of probability variations of the states: operational r_z, intermediate r_p and non-operational r_n, expressed by the sum (3) and respectively (in Fig. 1) by solid lines in green, orange and red

2.2. Entropy of state

Entropy H, in relation to the subject of this work, expresses the degree of ignorance about the dependability (reliability) of the functioning of an element, or the unpredictability of the performance of the task assigned to the element.

A study of the variability of the function of entropy (according to Shannon's formula) [10]:

$$H = -r_z \log_3 r_z - r_p \log_3 r_p - r_n \log_3 r_n \tag{4}$$

with:

$$r_z =^{iii} p = \frac{1-q}{1+3q} \tag{5}$$

$$r_p = 3q\frac{1-q}{1+3q} \tag{6}$$

$$r_n = q \tag{7}$$

distinguishes between two zero values and a maximum value in between (Fig. 2) each time.

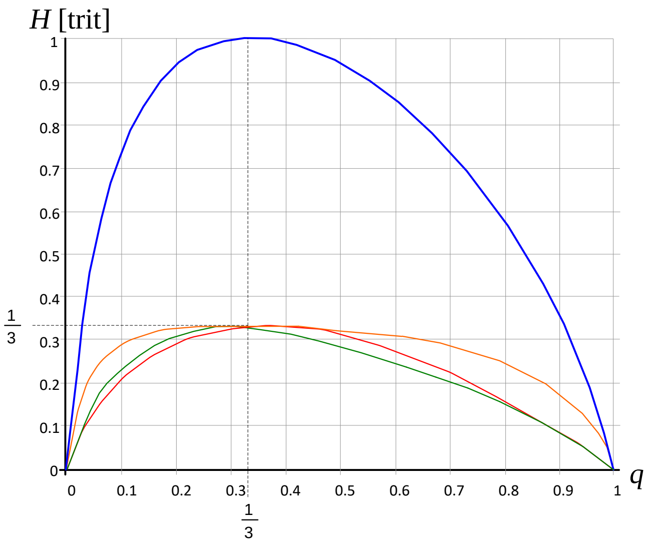


Fig. 2. The entropy of the **state of an element** with its three-valued assessment as a function of the non-operational state probability *q* (solid blue line), together with its components (solid green, orange and red lines) relating to the probabilities expressed respectively by the equations: (5), (6) and (7)

The first of the zeros (to the left of the maximum value) characterises an operational (fully serviceable) component, which deserves full confidence. The second zero, on the contrary, characterises an element as completely non-operational and devoid of performance capabilities. It should be noted that in both cases the question of the use of the element is predetermined.

The maximum value, which expresses the highest uncertainty, from a practical point of view expresses the average number of checks necessary to determine the diagnostic state of the component. For a single element this is 1 and refers to an equal probability of $q = 1/3$ for each state.

The logarithm base equal to 3 applies to three potential outcomes of this check, for example, *yes, no, difficult to say* or: *good, bad, "so-so"* or: *operational, partially operational, non-operational*; or: ... . For a three-valued state assessment, the unit of entropy is called the 'trit' (trenary digit). In other words, it can be stated the trit is a unit of information quantity that equals to the amount of the information obtained by realising one of three equally probable states.

2.3. Interstate transition intensities

The intensity of transition from state *x* to state *y*: λ_{xy}, is in other words an assessment of the increase or decrease in the rate of this transition. In this work, the transitions assessed are those highlighted in graphs G_1, G_2 and G_3 (Table 1). The relevant calculations were based on probabilities r_z (5), r_p (6) and r_n (7), the derivatives of these probabilities, namely r'_z (8), r'_p (9) and r'_n (10), and the corresponding systems of Chapman–Kolmogorov equations (Table 1).

$$\frac{dr_z}{dq} = r_z^{'} = \frac{-\frac{4}{9}}{\left(q+\frac{1}{3}\right)^2} \tag{8}$$

$$\frac{dr_p}{dq} = r_p^{'} = \frac{-\left(q-\frac{1}{3}\right)(q+1)}{\left(q+\frac{1}{3}\right)^2} \tag{9}$$

$$\frac{dr_n}{dq} = r_n^{'} = 1 \tag{10}$$

The derivatives (8), (9) and (10) express the rate of change of the probabilities r_z, r_p and r_n with respect to their argument, the probability of an element's non-operational state, *q*. Each time (for each value of *q*):

$$r_z^{'} + r_p^{'} + r_n^{'} = 0 \tag{11}$$

This property is illustrated in Fig. 3. Note also that the sum of the derivatives of functions is the derivative of the functions' sum. According to relation (3), the latter is equal to 1, therefore its derivative is equal to 0.

The zero values of the sums of the derivatives can also confirm the zero values of the sums of the right-hand sides of the systems of Chapman–Kolmogorov equations, which in turn can also confirm their correctness (Table 1).

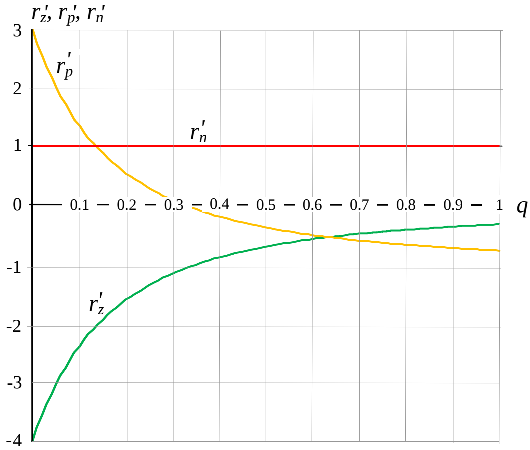


Fig. 3. Variation of the derivatives: r'_z, r'_p and r'_n as a function of the non-operational state probability q

The variability of λ_{zp1} seems to contradict the common perception of intensity values greater than zero. It is worth noting its relationship with the decrease in the values of r_p values $q>1/3$.

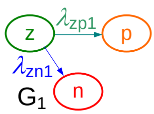
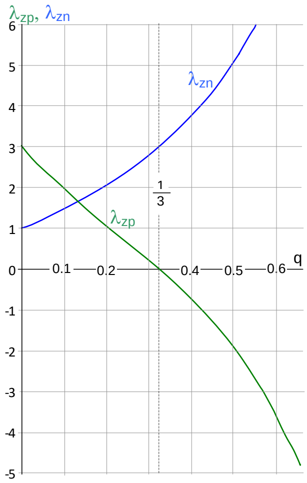
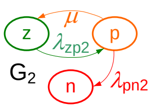
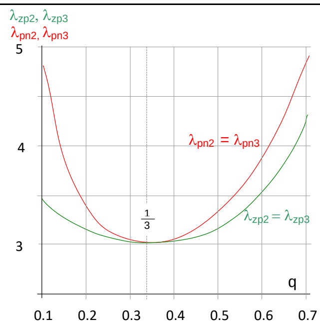
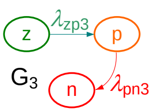
The nature of the changes in individual intensities was determined by the directions of transition assigned to them. Care was taken to ensure that they were in keeping with the natural essence of things. Therefore, from this point of view, there should be no doubt about the introduction of intensity μ in graph G_2, from the intermediate state to the operational state, along the lines of life concepts: from illness to health. It was decided to test its relevance as well as the advisability of introducing the intensity in [9, 15, 16, 17]. Comparisons of the solutions of the equation systems for graphs G_2 and G_3 showed no differences; the intensity μ was zero, and intensities $\lambda_{zp2} = \lambda_{zp3}$ and $\lambda_{pn2} = \lambda_{pn3}$ were equal to each other. In [9], the need for an intermediate state was noted for a battery-charger power supply, while in [16] it was noted for the flight of an aircraft.

Of particular note is the 'bathtub' variation in intensity:

$$\lambda_{zp2} = \lambda_{zp3} = \lambda_{zn1} + \lambda_{zp1} \tag{12}$$

There might be the impression that these intensities mathematically confirm the research results known in this context [4]. Attention is drawn to an analysis of the graph shown in [1].

Table 1. Examples of determination of interstate crossing intensities λ_{xy} for three-vertex state graphs

State graph	Chapman– Kolmogorov equations	Transition intensity λ_{xy}	Variation $\lambda_{xy}(q)$
	$\begin{cases} r_z' = -(\lambda_{zp1} + \lambda_{zn1}) r_z \\ r_p' = \lambda_{zp1} r_z \\ r_n' = \lambda_{zn1} r_z \end{cases}$	$\lambda_{zp1} = \frac{3(3q^2+2q-1)}{(q-1)(3q+1)}$ $\lambda_{zn1} = \frac{3q+1}{1-q}$	
	$\begin{cases} r_z' = -\lambda_{zp2} r_z + \mu r_p \\ r_p' = \lambda_{zp2} r_z - (\mu + \lambda_{pn2}) r_p \\ r_n' = \lambda_{pn2} r_p \end{cases}$	$\lambda_{zp2} = \frac{4}{(1-q)(3q+1)}$ $\mu = 0$ $\lambda_{pn2} = \frac{3q+1}{3q(1-q)}$	
	$\begin{cases} r_z' = -\lambda_{zp3} r_z \\ r_p' = \lambda_{zp3} r_z - \lambda_{pn3} r_p \\ r_n' = \lambda_{pn3} r_p \end{cases}$	$\lambda_{zp3} = \frac{4}{(1-q)(3q+1)}$ $\lambda_{pn3} = \frac{3q+1}{3q(1-q)}$	

2.4. Credibility of state assessment

In speaking about credibility, one can mean the quality of the attestation of a given *state* by its *evaluation*. By assigning to the *state* and the *assessment*, respectively, the probabilities of *occurrence r* and *accuracy s*, it can be said that *truth* and *falsehood* are defined by probabilities *rs* and (1 - *r*)(1 - *s*), respectively, and credibility is defined by this relationship:

$$W = \frac{rs}{rs + (1-r)(1-s)} \tag{13}$$

Figure 4 shows the credibility of the attestation of the operational, intermediate, and non-operational states as a function of the probabilities of their occurrence, r_z (5), r_p (6) and r_n (7) and the reliability (*accuracy*) s of their assessment. The accuracy is influenced by the quality of the research process and the accuracy of the control and measurement instruments.

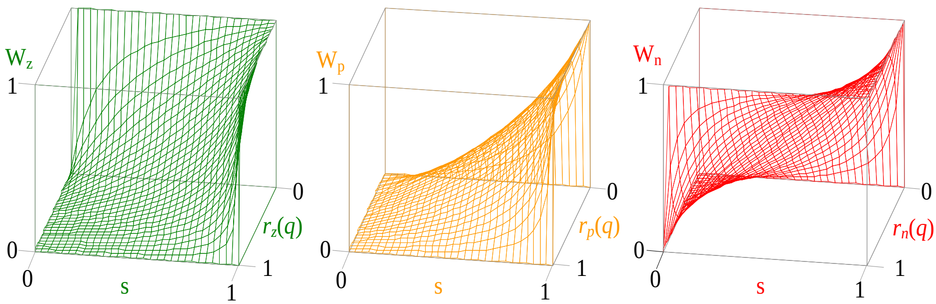


Fig. 4. The credibility of the attestation of the operational, intermediate, and non-operational states as a function of the probabilities of the states' occurrence and reliability (accuracy) of the states' assessment

In order to determine the trend of changes in credibility as a function of the reliability considered, an analysis of these changes was undertaken for two of its values: s_{95} = 0.95 and s_{90} = 0.90. The result is shown in Fig. 5.

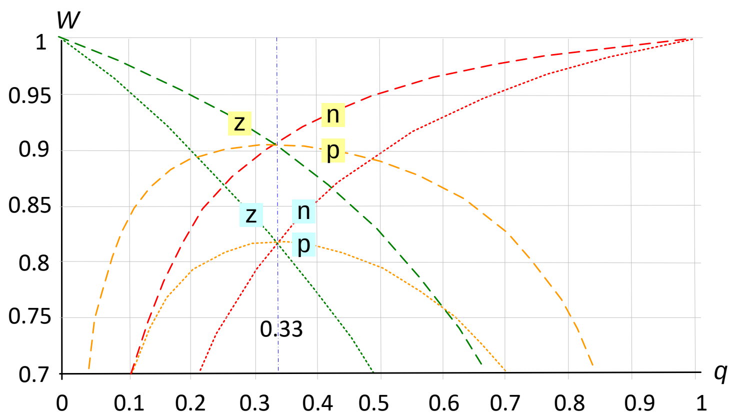


Fig. 5. Variation of the credibility of the attestation of the operational (z), intermediate (p) and non-operational (n) as a function of probability q and probabilities s_{90} and s_{95}, equal to, respectively: 0.90 and 0.95

Full credibility occurs when either of the probabilities *r* or *s* is equal to unity, and at the same time is greater than zero. In reality, although it is striven for, none of these probabilities reaches the 100% ceiling.

The trend analysis (Fig. 5) suggests that the *operational and non-operational states* can be determined with the highest reliability, while the *intermediate state* can be determined with the lowest reliability, where the maximum is found for q=0.33 (3).

In order to establish the *operational state*, it is advisable to set very high requirements for the element, narrowing the tolerances of its parameters.

2.5. Five-value qualification of element state

The five-value qualification can be discussed in the context of school grades, *excellent*, *very good*, *good*, *satisfactory* and *unsatisfactory*, but in comparison to the content of this paper, it is prudent to determine the intermediate probabilities to the intermediate state probabilities (Fig. 6a, turquoise and pink chart lines). This is another interference in the relation defined by equation (1) and expressed as follows:

$$ {}^{v}p + 25q\,{}^{v}p^{2} + 5\,{}^{v}pq + 25\,{}^{v}pq^{2} + q = 1 \tag{14}$$

Moving probability ${}^{v}p$ to the left of the equality sign:

$$ {}^{v}p = \frac{\sqrt{625q^{4} + 250q^{3} - 25q^{2} + 110q + 1} - 25q^{2} - 5q - 1}{50q} \tag{15}$$

gives rise to the need to determine and present the variation of the five components of equation (14) as a function of the non-functional state probability q (Fig. 6).

Somewhat as a consequence, the entropy variation was also determined (Fig. 7) not unlike as shown in Fig. 2. The logarithm base for each component of relation (14) is ‘5’, equal to the line count. Fivt is the name of a unit which was adopted here and is similar to the bit and trit, that are well known in the reference literature and correspond to bases ‘2’ and ‘3’ respectively.

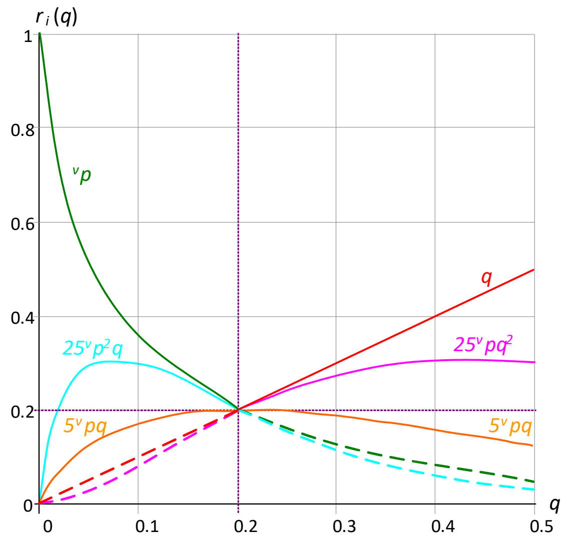


Fig. 6. Probabilities of intermediate states (turquoise and pink lines) to the intermediate state (orange line) and the extreme states (green and red lines)

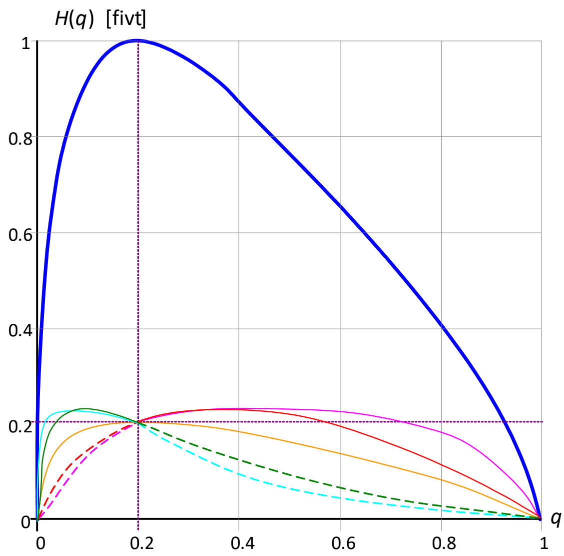


Fig. 7. The entropy of **an element's state** at its five-value evaluation as a function of probability q, including its components (solid green, turquoise, orange, pink, and red lines)

3. PROBABILISTIC PROPERTIES OF AN OBJECT

An object is an entity divisible into elements; it does not necessarily have a single output, and, like the elements, it does not have a reliability structure. The elements of an object need not be accessible[3] or real[4]. There are relationships between the elements of an object: *dependence*, *independence* and *interdependence*.

3.1. Probabilities of distinguishable diagnostic states

A determination of the particular probabilities of the distinguishable diagnostic states of an object must be preceded by an understanding of its constructional structure (a schematic diagram, a functional flowchart, technical documentation, etc.) with a determination of the number of the object's elements and identification of the interrelations between these elements.

In a rather truncated version, the language of graph theory can be used to state that the output of this identification is digraph D together with binary availability matrix **D**(D) [7].

Figure 8 is a form of "pictorial writing" illustrating an algorithm for processing a connection of three wires.

The binary availability matrix **D**(D) indicates with ones in its rows and columns all successors and predecessors of each vertex, the sets of which are called transitive $\widehat{F}(e_i)$ closures and antitransitive $\breve{F}(e_i)$ closures, respectively [11]. For example, for digraph D in Fig. 8:

$$\widehat{F}(e_1) = \{e_1, e_2, e_3\} \tag{16}$$

whereas:

$$\breve{F}(e_3) = \{e_3, e_1\} \tag{17}$$

The availability matrix **D**(D) had (by virtue of the following procedure) to assume the form of an upper triangular matrix by introducing the numbering of rows and columns consistent with the numbering of the vertices in the successive layers of digraph D. The first layer includes those vertices that have no predecessors. Layer two included vertices which would not have any predecessors with the layer one vertices removed from the digraph. Layer three and each successive layer included the vertices which would not have any predecessors with the preceding layers removed from the digraph.

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[3] Inaccessible elements describe the structure of multi-output creations, e.g., integrated circuits, modules, relays, etc.

[4] It is a reference to hazard-protection pairs [12].

Digraph D has two layers:

$$W_1 = \{e_1\} \tag{18}$$

$$W_2 = \{e_2, e_3\} \tag{19}$$

A translation of the availability matrix **D**(D) for the determination of the size and probabilities of diagnostic states is, in essence, a translation of the reliability diagnostics. This matrix is a description of the diagnostic structure of the object, so its symbols in each column can be assigned to the results of the checks[5]. If any of the parameters of a signal exceeded its limit, the check result is a failure. Otherwise, the check result is a pass. Both results have logical values assigned as follows: 1 and 0. The distribution of zeros and ones in the i^{th} line identifies the non-operational state of the i^{th} element.

With a three-valued assessment of element states, the argument above requires some modification. If the aim is to determine the full set of distinguishable states, the operational state of all elements and (in addition to the states resulting from single *non-operational instances* and *intermediate instances*) the states resulting from *multiple non-operational* and *intermediate instances* must be considered (Fig. 8d).

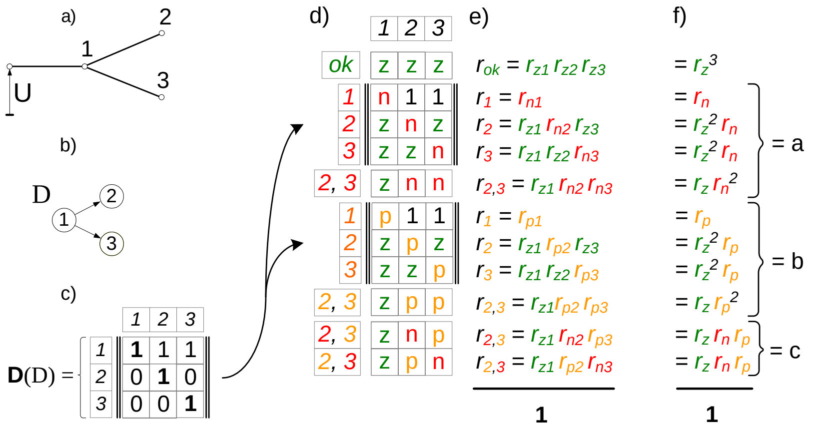
In the upper triangular availability matrix **D**(D), the diagonal should be noticed first of all. Its ones should be assigned the symbolism of the intermediate state (p) and the non-operational state (n), and for this reason it needs to be duplicated (see the arrows in Fig. 8). By convention, the two symbols are shown in orange and red, respectively (see sub-matrices *a* and *b* enclosed by brackets). The **D**(D) ones (to the right of the diagonal) embody the obscurity of diagnostic states and sums (1) and (3). The obscurity of the states refers to the elements defined by the vertical coordinates of said ones. The zeros to the right of the diagonal also have their own distinct meaning. Their two coordinates indicate the states that can be identified simultaneously, or the elements that operate independently of one another. The distribution of symbols in a row is then the result of summing the rows which have the numbers of these elements assigned (see the rows marked with orange and red numbers: 2 and 3; bracket *c*). The number of possible states is $3^3=27$, but there are only 11 distinguishable states.

The probabilities of the individual states of an object r_i are the products of the probabilities of the states of the individual elements, or the probabilities, which are the projections of the symbols contained in the individual rows. The projections *z*, *p* and *n* are the probabilities r_z (5), r_p (6) and r_n (7) respectively. The ones (the symbols of obscurity) are left with their own values.

Indexing is only done for the heterogeneous properties of elements. The indices then follow the vertical coordinates of the projected symbols.

[5] Check – an operation to examine the conformity of an output signal to a standard.

A test of the correctness of the performed operations can be the sum of the probabilities of individual distinguishable diagnostic states of the object (Fig. 8g), which is equal to 1.



g)

$$r_{ok} + r_1 + r_2 + r_3 + r_{2,3} + r_1 + r_2 + r_3 + r_{2,3} + r_{2,3} + r_{2,3} =$$

$$= r_{z1} r_{z2} r_{z3} + r_{n1} + r_{z1} r_{n2} r_{z3} + r_{z1} r_{z2} r_{n3} + r_{z1} r_{n2} r_{n3} + r_{p1} +$$

$$r_{z1} r_{p2} r_{z3} + r_{z1} r_{z2} r_{p3} + r_{z1} r_{p2} r_{p3} + r_{z1} r_{n2} r_{p3} + r_{z1} r_{p2} r_{n3} =$$

$$= r_{z1} (r_{z2} r_{z3} + r_{n2} r_{z3} + r_{z2} q_3 + r_{n2} r_{n3} + r_{p2} r_{z3} + r_{z2} r_{p3} + r_{p2} r_{p3} + r_{n2} r_{p3} + r_{p2} r_{n3}) + r_{n1} + r_{p1} =$$

$$= r_{z1} \left(r_{z2} \overbrace{(r_{z3} + r_{n3} + r_{p3})}^{1} + r_{n2} \overbrace{(r_{z3} + r_{n3} + r_{p3})}^{1} + r_{p2} \overbrace{(r_{z3} + r_{n3} + r_{p3})}^{1}\right) + r_{n1} + r_{p2} =$$

$$= r_{z1} \overbrace{(r_{z2} + r_{n2} + r_{p2})}^{1} + r_{n1} + r_{p1} = \overbrace{r_{z1} + r_{n1} + r_{p1}}^{1} = \mathbf{1}$$

Fig. 8. Illustration of the process of determining the full set of distinguishable diagnostic states of an object and the determination of probabilities of the occurrence of the states: (a) a schematic diagram of the object; (b) digraph, the object's model; (c) the availability matrix describing the digraph; (d) the algorithm for determination of the full set of distinguishable diagnostic states (see details in the text); (e) and (f) are the probabilities of particular states with non-uniform and homogeneous probabilities of the diagnostic states of particular elements of the object; (g) the test of the sum of particular probabilities with non-uniform probabilities of diagnostic states of particular elements of the object

The sum of the probabilities determined as above is always equal to 1. This happens irrespectively of digraph D, whatever the number of its vertices and the structure of vertex connections are.

Additionally, the probabilities which occur as specific factors and satisfying the following relation: $r_z + r_p + r_n = 1$, in order to obtain the value of this sum, may take any value, of any character of their changes and of any relation to each other. In this paper, the probability values naturally assume only the values in the interval of <0; 1>.

For standardised values of probabilities r_z, r_p and r_n, probabilities r_i assume the forms as shown in Fig. 8f and which greatly simplify further analysis. The implication of this simplification is, among other things, to study the variation of probabilities r_i as a function of probability q (Fig. 9)

$$a = (r_n + 2\,r_z^2\,r_n + r_z\,r_n^2) \tag{20}$$

$$b = (r_p + 2\,r_z^2\,r_p + r_z\,r_p^2) \tag{21}$$

$$c = 2\,r_z\,r_n\,r_p \tag{22}$$

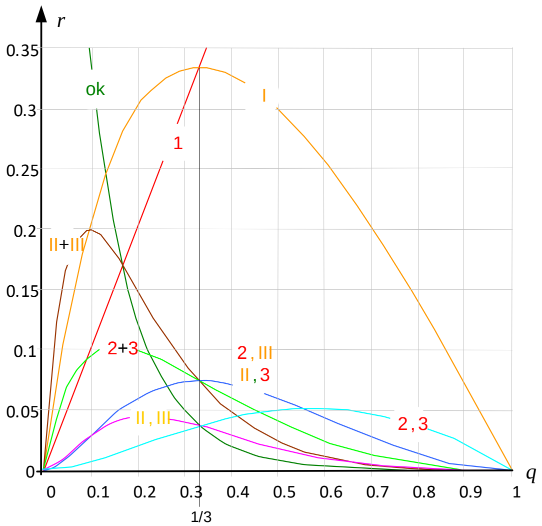


Fig. 9. Variation of probabilities r_i with homogeneous properties of the elements as a function of probability q, with:

No.	Designation		Value		
0	ok	→	r_{ok}	=	r_z^3
1	1	→	r_1	=	r_n
2	I	→	r_I	=	r_p
3, 5	2+3	→	r_2+r_3	=	$2r_z^2r_n$
4, 6	II+III	→	$r_{II}+r_{III}$	=	$2r_z^2r_p$
9, 10	2, III II, 3	→	$r_{2,\,III} + r_{II,\,3}$	=	$2r_zr_nr_p$
7	2, 3	→	$r_{2,\,3}$	=	$r_zr_n^2$
8	II, III	→	$r_{II,\,III}$	=	$r_zr_p^2$

Only two probabilities (rows 0 and 1)[6] achieve a full span of the values from 1 to 0 and from 0 to 1, respectively. For this reason, the other probabilities reach lower values. It must be known that the sum of all eight[7] ordinates, for any value of the abscissa *q*, is equal to unity. In addition to this sum, the noteworthy sums of the probabilities are in brackets *a*, *b* and *c* (Fig. 8), which respectively determine the need to undertake **replacement** (20), **prevention** (21) and simultaneous **replacement and prevention** (22).

Usually *the cost of prevention* is much lower than *the cost of replacement*. Figure 10 shows the variation of the probabilities, expressed in brackets, in the relationships (20) and (21).

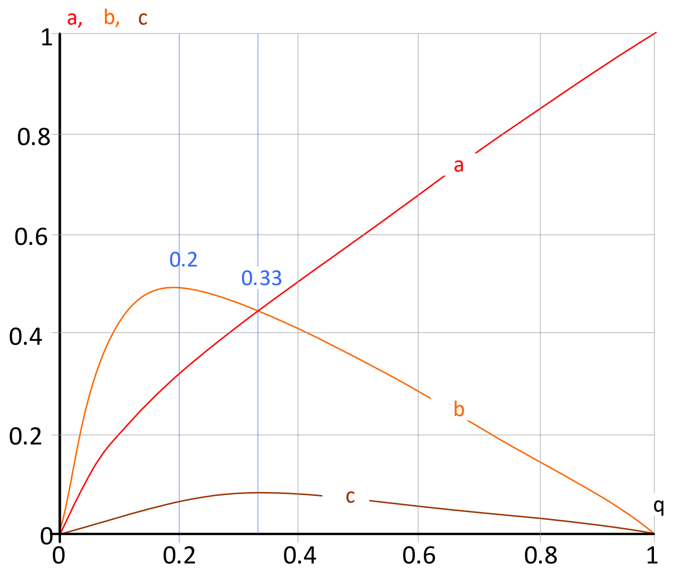


Fig. 10. Variation in the sum of the probabilities in brackets *a*, *b* and *c* in Fig. 11; the probabilities which determine the need for **replacement** (20), **prevention** (21), and simultaneous **replacement and prevention** (22) as a function of the non-operational state probability *q*

The expression of the variations shown in Fig. 10 confirms the intuitively understandable sense of preventive maintenance. It is encouraging to see that with diagnostic tests for $q < 1/3$, the probability of "intermediate states" is higher than the probability of "non-operational states". This usually translates into prevention which is faster than replacement and maintaining an object longer in the operational state of all its elements.

[6] From here on, the rows will be identified by their numbers.

[7] In fact, there are eleven ordinates; however, due to the similarity of three of them, their doubled values are presented (compare Fig. 11 to Fig. 12).

The inflection of line *b* is at $q \approx 0.2$. When this value is exceeded, the object's user should be prompted to halt the operation of the object and commence appropriate refurbishment.

3.2. Entropy of an object structure

Once the probabilities of the individual diagnostic states have been determined, the determination of the entropy of the structure is a mere formality; not unlike in section 2.2, the Shannon formula should be applied. Figure 11 shows a chart of entropy variation for the object depicted Fig. 8.

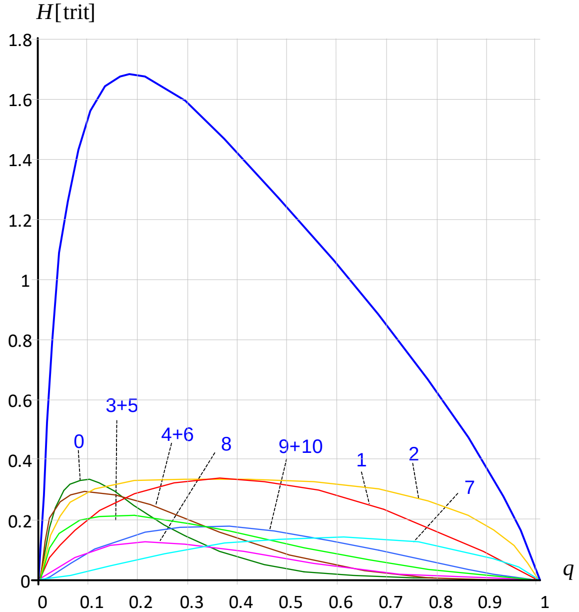


Fig. 11. The entropy of the **object structure** (Fig. 8b) as a function of the non-operational state probability *q* (solid blue line), together with its components, are summarised in the following table

No.	Ingredients H=			
0	$-r_z^3$	$\log_3$	r_z^3	+
1	$-r_n$	$\log_3$	r_n	+
2	$-r_p$	$\log_3$	r_p	+
3+5	$-2r_z^2 r_n$	$\log_3$	$2 r_z^2 r_n$	+
4+6	$-2 r_z^2 r_p$	$\log_3$	$2 r_z^2 r_p$	+
9+10	$-2 r_z r_n r_p$	$\log_3$	$2 r_z r_n r_p$	+
7	$- r_z r_n^2$	$\log_3$	$r_z r_p^2$	+
8	$- r_z r_p^2$	$\log_3$	$r_z r_p^2$	

The maximum of the entropy is at $q \approx 0.18$, which for this q can mean the average number of checks needed to determine the state of the object [6].

Understanding the entropy of an object is very important – it guides the decision to undertake or abandon activities related to its operation on one hand and the decision to rationalise the cost of its research on the other hand.

In line with the considerations made, it can be said that structure becomes key. Resources need to be distributed in a specific way when the structure is amorphous and the distribution is completely different when the structure is concentrated. An amorphous structure could be a bag of sweets, while a concentrated structure could be a mixture of blood tested for a dangerous virus. The former has the highest entropy, while the latter has the lowest entropy.

The structures of objects are characterised not only by the intricacy of the interconnection of their elements, but also the vast number of elements, as is common nowadays. However, this does not prove to be particularly important.

Describing the structure of an object using the language of graph theory allows the condensation of these elements, especially when the elements are deprived of access for measurement and monitoring or when they include feedback loops. Even with a personal computer, it should be noted that the number of interchangeable and available components is low. Billions of transistors and resistors are encapsulated in integrated circuits, which are housed in modules that are coated with lacquer and (usually) potted in resin.

Over the years, it seems that a relentless, real challenge is the study of objects with distributed structures, that is, where there are no functional connections between the elements. Here the imagination suggests human society with its health problems. At present, a database of statistical results should make it possible to identify the optimal solution, especially as this database can be enhanced with the probabilities of the intermediate states.

3.3. Transition intensities

An object, according to the nature of things, can move from an operational state into one of its intermediate or non-operational states. Each time the states in question are mapped by the vertices of a two-layer graph. For the three wire connection in Fig. 11a the state graph takes the form shown in Fig. 12a. Next to it, Figure 12b shows the variation of the different transition intensities:

$$\lambda_i = \frac{\frac{dr_i}{dq}}{r_z^{\,3}};\ i \in \{1, 2, 3, ..., 10\} \tag{23}$$

as a function of probability q. The numbers i are written in circles, which in turn are written in ovals, symbolising the distinguishable diagnostic states of the object.

The dark green graph, numbered 0, refers to intensity:

$$\lambda_0 = -\sum_{i=1}^{10} \frac{\frac{dr_i}{dq}}{r_z^3} = \frac{\frac{dr_{ok}}{dq}}{r_z^3} = \frac{\frac{dr_z^3}{dq}}{r_z^3} \tag{24}$$

Given the sum of transition rates for a full set of distinct diagnostic states is:

$$\lambda_0 + \sum_{i=1}^{10} \lambda_i = 0 \tag{25}$$

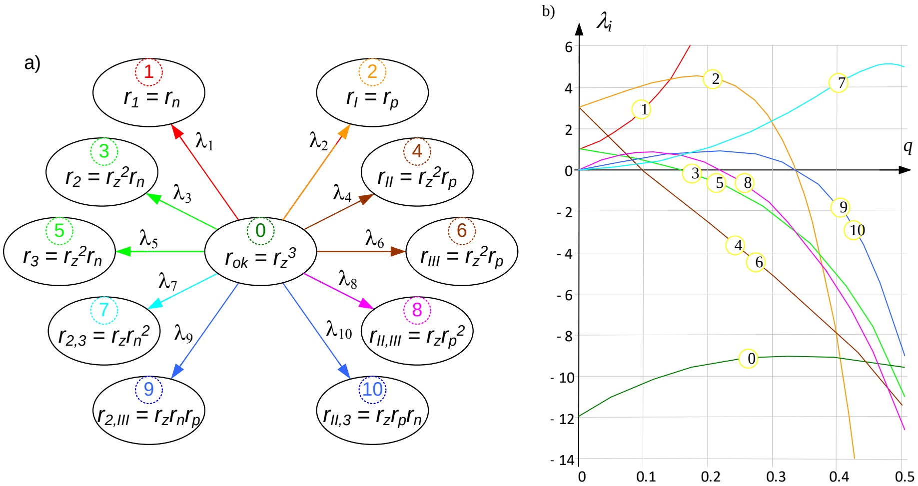


Fig. 12. (a) Graph of distinguishable reliability states of the object shown in Figs. 8a and 8b; (b) the variation of transition intensities as a function of probability q

Consideration of the area of transition rate λ provides a clearer image of the object's properties. The derivatives, which are measures of the rate of change of the values of the individual probabilities relative to changes in the unified probability of non-operational state of a single element, divided by probability r_{ok}, change very rapidly.

As the probability approaches zero, the transition rate values spread asymptotically towards $\pm\infty$. This became the reason why the q values were restricted to the interval <0; 0.5>. The analysis of intensity variation should be carried out in parallel with the view of the variation of the probabilities shown in Fig. 9. Each increase and decrease in these probabilities should be associated with, respectively, a positive and negative intensity value.

The intensity becomes greater with the difference between the probabilities of *non-operational and intermediate states* and the probability of *operational state* of all elements of the object.

3.4. Diagnostics with three-value assessment of check results[8]

The diagnostics which identify intermediate states requires a specific testing procedure that does not have the traditional distinction between identifying and locating the non-operational instances, both of which were called serviceability checks and fault location.

It makes no sense to examine the successors of an element in an intermediate state. It is difficult to assess their behaviour without understanding their sensitivities. It is also difficult to make any judgement about their condition in view of the known diagnostic phenomenon of non-operational instances obscuring other non-operational instances. The phenomenon of mutually compensating non-operational instances is also significant. Considering all of the foregoing, a natural consequence is to examine the object starting with the elements of the first layer and ending with those of the last layer. A formal notation of the method of this examination is **Algorithm 1**.

A property of the algorithm is its particular simplicity. On the one hand, the simplicity is due to the possibility of directly using the upper triangular availability matrix **D**(D), and on the other hand, to the possibility of reaching an immediate diagnosis. The set of elements in non-operational or intermediate states is determined by the numbering of the checks with negative and or unresolved (not fully clear) results. The declaration of the operational state of all parts of an object can only be made once all the checks have been passed with positive results.

Algorithm 1. ***Diagnostics with three-value assessment of check results***

1. Perform a check on the first row number in matrix **D**(D).
2. (a) If the check is negative or intermediate, remove from the matrix **D**(D) the rows and columns (except for the first row and the first column) defined by the vertical coordinates defined by the coordinates of ones in the first row.
   (b) If the check is positive, remove the first row and the first column from matrix **D**(D).
3. Repeat steps 1 and 2 until the dimension of matrix **D**(D) is not equal to zero.
4. Determine the set of elements that are a non-operational or intermediate state according with this relationship:

[8] An outline of the concept is included in the scientific communication: Paweł Szczepański: Application of three-value classification of element states in the diagnostics of engineering facilities. 30th Conference on Applications of Mathematics; Zakopane 2001 [13].

$$\tilde{E}_{nz} = \bigcup_{i:S_i^n} \{e_i\} \cup \bigcup_{j:S_j^p} \{e_j\} \tag{26}$$

with

$$S_i^n, S_j^p$$

respectively: negative and intermediate results of checks which by their numbering indicate the number of elements in a non-operational or intermediate state.

Relation (26) can indicate more than one non-operational or intermediate instance. Their number depends on the structure of the object. For an object with a serial structure of connections, only one element with specific characteristics can be specified, while for an object with an amorphous structure of connections, where the elements operate independently of each other, more elements can be specified.

The number of distinguishable object states of such a structure is many times greater than in a two-valued assessment of the state of elements.

In diagnostics with three-valued assessment, the declaration of a non-operational state or an intermediate state of a component is the result of an unacceptable or partially unacceptable output signal while the input signals are fully acceptable. At the same time, this means not testing an element's output signal when at least one input signal is unacceptable or partially unacceptable.

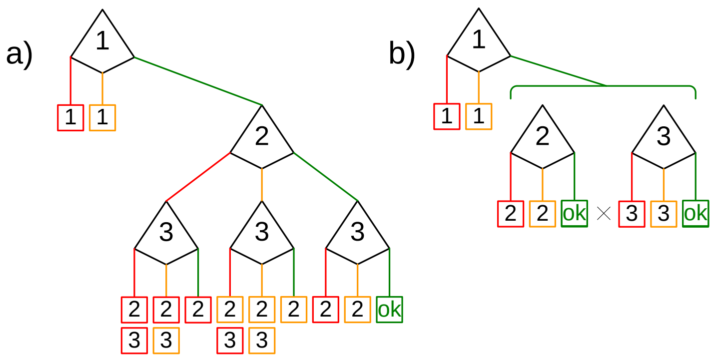


Fig. 13. Conditional diagnostic programmes for the connection of three wires (Fig. 8a) with a three-valued assessment of the check results with (a) one and (b) two control and measurement entities

Figure 13a shows a conditional program developed from Algorithm 1 for diagnosing the connection of three wires (Fig. 8a) with a three-valued assessment of the check results.

The deltoids indicate the checks, the deltoid branches indicate the results of these checks, while the rectangles indicate the sets of non-operational or partially operational elements of the object.

The number – depending on whether it is inside a deltoid or a rectangle – indicates the check number or the element number, respectively. Each left-hand branch of the dendrite is assumed to indicate a negative check result, the middle branch indicates an intermediate check result, and the right-hand branch indicates a positive check result. The colours green, orange and red are associated with the operational, intermediate, and non-operational states, respectively.

The test program presented is dedicated to single control and measurement entities, but it is easy to see that there is the possibility of parallel testing, or the simultaneous participation of two control and measurement entities (Fig. 13b). Thus, having found wire 1 to be operational, wires 2 and 3 can be tested. An economic analysis of the use of a parallel or sequential diagnostic process requires an estimation of its time and labour intensity (this estimation was not undertaken here).

4. CONCLUSIONS

The main theme of this paper is the analysis of the application of the properties of a full set of distinguishable diagnostic states of an object in the aspect of a three-valued assessment of the diagnostic states of the individual elements of the object. It was considered that a single element, in addition to being either in an operational or non-operational state, may also be in an intermediate state, which is the conjunction of the two former states. The probability value of this state (given by a function), together with the probabilities of the associated states (also given by functions) add up 1. The only argument of these functions is the non-operational state probability q of a single element, and a specific check for the correctness of the reasoning is the entropy value equal to one trit.

The probabilistic properties of the elements of the full set of distinguishable diagnostic states depend on the constructional structure of the object. It is demonstrated that the relevance of the probability of any of the states of an object – resulting from either the non-operational state or the intermediate state of an element – is greater the smaller the power is of the antitransitive closure of that element. In other words, there is confirmation of the common sayings “the example comes from the top” or “a fish goes bad from the head”. The greater the number there is of elements of individual transitive closures, the smaller is the number of distinguishable diagnostic states. Only for an amorphous structure is it equal to 3^n, with n being the number of elements of the object.

By analogy, as is the case for a single element, the sum of all the probabilities of the individual states of an object is equal to unity. This is so regardless of the value of the argument of q. In this paper, this value is clearly in the interval <0; 1>.

The main theme of this paper is the presentation of an algorithm for diagnosing an object in terms of intermediate states of its individual elements. *It can be stated with a great confidence that for a three-state classification of checking results, there is no more reasonable alternative to this algorithm. It is hoped that this statement will provide a sufficient reason to challenge the solutions presented in papers* [2, 3, 5, 8]. Given the phenomenon of obscuring states and mutually compensating faults, an object should be diagnosed starting from the elements of the first layer. Positive results from all checks indicate that all elements of the object are operational. Here the traditional division of the diagnosis process into identification and location of non-operational instances does not apply. Insofar as it would be possible to speak of either of these activities at all, it would have to be said that the first should be the second. Depending on the constructional structure of the object, it was suggested it could be possible to indicate more than one non-operational or partially operational element.

It was confirmed that the credibility of a diagnosis is higher the higher the probability is of the state determined by the diagnosis.

An extensive arc of this work is the examination of transition intensities by application of Chapman-Kolmogorov equations. As a result of the analyses, the bathtub curve, which is known from the theory of reliability, has been externalized. Its analytical form can provide an impetus not only for broader research, but it can also confirm its present meaning. It is worth noting that transition intensities, contrary to the opinions of many contemporary researchers, are not always constant or positive quantities. Such discourse, though very necessary, requires a separate work.

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Prawdopodobieństwo stanu pośredniego

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Streszczenie. W pracy zaingerowano w postać powszechnie znanej sumy: $p + q = 1$ sumy prawdopodobieństw zdarzeń przeciwnych, w tym zwłaszcza: sumy prawdopodobieństw stanów: zdatności i niezdatności pojedynczego elementu (tworu charakteryzującego się jednym wyjściem i dowolną liczbą wejść). Okazało się, że bez istotnego uszczerbku dla dokładności dotychczasowych analiz można wprowadzić do rzeczonej sumy dodatkowy składnik: ${}^{iii}pq3$; składnik uosabiający tytułowe prawdopodobieństwo stanu pośredniego. Przy zachowaniu stałej wartości sumy rzeczonych składników określono ich zmienności w funkcji prawdopodobieństwa *q*, po czym w funkcji tej samej zmiennej zbadano zmienność entropii stanu elementu *i* z wykorzystaniem równań Chapmana –Kołmogorowa – intensywności przejść od stanu zdatności do stanów: niezdatności i pośredniego oraz: od stanu pośredniego do stanu niezdatności. Znaczenie prawdopodobieństwa pośredniego odniesiono także do obiektu: jego programu diagnozowania, entropii struktury, pełnego zbioru rozróżnialnych stanów, stosownych intensywności przejść. Nieodzowny stał się w tym względzie opis obiektu językiem teorii grafów, w którym: warstwy i macierz osiągalności są podstawowymi pojęciami. Należy zauważyć, że obiekt jest tworem stanowiącym zbiór pojedynczych elementów, o liczebności i strukturze połączeń zgodnej z przeznaczeniem tego tworu.
Słowa kluczowe: stan pośredni, element obiektu, element zbioru stanów, intensywność, pełny zbiór, entropia, wiarygodność, program diagnozowania