PROBLEMY MECHATRONIKI
UZBROJENIE, LOTNICTWO, INŻYNIERIA BEZPIECZEŃSTWA

PROBLEMS OF MECHATRONICS
ARMAMENT, AVIATION, SAFETY ENGINEERING

ISSN 2081-5891; E-ISSN 2720-5266 **https://promechjournal.pl/**

The Complete Set of Distinguishable Diagnostic States for a Centrifugal Pump System

Paweł SZCZEPAŃSKI[1*] (paszczep@wp.pl)
Robert KIJAK[2] (kijakr@gmail.com)

[*]Corresponding author
ORCID: https://orcid.org/0000-0002-9759-5498

[1]*Military University of Technology;*
Faculty of Mechatronics, Armament and Aerospace,
2 Sylwestra Kaliskiego Str. 00-908 Warsaw, Poland
(professional status of the Author: retired)
[2]*University of Economics and Innovation in Lublin*
4 Projektowa Str., 20-209 Lublin, Poland

Received: September 26, 2022 / Revised: December 22, 2022 / Accepted: February 12, 2023 / Published: September 30, 2023.

2023, 14 (3), 95-118; https://doi.org/10.5604/01.3001.0053.8822

Cite: Chicago Style

Szczepański, Paweł, and Robert Kijak. 2023. "The Complete Set of Distinguishable Diagnostic States for a Centrifugal Pump System". *Probl. Mechatronics. Armament Aviat. Saf. Eng.* 14 (3): 95-118. https://doi.org/10.5604/01.3001.0053.8822

Abstract. In this article, the concept of determination of a complete set of distinguishable diagnostic states has been utilised, which has been well known since the 1980's and, in essence, could refer to any item with any application and any structural features. The only condition set for this item, is its ability to be described by a structure consisting of single exit-point elements. As a consequence, it can become possible to assess, amongst other things, its layers, strong cohesions, transitive and antitransitive closures. Typical tools of such assessment are matrices: transition and reachability. The reason that a centrifugal pump has been selected is not only its importance and common use, but also specifics of interaction of its elements. It is possible to identify for them: independence, dependence, and interdependence. For particular distinguishable diagnostic states, the probabilities of its occurrence have been determined. These probabilities have become a starting point to estimate structure entropy and intensity of transition from the health to any particular failure state. Results of such estimation have been depicted by relevant graphs. A reference has been made to the Bayesian methods and the inseparable 'serial reliability structure', to identify their flaws that result particularly from simplifications leading to false and ambiguous diagnoses. The probabilities of health for particular elements have been differentiated by the selection of adequate Weibull indices.
Keywords: digraph, Weibull index, transition matrix, reachability matrix, transition intensity

1. INTRODUCTION

The concept of a complete set of distinguishable diagnostic states can refer to any item, but on one condition: its structure and operational principles must be described by a structure consisting of single exit-point elements. It is permitted to introduce elements without instrumentation and monitoring accessibility. It is also possible to introduce virtual elements [16]. The meticulousness is beneficial for the final outcome in: the assessment of probabilistic characteristics, the implementation of a diagnostic program, or the identification of structural features of an item. It should be remembered that:

- firstly, the element of an item is considered to be in the failure state if it's all entry signals are acceptable, but the exit signal is unacceptable; and
- secondly, even if there is only one unacceptable entry signal, this results in the unacceptable signal at its exit.

The determination of a set of elements and its mutual interaction becomes the beginning of a mathematical (but also machine) description of an item. In the entry data, it is sufficient to provide successors of particular elements. In such way, binary transition matrix **P** is being developed, which becomes reachability matrix **D** as a result of exponentiation. All rows of the latter indicate all elements that are within the influence of particulate elements of an item. Sets of elements of this influence are named as transitive closures. After this stage, rows and columns that refer to elements without the instrumentation and monitoring accessibility, could be removed from reachability matrix **D**. They have fulfilled their task during determination of the mentioned transitivities.

The reachability matrix identifies also strong cohesions (closed loops) that are revealed by the presence of identical rows and columns. In this instance, they should be left (for a specific loop) only one each. The closed loop determines the set of elements that demonstrate a strong mutual interaction. However, the first impression is that there is actually no certainty, as with a trivial example of the serial connection for light bulbs [9].

The next step (not discussed in this article) can be finding the roots of the reduced reachability matrix $\mathbf{D_R}$ to obtain the reduced transition matrix $\mathbf{P_R}$. The latter allows for putting the depicted functional structure of an item in order, primarily: identifying its layers and removing redundant arches – arches that connect directly two elements and they are connected in a transient closure indirectly by other elements. You could say that a digital copy of a physical item is being created.

Subsequently, there is a step involving the determination of a complete set of distinguishable diagnostic states for an item. The distinguishable state can be such state that results from: no failures, a single failure, but also - in the extreme case – from all failures with their number resulting from the number of independently working elements. It is worthwhile noting that the number of states resulting from two failures is determined by the number of zeros located above the diagonal of upper triangular matrix $\mathbf{D_R}$.

The subject completeness is being reached through supplementing upper triangular matrix $\mathbf{D_R}$ by zero-one rows until it becomes impossible to supplement these rows by another distinguishable row. Assigning particular 0's and 1's respective probabilities of health and failure states assists the development of formulas for probabilities of the occurrence of particular diagnostic states. Their sum that equals unity, confirms the correctness of all completed (preceding) steps. The system consisting of all formulas for probabilities of particular diagnostic states could be compared to the system of connected vessels where any change to a value, that is their probabilities of health state (or failure state) for elements, do not affect the sum result. The results always equal unity.

The determined probabilities could assist with the determination of entropy of the structure, transition intensity and – not discussed in this work – assessment of safety, maintenance risk and compensation risk, but also the diagnosis cost analysis. There is no need to determine the mentioned probabilities for the purpose of determination of diagnostic programs – the upper triangular matrix $\mathbf{D_R}$ is completely sufficient. The application of three-state assessment for results of the checks, which are consistent with the numbering of elements in the consecutive layers of the diagraph, provides an opportunity of the immediate identification of the set of faulty elements.

The more than 40 year old concept of a complete set of distinguishable diagnostic states is an alternative that is worthwhile considering, to the contemporary reliability methods based on the Bayes' formula, so-called the 'serial reliability structure'.

It does not have drawbacks that are particularly caused by simplifications leading to the false and ambiguous diagnoses. Such view has been justified in, *inter alia*, in previous works [13] [14].

The vibration monitoring methods can constitute a serious competition (particularly for mechanical items including the subject pump system) where faults are investigated when anomalies occur in the spectrum of the acoustic signal [2, 8, 10, 11]. However, a potential problem could be the interference and relatively low credibility of the process of assigning this anomaly to a specific fault. The subject problem is potentially making the Bayesian methods more preferred. However, the most appropriate seems to be in such situation the assessment of each element separately. The latter approach changes actually the structure of an item, but creates a situation where the *number of distinguishable diagnostic states* equals the *number of all potential states* of an item. It can be added that such assessment involves testing with defectoscopes and computer tomographs, and using deformation and stress analysis methods. There is a hope that in the near future, it could be possible to apply machine learning methods (artificial intelligence) that allow for the identification of non-linear and non-stationary anomalies.

2. PUMP STRUCTURE AND OPERATIONAL PRINCIPLES

A diagram of the subject pump is presented in Fig. 1. Centrifugal pumps have a wide and various applications in the municipal and industrial water engineering, that is:

a) in water supply systems for towns, districts and industrial plants;
b) in thermal energy generation – to supply steam boilers with hot water of a temperature up to approximately 250°C, pumping cooling water (steam condenser), removal of condensate, etc.;
c) in hydroelectric power stations as tank pumps;
d) in mining – to drain a flooded mine
e) in the industry: chemical and food manufacturing – pumping of any liquid products, etc.

They convey clean, contaminated and reactive liquids, and solids mixed with water. They are suitable to the direct drive by high-speed motors (typically electrical).

The main working element is impeller 7, equipped with a number of blades that are curved opposite to the direction of rotation of the shaft. The rotor mounted on shaft 5 that is rotating at a high speed and located in a spiral cover (hull 10), turning into a conical diffuser that is also an outlet connection marked as 6 with the attached discharge pipe 8. The fluid flows into the pump through suction pipe 3 and suction pipe 4. At the beginning of the suction pipe, there is strainer 1 with check valve 2.

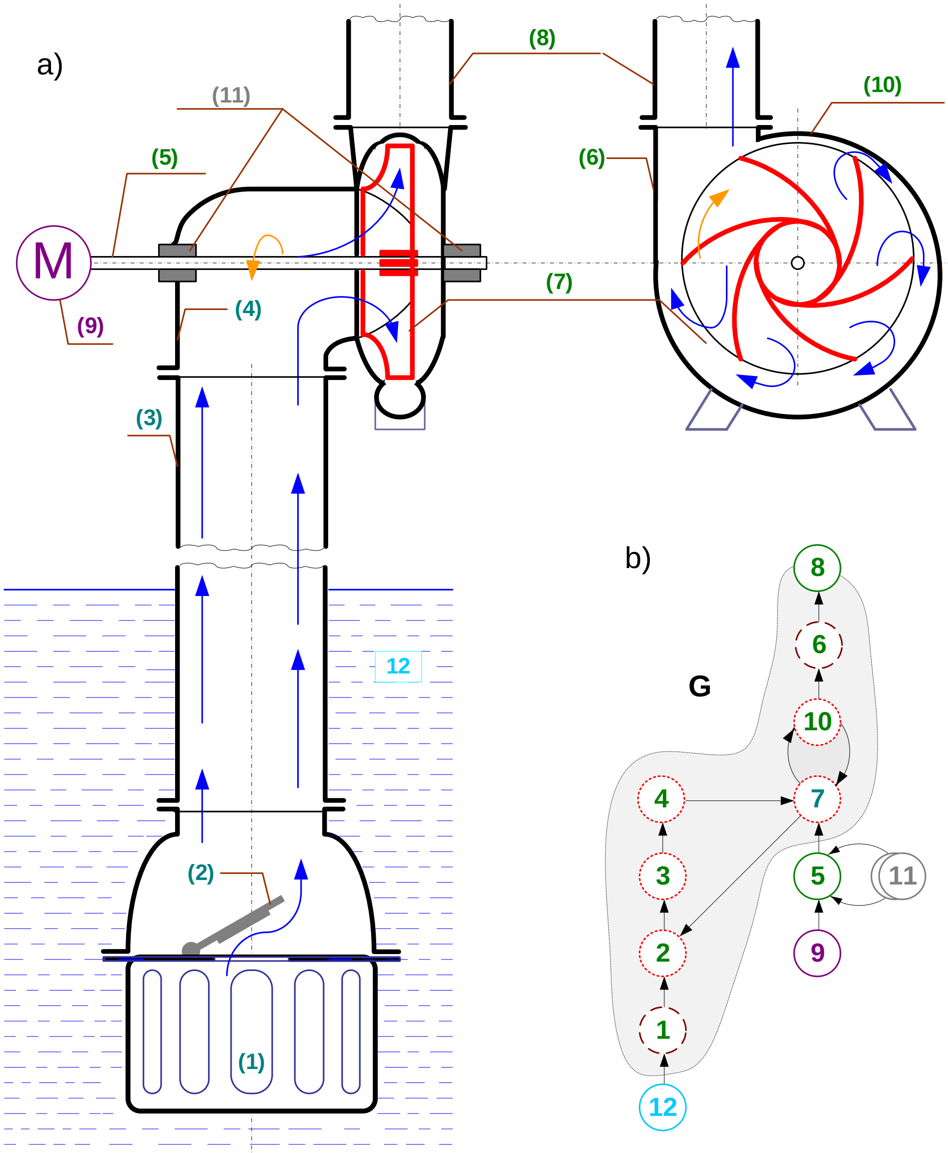

Fig. 1. A single stage centrifugal pump (this figure has been prepared based on a figure contained in [7]):

a) diagram: 1 – strainer, 2 – check valve, 3 – suction pipe, 4 – suction connection, 5 – shaft, 6 – pressure connection, 7 – single-suction impeller, 8 – discharge pipe, 9 – motor, 10 – hull, 11 – bearings, 12 – input fluid

b) digraph (description of the interaction of pump elements): the grey area shows a set of components for element 8’, but components highlighted by a red dotted line demonstrate a strong cohesion

Starting the pump requires its previous flooding, that is, filling with fluid: impeller 7 and suction pipe 3 with suction connection 4. Otherwise, impeller 7 would ‘rotate in the air’ and would be able to create a vacuum for impeller 7 to be able to suck in the fluid. The pump flooding is only possible when suction pipe 3 has check valve 2 installed.

When the pump has been flooded and started, fluid particles influenced by the rotation of the blades that generate a centrifugal force, move from the centre of impeller to the outer part with the fluid velocity and pressure increasing during the flow through the impeller. The velocity of fluid flowing out of the impeller decreases slightly in a spiral channel surrounding the impeller and in a conical diffuser that ends the spiral. As a result, kinetic energy is converted into pressure energy.

The pressure difference acting on the surface of the fluid in tank 12 and pressure at the impeller outlet causes the fluid to flow through suction pipe 3 to the impeller. During the flow through the impeller, angular momentum that is a momentum of fluid in motion, increases from the energy of motor 9 driving the pump.

3. A MATHEMATICAL DESCRIPTION OF A PUMP

In general, the mathematical description of an item is assisted by the language of graph theory together with its digraphs and binary transition **P**(G) and reachability matrices **D**(G) [1].

P(G) =

	1	2	3	4	5	6	7	8	9	10	11	12
1		1										
2			1									
3				1								
4							1					
5							1					
6								1				
7		1								1		
8												
9					1							
10						1	1					
11					1							
12	1											

(1)

The binary transition matrix **P**(G) marked as (1), is a mathematical description of digraph G structure from Fig. 1.b). Vertical and horizontal coordinates of ones in particular rows and columns mean respective direct successors and predecessors of particular vertices. The empty cells mean zeros. This comment applies also to the remaining matrices.

The binary reachability matrix **D**(G) marked as (2), indicates by ones in its rows and columns, all direct, but also indirect successors and predecessors of particular digraph G's vertices. In the language of graph theory, sets of these successors and predecessors are named as closures: transitive and antitransitive.

D(G) = (2)

	1	2'	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1		1	1	1		1		
2'		1	1	1		1	1	1		1		
3		1	1	1		1	1	1		1		
4		1	1	1		1	1	1		1		
5		1	1	1	1	1	1	1		1		
6						1		1				
7		1	1	1		1	1	1		1		
8								1				
9		1	1	1	1	1	1	1	1	1		
10		1	1	1		1	1	1		1		
11		1	1	1	1	1	1	1		1	1	
12	1	1	1	1		1	1	1		1		1

Theoretically, reachability matrix is defined as:

$$\mathbf{D}(\mathrm{G}) = \sum_{k=0}^{n-1} \mathbf{P}^{k}(\mathrm{G})\,;\; n = card\ \mathbb{E} \tag{3}$$

where: $\mathbb{E}$ – the set of digraph's vertices. Symbols: sum and multiplication mean alternative and conjunction respectively.

Typically, there are conducted mutual comparisons of columns (or rows) in a matrix determined as above, for the purpose of identification of strong cohesions (closed loops). The numbering of vertices (elements) of these cohesions is determined by the identical columns (or rows). Regardless what is the numbering of vertices in strong cohesions, they are replaced by single vertices and consequently, the initial digraph form is transformed to the Hertz graph [1].

From the **D**(G) matrix (2), it can be selected only one strong cohesion:

$$\mathbb{E}_{sp} = \{e_2, e_3, e_4, e_7, e_{10}\} = \{e_{2'}\} \tag{4}$$

A practical application of **D**(G) matrix can occur after the reduction of rows and columns referring to elements without instrumentation and monitoring accessibility. In the considered case, this refers to: elements of a strong cohesion ***2'***, strainer 1 and discharge pipe 8.

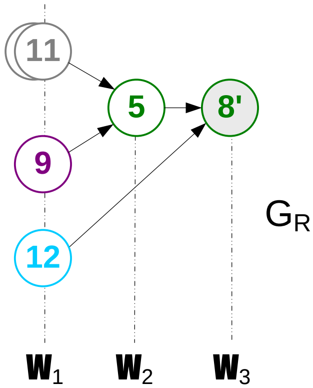

Fig. 2. Reduced G_R digraph of a digraph depicted in Fig. 1b).

Expressions have been explained in the below text

$\mathbf{D_R}(G_R) =$ (5)

	9	*11*	*12*	*5*	*8*
9	1			1	1
11		1		1	1
12			1		1
5				1	1
8					1

The reduced reachability matrix $\mathbf{D_R}(G_R)$ (5) has a form of an upper triangular matrix as a result of in the introduction of rows and columns numbering that is consistent with the numbering of vertices in the following digraph layers [1]. In the first layer, there are included these vertices that do not have their predecessors. In the second layer, there are included those vertices that would not have their predecessors after deleting from the digraph all vertices from the previous layers.

Digraph G_R (Fig. 2.) has five layers:

$$\mathbb{W}_1=\{e_9, e_{11},e_{12}\}$$
$$\mathbb{W}_2=\{e_5\}$$
$$\mathbb{W}_3=\{e_{8'}\}$$

If determination of the reachability matrix $\mathbf{D_R}(G_R)$ for a pump described by a digraph G [see Fig. 1.b.)] caused no problems, such determination for a water utility company, a manufacturing plant or a local Council would perhaps do. The latter could also be described in the graph theory language. Taking this into consideration, there are a number of additional approaches. The most of spectacular procedures include the exponentiation of powers of the upper triangular matrix $\mathbf{B = P + I}$, where is a unit matrix. As a result, the number of required operations is six times lower than the exponentiation of its square form. The exponentiation of matrix **B** is being conducted until its next power equals to the previous: $\mathbf{B}^{k+1} = \mathbf{B}^{k}$.

Nowadays, almost all procedures related to the determination of reachability matrix, but also to its application, are carried out with specialised software. However, it is worthwhile noting that the substantial effort is to be made at an early stage of the graphical representation of the item structure: conducting aggregation of elements without the instrumentation and monitoring access and condensing each closed loop that is called in the graph theory language as a strong cohesion.

4. A PRACTICAL APPLICATION OF THE PROPOSED METHOD OF STRUCTURE DESCRIPTION

The subject considerations require the conversion of the reachability matrix $\mathbf{D_R}(G_R)$ for the diagnostics purposes. Therefore, particular columns are allocated **checks,** that is steps focused on the confirmation of compliance of the elements' exit signals with their standard pattern. It is assumed that, if any parameter of the given signal exceeds the permitted standard, the relevant check result is then negative. Otherwise, the result is positive. Both results are allocated logical values **1** and **0** respectively.

The above process can assist with the determination of the complete set of distinguishable diagnostic states for a pump, as depicted in Fig. 3.

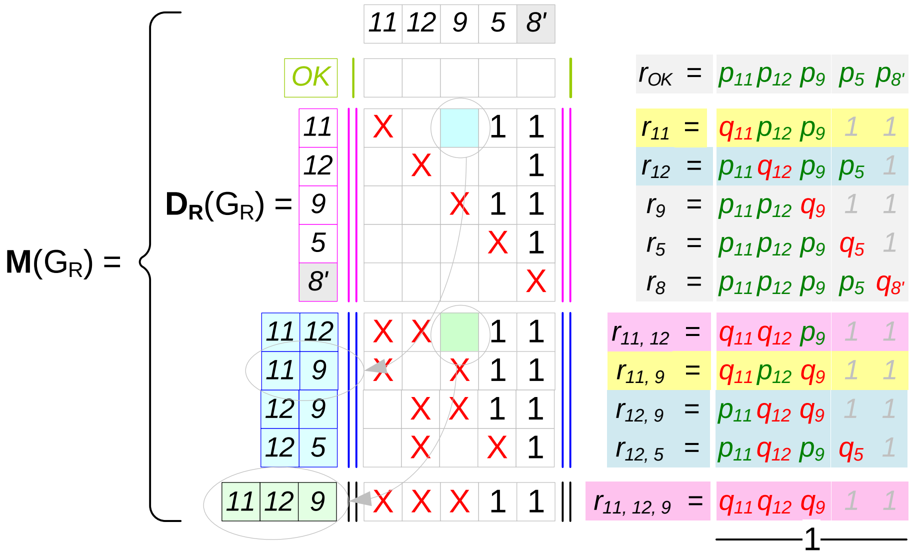

Fig. 3 An illustration of the process of determination of the complete set of distinguishable diagnostic states of an item and estimation of the probabilities of occurrence of elements of this set.

The description is in the below text.

For the understanding of this step's meaning, it is important to understand the meaning of ones within the diagonal of the upper triangular matrix $\mathbf{D_R}(G_R)$. It is worthwhile noting that the identification of a state is made easy due to determination of the position of the first one on the left-hand side in a row vector of all results of checks being done. Further, if a row of $\mathbf{D_R}(G_R)$, which has this position number, has a distribution with these results' distribution, the indicated faulty element is the only cause of this distribution. Each cause (fault) will be marked with: X and - what should be easily understood – identified its position (vertical coordinate of this symbol in matrix $\mathbf{D_R}(G_R)$).

The indication of second X symbols in rows, which means zero-one distributions of two distinguishable faults, requires summation of rows determined by coordinates of zeros located on the right-hand side of diagonal (colour coded in blue). An example of the summation (expressed with an arrow) refers to rows: 11 and 9.

The determination of subsequent distinguishable states of an item is possible amongst zeros located on the right-hand side in relation to the far-right X's. On each occasion, the horizontal coordinate of every added value is being supplemented by the horizontal coordinate of this value (addend). The summed number of components of a particular row is determined by the number of X's in this row.

An example of the summation of rows that determine zero-one distributions for three causes (faults), refers to rows: 11, 12 and 1 (colour-coded green).

The summation process ends at a time when all zeros that have been used located on the right-hand side in relation to the far-right X's. It is worthwhile noting that each addend is a row of $\mathbf{D_R}(G_R)$ matrix. The last step is adding a row above the upper triangular matrix $\mathbf{D_R}(G_R)$, which only contains zeros – a row that symbolises occurrence of no causes of the elements' faults.

In this way, $\mathbf{M}(G_R)$ matrix has been developed with rows that determine a unique zero-one sequences that are assigned particular elements of the complete set of distinguishable diagnostic states. In addition to the matrix, there have been presented probabilities ***r*** of the occurrence of elements of this set. The probabilities have then become projections of these sequences. X's and zeros (empty cells) are assigned respective q and p probabilities, that is probabilities of health and failure with indices of checks (vertical coordinates) for these X's and zeros. Ones are left their own values, that is values that identify probabilities of the states of certainty – states that are obscured by the identified of failure states.

The sum of probabilities that have been identified in this way, equals one. The check of this sum is based on the following relationship:

$$p_{11}p_{12}\overbrace{\left(p_9\overbrace{\left(p_5\overbrace{\left(p_{8'}+q_{8'}\right)}^{1}+q_5\right)}^{1}+q_9\right)}^{1} + p_{11}q_{12}\overbrace{\left(p_9\overbrace{\left(p_5+q_5\right)}^{1}+q_9\right)}^{1} + \qquad (9)$$

$$+\,q_{11}p_{12}\overbrace{\left(p_9+q_9\right)}^{1} + q_{11}q_{12}\overbrace{\left(p_9+q_9\right)}^{1} = p_{11}p_{12}+p_{11}q_{12}+q_{11}p_{12}+q_{11}q_{12} =$$

$$= p_{11}\overbrace{\left(p_{12}+q_{12}\right)}^{1} + q_{11}\overbrace{\left(p_{12}+q_{12}\right)}^{1} = p_{11}+q_{11} = \mathbf{1}$$

This value could be demonstrated in various ways. Here, states have been initially colour-coded, which probabilities are characterised by the same factors (see Fig. 3).

Probabilities r_i of the occurrence of states from <u>the complete set of distinguishable diagnostic states</u> (see Fig. 3) can be used to determine: structure's entropy and renewal risk; optimalisation of conditional diagnostic programs; and calculation of the intensity of transfer from the health to any failure state.

Those who do not favour the probabilistic approach, could directly use the reachability matrix $\mathbf{D_R}(G_R)$ (5)8 for the determination of, at least, a conditional diagnostic program (see Fig. 4).

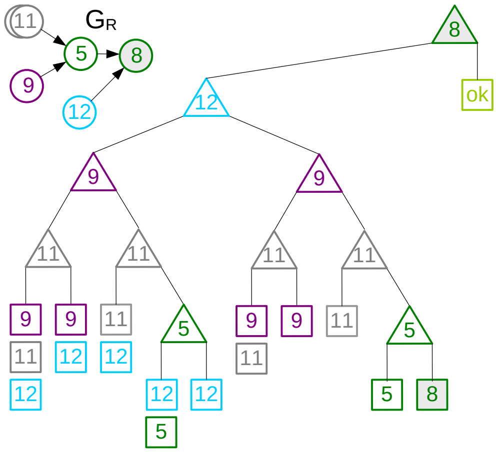

Fig. 4. A conditional program for diagnosing a centrifugal pump with G_R digraph structure, where:

- Pump elements, checks and faults have been identified as circles, triangles and squares respectively.
- The negative and positive results of checks have been identified as the respective left and right branches from the triangles.

The priority has become the assessment of health of the complete pump through the water flow check (check 8). It is sufficient to say that the next checks were determined based on the difference in the number of transitive and antitransitive closures for the particular digraph vertices. The average number of checks is: $\frac{1 \cdot 1 + 4 \cdot 6 + 5 \cdot 4}{11} = 4.09(09)$. For checks at the exits from elements: 8, 12, 9, 11, and 5, a flowmeter, liquid level sensor, tachometer, thermometers, and again a tachometer could be used respectively.

Instrumentation and monitoring apparatus is often an integral part of an item being examined, in particular when important from the water engineering viewpoint. Its availability allows us to adjust and control the item's performance in an optimal way.

Figure 5 depicts a conditional and "forward" diagnostic program that involves an examination of a pump from the entry to exit elements [3].

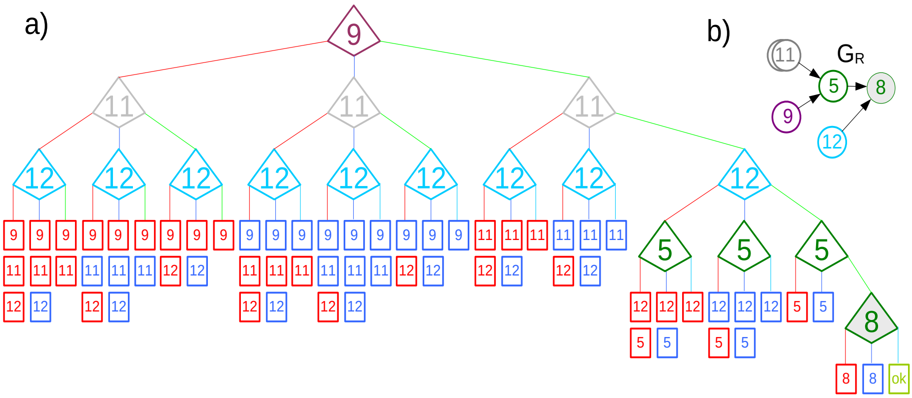

Fig. 5 A conditional and "forward" diagnostic program for a pump that is depicted as G_R in a b). a) is a dendrite in which each deltoid means a check with the triple (three-state) assessment of its result. Each rectangle means a diagnosis: failure, interim state and health where each left, middle and right branch negative, interim and positive check results respectively

The basis of preparation of a program in Fig. 5 has become an upper triangular matrix $\mathbf{D_R}(D_R)$ marked as (5). The upper section of the program (order of checks after their positive results) is consistent with the order of numbering of rows and columns of the matrix, that is, consistent with the numbering of vertices of G_R digraph, which are present in its consecutive layers: from the first to the third layer. Therefore, the name: a conditional and 'forward' diagnostic program, results from the latter characteristic.

The conditional and forward diagnostic program is not the most effective from the time and effort viewpoint. However, the program has a unique benefit, that is, the number of each failure and interim state is consistent with the number of a check that determines them. The number of diagnosis L_d for the three-state assessment of check results are expressed by the following relationship:

$$L_d = \sum_{n=0}^{n_{max}} a_n 2^n \qquad (10)$$

where: a_n – the number of distinguishable diagnostic states resulting from faults of n elements; – maximum number of faults, which could be identified in the diagnostic process

From Fig. 3, it can be concluded that a_0, a_1, a_2, a_3, and n_{max} are equal to: 1, 5, 4, 1, and 3, respectively, that is, consistent with the relationship (10), $L_d = 1 \cdot 1 + 5 \cdot 2 + 4 \cdot 4 + 1 \cdot 8 = 35$.

From the dendrite of checks (see Fig. 5), it seems that after completion of: three checks, 24 diagnoses could be identified; four checks – eight diagnoses; five checks – three diagnoses. Therefore, the average number of checks equals: $\frac{3\cdot 24+4\cdot 8+5\cdot 3}{35} = 3.4$.

For diagnosis with three-state assessment of check results, there is applicable a rule applicable, which says that the element's **failure** or **interim state** is decided by its unacceptable and acceptable exit signal for acceptable entry signals. Consequently, there is adopted a rule that no check is done for the entry signal with its unacceptable or not fully acceptable exit signal

This resignation from checks for the entry signal with its unacceptable or not fully acceptable exit signal has a rational justification. It should be noted that diagnoses and renewals are intertwined and such resignation eliminates the occurrence of mutually compensating faults and reduces Type I and II's type errors[3].

5. CALCULATION OF SELECTED CHARACTERISTICS

Such calculation begins with the probabilities *r* of distinguishable diagnostic states that are presented in Fig. 3. However, their application requires determining at least, indicatively) probabilities of the health of particular pump elements (see Fig. 1). A rational should the knowledge of structure and nature of these elements in this instance. A proposition of the conducted assessment has been presented in Table 1. For example, the probability p_7 of the allocated impeller's health state has been given a respective exponent that equals 6 as only one blade could be faulty, but probability of the motor p_9 – 4 due to the potential faultiness of its components and potential power outages.

It is noted that Fig. 3 shows also the failure probabilities q_i. Their values result from a relationship: $q_i = 1 - p_i$ and $q_{8'} = 1 - p^{14.7}$.

The probabilities of the failure *q* can be distribution functions of any probability distributions. Most commonly, the exponential distribution is used for this purpose with failure intensities that are characteristic for this distribution. It is not necessary that these intensities must depend on time [12]. For a pump, the intensity can depend on, for example, volume of the pumped water.

Table 1. Values of probabilities of the health of particular pump elements

Symbol	$p_{8'}$	p_5	p_9	p_{11}	p_{12}
Value	$p_1p_2p_2p_3p_4p_6p_7p_{10}p_8 = p^{0.7}p^3p^{0.5}p^1p^1p^6p^{1.5}p^1 = p^{14.7}$	p^1	p^4	p^2	$p^{0.3}$

Variation of the identified probabilities are shown in Fig. 6.

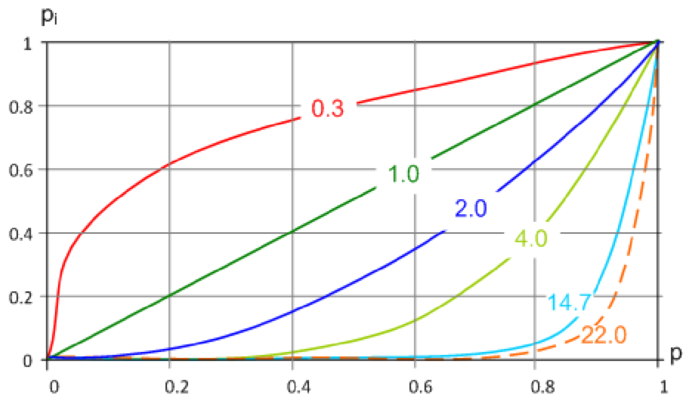

Fig. 6. Variations of probabilities p_i that depend on the magnitude of exponents: 0.3, 1.0, 2.0, 4.0, 14.7, and 22.0, that are allocated to respective: water level 12, shaft 5, bearings 11, motors 9, elements without instrumentation and monitoring access 8', but also the pump itself (dashed line)

If this has not yet been noticed, it should be stated that the greater exponent of power the greater probability of failure. The increase in exponent of power from zero to infinity could be seen as the transition of infinite number of components from their parallel reliability structure to the serial reliability structure. In the first case, an element should be recognised as unbreakable, but in the second case, as completely broken down, but only when p for all components will be slightly lower and does not equal one. The presented approach has Weibull distribution characteristics.

Characteristics of the variability of probability states and the intensities of transition from the health state to particular failure states, could be examined through the examination of variability of a function presented in Table 2.

Trajectories of variability of the selected distinguishable diagnostic states r_i (see Fig. 2) become intuitively understood if they accommodate that a state could be obscured by the other. When Fig. 5.b) is considered, it can be noticed that the magnitude of the probability $r_{8'}$ is dependent on the magnitude of probabilities of health for all remaining elements. The elevated probabilities of $r_{11,\,9}$ result from the concurrent faults of motor 9 and bearings 11 could be understood if their total independence from elements in their transitive closures is noticed. This elevation will occur only when p is lower than 0.5. For this and the lower values p, other states resulting from multiple faults will have a similar characteristic and they even become more common than single faults as a rule.

Table 2. Functions of probabilities of distinguishable diagnostic states and intensities of transition from the health state r_{OK} to the particular failure state $r_{k \neq OK}$; where: i – the state's symbol

i	$r_i(p)$	$\lambda_i(p) = \frac{r_i'(p)}{r_{OK}}$
OK	$p^2 p^{0.3} p^4 p^1 p^{14.7} = p^{22}$	$\frac{22}{p}$
11	$(1-p^2)p^{0.3}p^4$	$\frac{43-63p^2}{10p^{18.7}}$
12	$p^2(1-p^{0.3})p^4 p$	$\frac{70-73p^{0.3}}{10p^{16}}$
9	$p^2 p^{0.3}(1-\ p^4)$	$\frac{23-63p^4}{10p^{20.7}}$
5	$p^2 p^{0.3} p^4(1-\ p)$	$\frac{p^{5.3}(63-73p)}{10p^{22}}$
8'	$p^2 p^{0.3} p^4 p^1(1-p^{14.7})$	$\frac{73-220p^{14.7}}{10p^{15.7}}$
11, 12	$(1-p^2)(1-p^{0.3})p^4$	$\frac{63p^{2.3}\ -60p^2-43p^{0.3}+40}{10p^{19}}$
11, 9	$(1-p^2)p^{0.3}(1-p^4)$	$\frac{63p^6-43p^4-23p^2+3}{10p^{22.7}}$
12, 9	$p^2(1-p^{0.3})(1-p^4)$	$\frac{63p^{4.3}-60p^4-23p^{0.3}+20}{10p^{21}}$
12, 5	$p^2(1-p^{0.3})p^4(1-p)$	$\frac{73p^{1.3}\text{-}70p\text{-}63p^{0.3}+60}{10p^{17}}$
11, 12, 9	$(1-p^2)(1-p^{0.3})(1-p^4)$	$\frac{-63p^6+60p^{5.7}+43p^4-40p^{3.7}+23p^2-20p^{1.7}-3}{10p^{22.7}}$
Sum:	**1**	**0**

In relation to variabilities of the intensity of transition [4] from the health state $r_{OK.}$ to the failure states (see Fig. 8), it is worthwhile noting that the sum of all derivatives from Table 2 equals zero because the sum of probabilities equals one (9). For p = 1, each intensity related to a single fault has its beginning consistent with the negative value of exponent of power. All remaining intensities (taken as a sum) begin their 'run' from zero. The increase or decrease in particular intensities depends on the pump structure, but basically on the extent of being obscured by other states. The intersection of $\lambda_{8'}$ intensity with abscissa for p = 0.928 indicates the extremum of probability state $r_{8'}$ for the same value p. To highlight the nature of runs, considerations are limited to $p \in< 0.9;\ 1 >$.

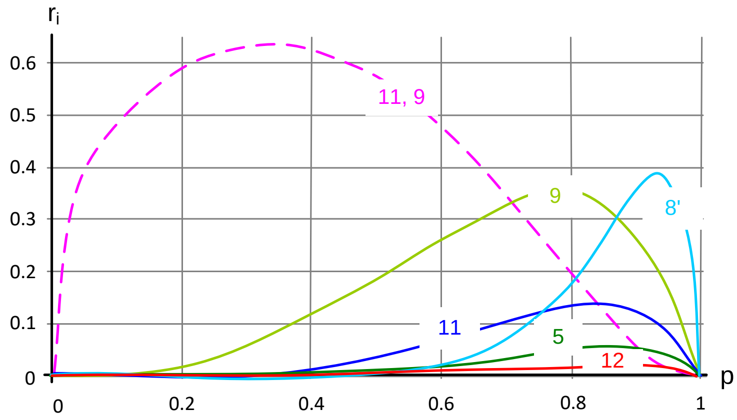

Fig. 7. Variabilities of selected probabilities of the distinguishable diagnostic states resulting from faulty water level 12, shaft 5, bearings 11, motors 9, elements without instrumentation and monitoring access 8' and a concurrent fault to bearings and motor 11, 9 (purple dashed line)

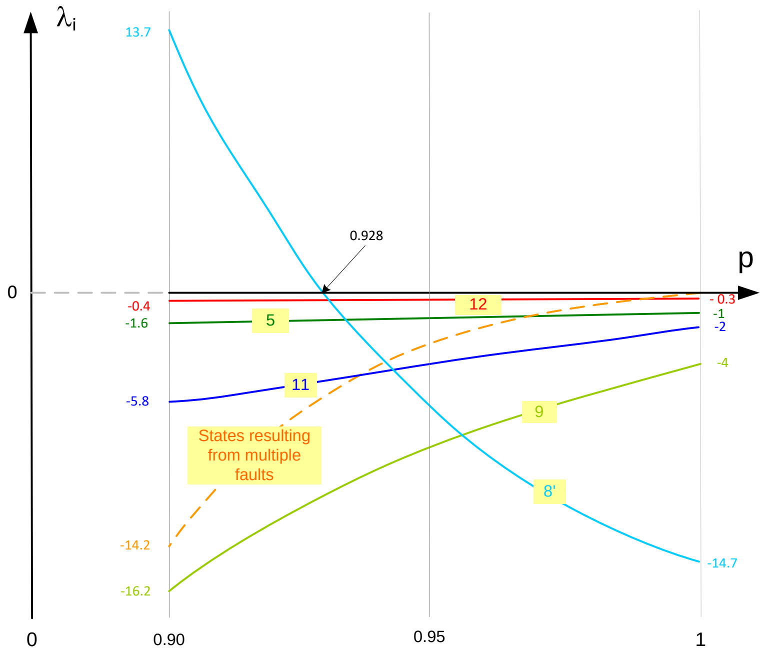

Fig 8. Variability of the intensity of transition from the health OK to the particular distinguishable diagnostic states resulting from faulty water level 12, shaft 5, bearings 11, motors 9, elements without instrumentation and monitoring access 8' and the remaining states all together that are a product of multiple faults (orange dashed line)

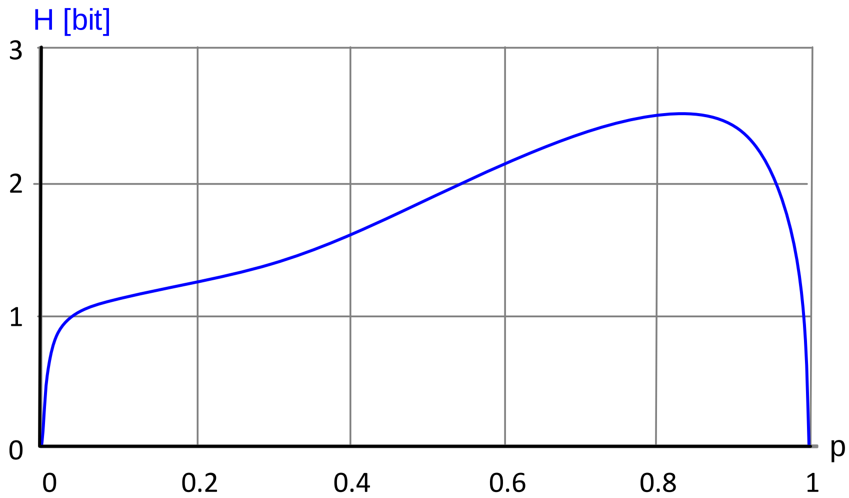

Fig. 9. An entropy of the pump structure from Fig. 1

The structure of entropy [5] for a pump system (see Fig. 9) has been determined based on the Shannon entropy formula [6], which is in this work as follows (11):

$$H = \sum_i -r_i \, log_2 \, r_i \quad \text{where: } i \text{ – the state symbol (see Table 2)} \tag{11}$$

On each occasion, entropy identifies two zero values and the maximum value that occurs between the former values. The first zero (left hand side of the maximum value), characterises a pump with no value in operation. On the contrary, the second zero characterises a pump with the complete operational certainty. (It should be noted that the state of a pump has been predetermined in both cases.)

The maximum value that expresses the maximum uncertainty, could express from the practical viewpoint, an average number of checks necessary for the determination of pump state. The logarithm base suggests binary values of responses to these checks, for example: *yes*, *no* or *good*, *bad*.

The knowledge of an item entropy is very important. It affects the decision to undertake or abandon activities associated with its maintenance on one side, and rationalise costs for its examination on the other side. Based on the conducted considerations, it can be concluded that the key issue here is becoming – previously being unnoticed - structure. Resources are differently distributed when it is dispersed and totally differently when aggregated. The first could refer to drinking water samples, but the second to their mixture that is analysed for the presence of a rare contaminant.

6. CONCEPT OF THE COMPLETE SET OF DISTINGUISHABLE DIAGNOSTIC STATES VERSUS THE BAYESION METHODS

Figure 11 depicts one of potential applications of this theory to the pump system. After the required reduction of information resulting from the presence of closed loops and a limited access to the monitoring and instrumentation accessibility, the starting point for the diagnostic program become a matrix presented in formula (5).

The assessment is conducted in two steps. The first step involves the identification of fault (R. n), but the second step involves the locating of fault (L. n.) [15]. The second step follows the negative result of the first step. For multi exit-point structures of an item (e.g., television set and the family), there is doubling of checks in the first step.

The optimisation of fault locating is made based on the conditional probabilities. In the context of pump system, when the coincidence are preserved with health state probabilities presented in Table 1, the subject conditional probabilities (being, in fact, relative probabilities) are described by the below relationships:

$$q'_{8'} = \frac{1 - p^{14.7}}{5 - (p^{14.7} + p^1 + p^4 + p^2 + p^{0.3})} \tag{12}$$

$$q'_5 = \frac{1 - p^1}{5 - (p^{14.7} + p^1 + p^4 + p^2 + p^{0.3})} \tag{13}$$

$$q'_9 = \frac{1 - p^4}{5 - (p^{14.7} + p^1 + p^4 + p^2 + p^{0.3})} \tag{14}$$

$$q'_{11} = \frac{1 - p^2}{5 - (p^{14.7} + p^1 + p^4 + p^2 + p^{0.3})} \tag{15}$$

$$q'_{12} = \frac{1 - p^{0,3}}{5 - (p^{14.7} + p^1 + p^4 + p^2 + p^{0.3})} \tag{16}$$

but their runs, as the probability function p (see Fig. 10). Absolute values for particular faults are the numerators within relationships (12-16). A graph of the latter could be Fig. 10. when the abscissa axis has been labelled by a value: 1-q.

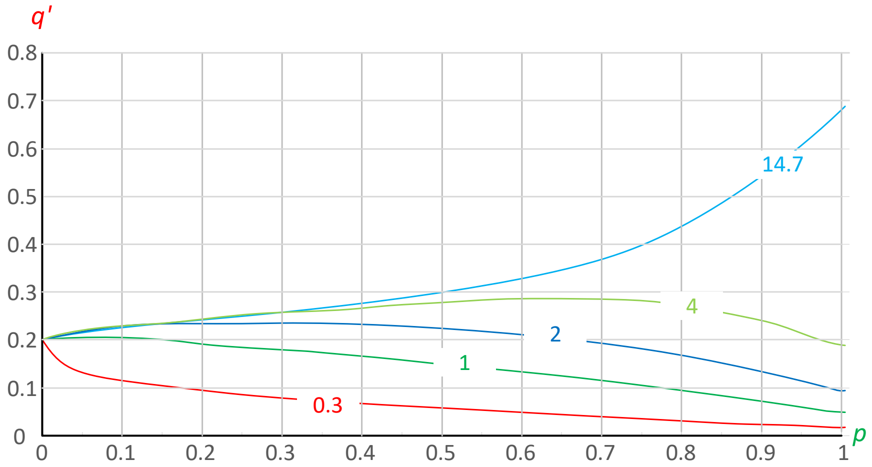

Fig 10. Variability of conditional probabilities for faults of the pump elements, for exponents of powers expressing the health s state probabilities included in Table 1

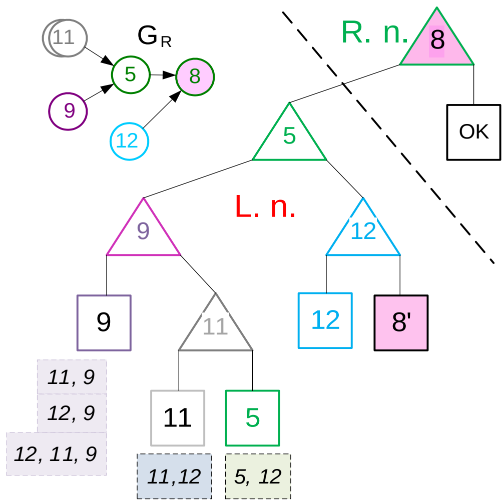

Fig. 11. Conditional program for diagnosing a centrifugal pump with the digraph structure G_R, according to the Bayesian method, assuming its serial reliability structure (location of single faults), where:

- Circles, triangles and squares are used to mark pump elements, checks and faults respectively.
- Left and right branches that are protruding from the triangles, are negative and positive results of the checks respectively.

Abbreviations: R. n. and L. n. mean the identification of fault and the locating of fault respectively.

NB. In the coloured rectangles, alternative diagnoses have been placed, which could result from multiple faults.

The sense of application of conditional probabilities exists only in relation to the locating of single faults, for example, in accordance with programs based on the 'probabilistic effectiveness' or 'informative effectiveness' of the checks. A program presented in Fig. 6 has been prepared using a 'half-division' method.

Regardless what method is used, there is an important problem when the fault identification instead of being positive will be negative. The false alarm might result in the false diagnosis. When compared to a program depicted in Fig. 4, a program shown in Fig. 11 becomes ambiguous for some sequencies of results of the checks.

In the proposed method, the state of health for an item is considered as being equal to the other states. All of them are named as the diagnostic states, what has not consequently resulted in the division of assessment into the fault identification and the fault locating. Therefore, the diagnostic process depicted in Fig. 5 is considered as uncommon. The use of intermediate results of the checks increases substantially its credibility. It should be noted that the subject intermediateness contributes to undertaking of predictive activities.

7. CONCLUSIONS

The structure of items is characterized by not only by its complexity of mutual connections of its elements, but presently also by the multiplicity of the latter. This does not seem to be particularly important. As a result, the description of the structure of an item using the graph theory language allows for reducing the number of these elements in a situation when the elements are not accessible to the monitoring and instrumentation apparatus and when they are subject to the closed loops.

In this article, the probabilities of the distinguishable diagnostic states r_i, conditional probabilities q', the transition intensities λ_i and the entropy H have been presented as a function of a single variable – the probability of the health state p, which replaces well known from the literature value $R(t)$. This has been made that this probability could become a function of revolutions, moto-hours, cycles, gunshots…, and, in the case of a pump system – the volume of water being pumped. Moreover, the probability p could results from any distribution. However, the exponential distribution is most commonly being used for this purpose, and if something were dependent on 't', it would be necessary to make such introduction: $p = \mathrm{e}^{-\mathrm{a}t} \rightarrow t = -\frac{\ln p}{\mathrm{a}}$; where: a[1] – fault intensity.

[1] Traditionally, the failure intensity is expressed by the symbol λ. In this article, this symbol has been used for the transition intensity (see Table 2.); 'a' is a dimensioned , but – 'λ' dimensionless value.

The criticism of Bayesian methods and the ‘serial reliability structure’ when applied to an item, has no justification for single elements in the context of connectivity of their components.

Over years, it seems that there has been an on-going challenge to conduct examinations into items with dispersed structures, that is items that have no functional connectivity between their elements. Our imagination can indicate our community with its public health related problems resulting from the quality of natural environment, particularly water [10, 11]. A comprehensive current data base of statistical assessment results assists with identification of optimal solutions. Can this work’s conclusions assist with further research to find relevant solutions?

FUNDING

The authors received no financial support for the research, authorship, and/or publication of this article.

REFERENCES

[1] Korzan, Bohdan. 1978. *Elementy teorii grafów i sieci, Metody i zastosowania*. Warszawa. Wydawnictwo Naukowo-Techniczne.
[2] Cempel, Czesław. 1989. *Diagnostyka wibroakustyczna maszyn*. Warszawa: Wydawnictwo Naukowo-Techniczne.
[3] Szczepański, Paweł. 2021. “Probability of an Intermediate (Reduced Operational) State”. *Probl. Mechatronics. Armament Aviat. Saf. Eng.* 12 (3) : 71-96.
[4] Szczepański, Paweł, and Józef Żurek. 2018. “Chapman-Kolmogorov Equations for a Complete Set of Distinct Reliability States of an Item, State”. *Probl. Mechatronics. Armament Aviat. Saf. Eng.* 9 (4) : 49-70.
[5] Szczepański, Paweł. 2009. “Entropia struktury obiektu w ujęciu addytywnym”. *Biuletyn WAT* 3 : 169-192.
[6] Shannon, Claude E. 1948. “A mathematical theory of communication”. *Bell System Technical Journal* 27 : 379–423 and 623–656.
[7] *Mała encyklopedia techniki*. 1969. Warszawa: Wydawnictwo Naukowo-Techniczne, pp. 852.
[8] Nowicki, Ryszard. 2019. “Monitorowanie stanu technicznego pionowych agregatów pompowych”. *Pompy, Pompownie* 1 : 69-83.
[9] Szczepański, Paweł. 2003. “Funkcjonalna struktura niezawodnościowa obiektu na przykładzie diagnozowania szeregowo połączonych elementów”. *Diagnostyka* 28 : 53-62.

[10] Kijak, Robert. 2022. Water 4.0: enhancing climate resilience. In: Brears R.C. (eds) *The Palgrave Handbook of Climate Resilient Societies*. London: Palgrave Macmillan, Cham,

[11] Kijak, Robert. 2021. Defining Water 4.0. In: *Water Asset Management in Times of Climate Change and Digital Transformation*. Palgrave Studies in Climate Resilient Societies. Palgrave Macmillan, Cham.

[12] Kapur, Kailash C., and Leonard R. Lambertson. 1977. *Reliability in Engineering Design*. Department of Industrial Engineering and Operations Research, Wayne State University Detroit, Michigan 48202.

[13] Szczepański, Paweł. 2004. "Nie dzielmy diagnozowania na rozpoznanie i lokalizowanie niezdatności". *Diagnostyka* 30 : 117-121.

[14] Szczepański, Paweł. 2011. "Erroneous use of Bayes' theorem in technical diagnostic". *Diagnostyka* 57 : 47-54.

[15] PN-93/N-50191 *Niezawodność; jakość usługi. Słownik terminologiczny elektryki.* Warszawa: Polski Komitet Normalizacyjny.

[16] Szczepański, Paweł. 2014. "Para zagrożeniowo-ochronna jako element szacowania bezpieczeństwa obiektu i ryzyka". *Biuletyn WAT* 4 : 233-257.

Pełny zbiór rozróżnialnych stanów diagnostycznych pompy wirowej

Paweł SZCZEPAŃSKI[1], Robert KIJAK[2]

[1]*Wojskowa Akademia Techniczna,*
Wydział Mechatroniki, Uzbrojenia i Lotnictwa,
ul. gen. Sylwestra Kaliskiego 2, 00-908 Warszawa
(status autora: emeryt)
[2]*Akademia WSEI, ul. Projektowa 4, 20-209 Lublin*

Streszczenie. W artykule posłużono się, znaną od lat osiemdziesiątych ubiegłego wieku, koncepcją wyznaczania pełnego zbioru rozróżnialnych stanów diagnostycznych, która w istocie rzeczy może odnieść się do dowolnego obiektu o dowolnym przeznaczeniu i dowolnych cechach konstrukcyjnych. Jedyny warunek jaki stawia się przed obiektem to możliwość jego opisu strukturą połączeń elementów jednowyjściowych. Konsekwencją stanie się wtedy możliwość poznania m.in. jego: warstw, silnych spójności, zamknięć tranzytywnych i zamknięć antytranzytywnych. Typowymi narzędziami tego poznania są macierze: przejść i osiągalności. Powodem wybrania pompy wirowej jest nie tylko jej ważkość i powszechność użycia, ale i specyfika współdziałania jej elementów. Można wśród nich wyróżnić: niezależność, zależność i współzależność. Poszczególnym rozróżnialnym stanom diagnostycznym wyznaczono prawdopodobieństwa ich wystąpienia. Owe prawdopodobieństwa stały się przyczynkiem do kalkulacji entropii struktury i intensywności przejść od stanu zdatności do poszczególnych stanów niezdatności. Wyniki kalkulacji zobrazowano stosownymi wykresami. Odniesiono się do metod bayesowskich i nierozłącznej z nimi „szeregowej struktury niezawodnościowej” wykazując ich wady, spowodowane zwłaszcza uproszczeniami prowadzącymi do fałszywych i niejednoznacznych diagnoz. Prawdopodobieństwa zdatności poszczególnych elementów zróżnicowano doborem adekwatnym im wykładników Weibulla.

Słowa kluczowe: digraf, wykładnik Weibulla, macierz przejść, macierz osiągalności, intensywności przejść