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MATHEMATICAL MODEL AND ANALYSIS OF SINGLE-LEG MATRIX CONVERTER

Abstract

The paper deals with analysis and modelling of a new type of single-phase supplied AC/AC converter with two phase outputs. It consists of single-leg half-bridge matrix converter loaded by resistive-inductive load in series connection. There are two problems solved in the paper: a) creating of second output phase shifted by 90 degrees against original one, and b) providing of frequency control of both orthogonal phase systems able supplying a two-phase electrical machine. As simulation results are given: harmonic analysis of the voltage of both phases; maximal possible reachable current waveform under R-L load with parametric changes of the time constant of the load. The simulation is resulting to recommendation for fair and right design of the converter, and demands to single- or two phase input supply voltage, respectively, under passive R-L or motoric load.

1. INTRODUCTION

The matrix converter topology has become well known after substitution of thyristor-devices in cycloconverters by switched-off elements acting in high frequency range, in 70-80-years [1]-[3], [9]. Matrix converter replace double energy conversion by single only, because within converter is not any energy storage element. Classical electric conversion uses DC link converters with rather bulky smoothing capacitors, since direct matrix converters operate without of DC-link circuit. One of the main advantages of that is unity power factor on its input side. Another advantage is that this converter offers sinusoidal input and output harmonic quantity and bi-directional energy flow [5]-[7].

Thanks to absence of any energy storage element, the instantaneous power on input must be the same as the power on output side. Unfortunately reactive power input does not have to equal the reactive power output. So, the input could be two-phase AC and output DC, or both could be DC, or both could be AC. To save the number power switching elements is also possible to use one-leg connection of the converter [4], [10]-[11].

2. SINGLE LEG MATRIX CONVERTER

From single leg VSI - voltage source inverter [4] was de-rived schematics for single leg matrix converter (Fig. 1), [10].

The matrix converter requires special semiconductor switches. The matrix converter requires a bidirectional switch, capable of blocking voltage and conducting current in both directions |the energy flow. These bidirectional switches, consisting of a pair of devices with turn-off capability, can be reverse blocking RB_IGBTs or more usually IGBTs with anti-parallel diodes, connected in either a common collector or a common emitter back-to-back arrangement [7].

Similarly to single leg voltage source inverter [4], single leg matrix converter also works in to two operation modes: nominal frequency regime, and variable frequency regime.

A. Nominal (Fixed) Frequency Regime

In this case (Fig. 1) the auxiliary phase will be created by 90 degrees one against original supply voltage, Fig. 2.

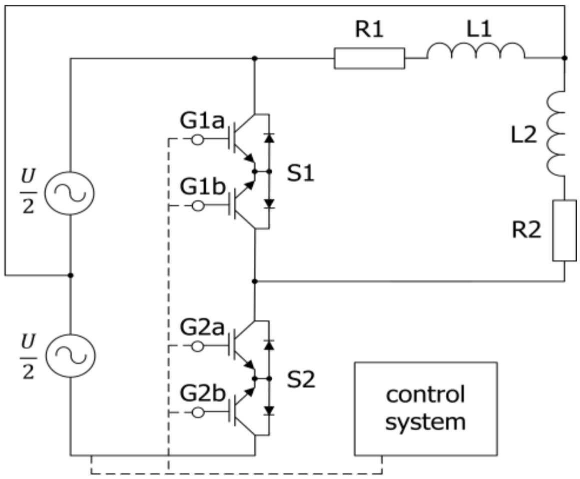
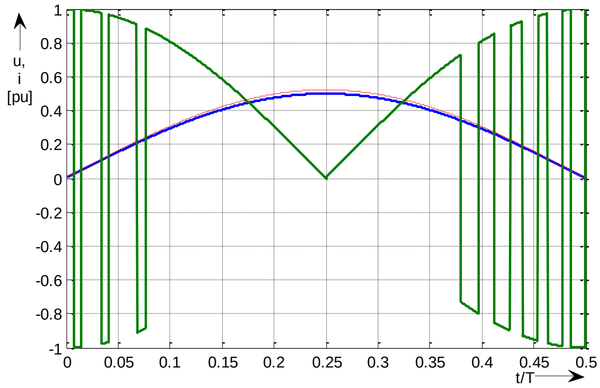


Fig. 1. *Single leg matrix converter – basic connection, nominal ferquency*



x: time [*n*/T]; y: *g*-input voltage; *r*-reference; *b*-fundamental harmonic

Fig.2. Creating auxiliary phase from the original one.

One of the tasks – the first question is how value the fundamental harmonic of auxiliary phase will reached. Using Fourier analysis of the one fourth of the waveform, Fig. 3, one can write

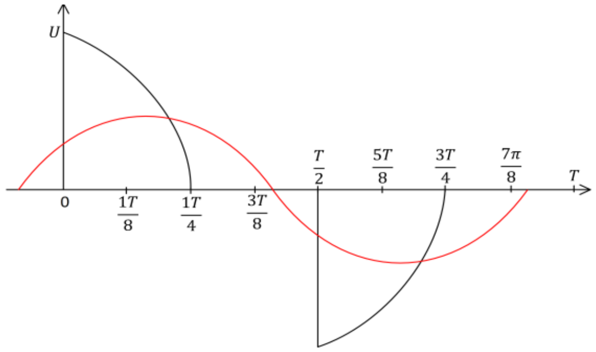


Fig.3. Harmonic analysis of the first part of the waveform.

Definition relations

$$f_1(t) = a_1\cos(\omega_1 t) + b_1\sin(\omega_1 t) = A_1\sin(\omega_1 t + \varphi_1)$$

$$A_1 = \sqrt[2]{{a_1}^2 + {b_1}^2};\quad \varphi_1 = \mathrm{arctang}\frac{b_1}{a_1}$$

$$a_1 = \frac{2}{T}\int_0^T f(t)\cos(\omega_1 t)\mathrm{d}t\,;\quad b_1 = \frac{2}{T}\int_0^T f(t)\sin(\omega_1 t)\mathrm{d}t$$

Then

$$a_1 = \frac{2}{T}\int_0^{\frac{T}{4}} \cos^2(\omega_1 t)\,\mathrm{d}t = \frac{2}{T}\int_0^{\frac{T}{4}} \frac{1}{2}[1+\cos(2\omega_1 t)]\,\mathrm{d}t =$$

$$= \frac{1}{T}\int_0^{\frac{T}{4}} [1+\cos(2\omega_1 t)]\,\mathrm{d}t = \frac{1}{T}\int_0^{\frac{T}{4}} 1\,\mathrm{d}t + \frac{1}{T}\int_0^{\frac{T}{4}} \cos(2\omega_1 t)\,\mathrm{d}t =$$

$$= \frac{1}{T}[t]_0^{T/4} + \frac{1}{T}\frac{1}{2\omega_1}[\sin(2\omega_1 t)]_0^{T/4} = \frac{1}{4}.$$

$$b_1 = \frac{4}{T}\int_0^{T/2} f(t)\sin(\omega_1 t) = \frac{2}{T}\int_0^{\frac{T}{4}} \cos(\omega_1 t)\sin(\omega_1 t)\mathrm{d}t$$

$$= \frac{2}{T}\int_0^{\frac{T}{4}} \frac{1}{2}[\sin(2\omega_1 t)]\,\mathrm{d}t = \frac{1}{T}\int_0^{\frac{T}{4}} [\sin(2\omega_1 t)]\,\mathrm{d}t =$$

$$= \frac{1}{T}\frac{1}{2\omega_1}[-\cos(2\omega_1 t)]_0^{T/4} = \frac{1}{4\pi}[\cos(2\omega_1 t)]_{T/4}^{0} = \frac{1}{2\pi}.$$

Fundamental harmonic waveform

a) $u_1(t) = \frac{1}{4}\cos(\omega_1 t) + \frac{1}{2\pi}\sin(\omega_1 t) =$

b) $= \sqrt[2]{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2\pi}\right)^2}\,\sin\left(\omega_1 t + \mathrm{arctag}\frac{\frac{1}{2\pi}}{\frac{1}{4}}\right) =$

c) $= 0.296\sin(\omega_1 t + 32.48°).$

The value if fundamental harmonic at the middle of half-period

$$A_1|_{\frac{\pi}{2}} = 0.296\cos(32.48°) = 0.249$$

The contribution from the second part of auxiliary phase waveform will be the same, Fig. 4. So, this means that maximal magnitude of auxiliary phase fundamental harmonic is

$$2A_1|_{\frac{\pi}{2}} = 2\,\mathrm{x}\,0.249 = 0.498 \approx 0.5\,.$$

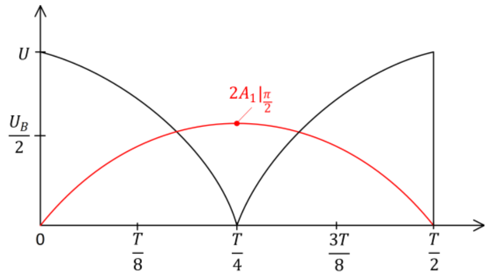


Fig.4. Fundamental harmonic of auxiliary phase waveform.

Thus, the RMS value of the output voltage of the one-leg converter should be two-times greater than requested voltage of the main phase of the system.

The maximal current reference magnitude is

$$I_{1\mathrm{REF}} = \frac{2A_1|_{\frac{\pi}{2}}}{|Z_1|}$$

and corresponding current reference waveform

$$i_{1ref}(t) = I_{1ref}\sin(\omega_1 t - \varphi_1)$$

where

$$|Z_1| = \sqrt[2]{{R_1}^2 + (\omega_1 L)^2};\quad \varphi_1 = \mathrm{arctag}\frac{\omega_1 L}{R}$$

Simulation experiments with various loading and current loop are shown in Chap. III.

B. Variable frequency regime

In this regime both phases are frequency controlled by one leg matrix converter switching in the range from zero up to nominal frequency. Basic scheme is depicted in Fig. 5, [11].

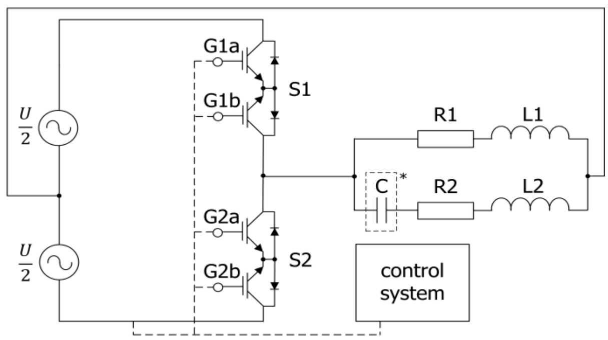
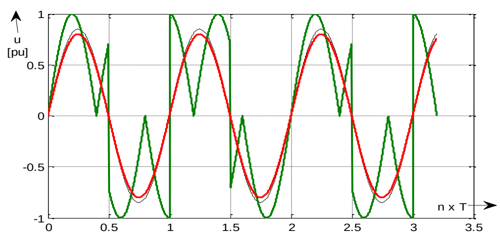
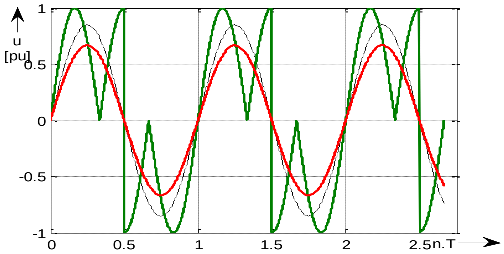


Fig.5 Single leg matrix converter – basic connection, variable frequency; C* - for motoric load.

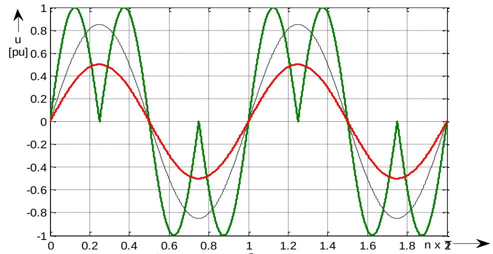
Principally, the possibilities of frequency control of the converter are shown in following Figs. 6a-6e.



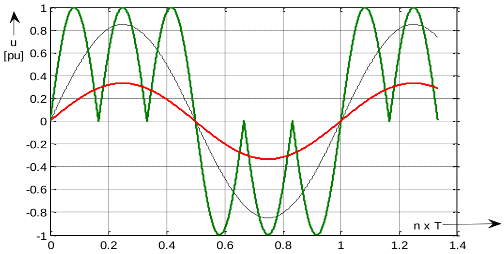
a) $f_b = 40$ Hz



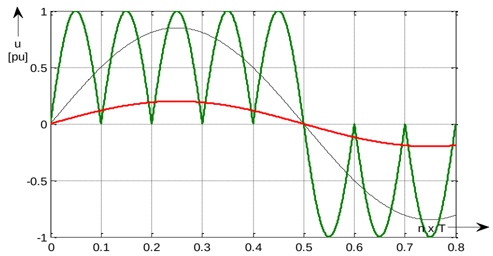
b) $f_b = 33$ Hz



c) f_b =25 Hz



d) f_b =16.66 Hz



e) f_b =10 Hz

Fig.6. Principle voltage waveforms g, and their referenced fundamental harmonics r at basis frequency f_b of 40 Hz (a), 33.33 Hz (b), 25 Hz (c), 16.66 (d), and 10 Hz (e). Note: input supply voltage should be switched as shown in Fig. 7 for 25 Hz!

C. Analysis of the output voltage of converter operated at 25 Hz.

At first, maximal reachable fundamental harmonic voltage will be calculated using Fourier analysis.

Example of calculation fundamental harmonic magnitude for half basis frequency (25 Hz) using Fourier analysis with following integration calculus [8]

$$A_1 = \frac{8}{T}\int_0^{T/4} \cos(2\omega t).\cos(\omega t)\,dt =$$

$$= \frac{8}{T}\int_0^{T/4} \frac{1}{2}[\cos(\omega t) - \cos(3\omega t)]\,dt =$$

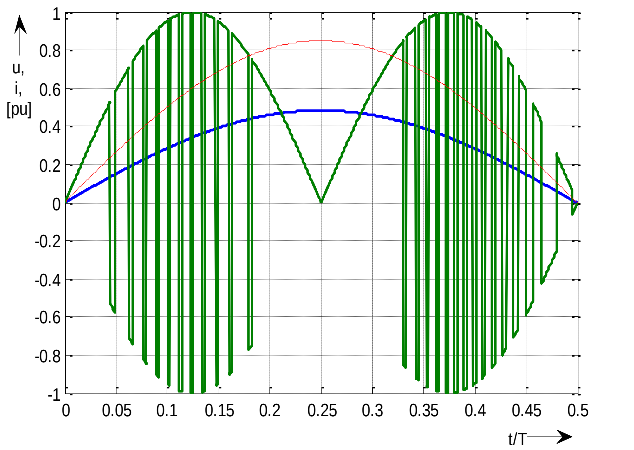
$$= \frac{4}{T}\left\{\frac{1}{\omega}[\sin(\omega t)]_0^{\frac{T}{4}} - \frac{1}{3\omega}\left[\sin(3\omega t)_0^{\frac{T}{4}}\right]\right\} =$$

$$= \frac{4}{T\omega}\left\{\left[\sin\left(\omega\frac{T}{4}\right) - \sin(0)\right] - \frac{1}{3}\left[\sin\left(3\omega\frac{T}{4}\right) - \sin(0)\right]\right\} =$$

$$= \frac{2}{\pi}\left[(1-0) - \frac{1}{3}(-1-0)\right] = \frac{2}{\pi}\left(1+\frac{1}{3}\right) = \frac{2}{\pi}\cdot\frac{4}{3} = \approx 0.85 \Rightarrow 85\ \%$$

This value is varied in full range of the basis frequency (10-50 Hz) from 0.82 to 0.9. So, the input voltage waveform should be switched by switching frequency f_{sw} to be the fundamental harmonic magnitude corresponded to basis frequency of the system.

The switched waveform of the output voltage to be reached corresponding value i.e. one half of the original is depicted in Fig. 7.



x: time [t/T]; y: g-input voltage; r-reference; b-fundamental harmonic

Fig.7 One-leg MxC fundamental harmonic reaching 0.85 p.u. of the input volatege magnitude at 25 Hz operational basis frequency.

3. SIMULATIONS

All simulations were calculated using the Matlab-Simulink package under source voltage 2 x 115 VRMS, 50 Hz.
Parameters of the system:
R-L load: R=1 Ω; L=2.8e-3 H; cos φ1=0.82 (was changed)
Operating frequency: 50 – 40 – 33.33 – 25 – 16.66 - 10 Hz
Simulation step: 1e-5 sec.

Direct connection of the R-L load to power supply auxiliary phase at basis frequency 50 Hz, Fig. 2 without PWM

Following differential equation for current is valid

$$u(t) = R.i(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \text{ or}$$

$$\frac{\mathrm{d}i(t)}{\mathrm{d}t} = -\frac{1}{\tau}i(t) + \frac{1}{\tau}\frac{u(t)}{R}, \text{ respectively.}$$

Using of numerical integration (e.g. Euler explicit method) yields

$$i_{n+1} = i_n + \Delta T\left[-\frac{1}{\tau}i_n + \frac{1}{\tau}\frac{u_n}{R}\right].$$

Taking supply voltage as in Fig. 2 the calculated current waveform is depicted in Fig. 8.

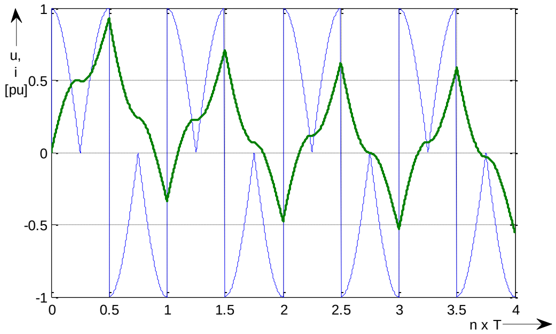


Fig. 8. Auxiliary voltage and current waveforms without PWM control.

It's clear that current time waveform is strongly non-harmonic. To be the current sinusoidal under any load it should be used some of PWM methods; we have used the current controlled PWM (hysteresis CC-PWM) with current feedback closed loop, as shown in Fig. 9.

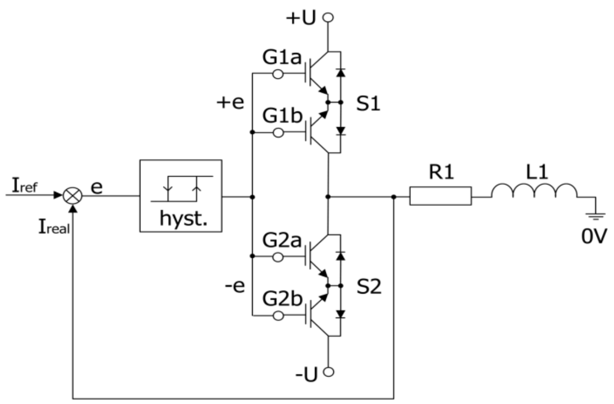
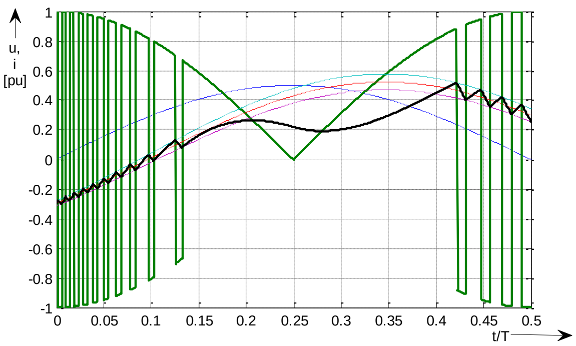


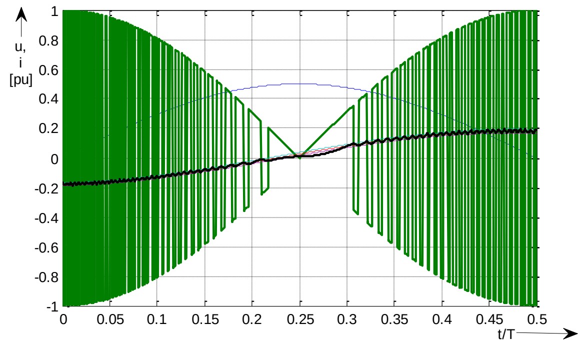
Fig.9. Principle of used CC_PWM feedback loop.

Then, using hysteresis controller the current waveforms of auxiliary phase are shown in Fig. 10 and 11 under 75 % and 15 % loading at steady state operation. The reference value of auxiliary phase current should be generated by power controller superior to current hysteresis one.



x: time [t/T]; y: g-input voltage; r-reference; b-fundamental harmonic

Fig.10. Auxiliary voltage and current waveforms with CC_PWM control under 75 % loading.



x: time [t/T]; y: g-input voltage; r-reference; b-fundamental harmonic

Fig. 11. Auxiliary voltage and current waveforms with CC_PWM control under 15 % loading.

Current time waveforms in figures are shown under 75- and 15 % loading at steady state operation.

Direct connection of the R-L load to power supply network at operational frequency 25 Hz, Fig. 6c

Taking supply voltage as in Fig. 6c the calculated current waveform is depicted in Fig. 12.

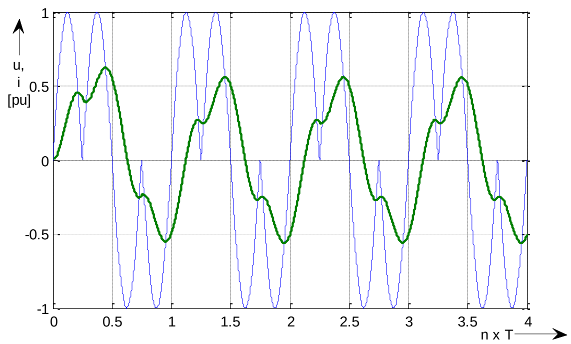
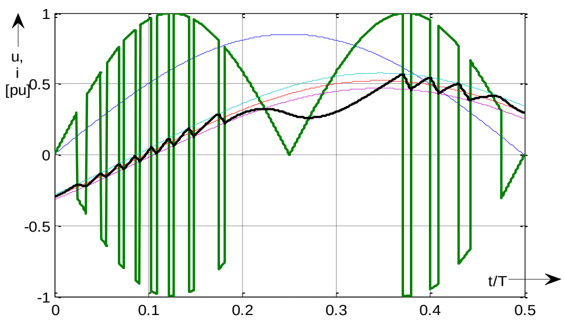


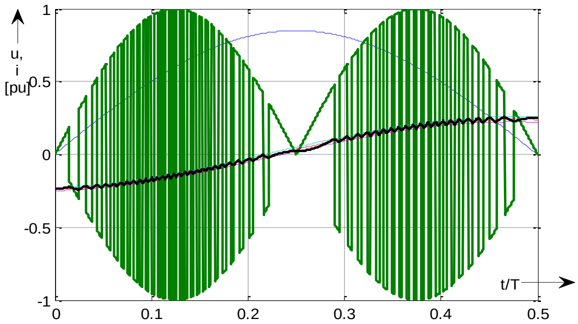
Fig.12. One-leg MxC fundamental harmonic output volatege and current waveforms without CC_PWM at 25 Hz operational basis frequency.

As in previous case the current is also non-harmonic one. Using hysteresis CC-PWM the currents will be again sinusoidal, Fig. 13 and 14.



x: time [t/T]; y: g-input voltage; r-reference; b-fundamental harmonic

Fig. 13. One-leg MxC fundamental harmonic output volatege and current waveforms with CC_PWM at 25 Hz under 75 % loading.



x: time [t/T]; y: g-input voltage; r-reference; b-fundamental harmonic

Fig.14. One-leg MxC fundamental harmonic output voltage and current waveforms with CC_PWM at 25 Hz under 15 % loading.

Current time waveforms in figures are shown under 75- and 15 % loading at steady state operation.

The Figs. 10 and 13 show that time constant should be higher that the waveforms were in the hysteresis bands (± 5 %) or wider hysteresis is can be used if possible.

CONCLUSION

The paper brings analysis, modelling and computer simulation of an one-leg matrix converter supplied by single-phase network with neutral point (as e.g. 2 x115 V). Analysis and worked-out simulation experiment results have shown that maximal reachable fundamental harmonic magnitude of auxiliary phase created from original single-phase is just 50 % of the network one. Thus, the supply voltage must be two-multiply of nominal values of the loading appliances. Therefore, the switching elements of the converter should be sized to double of input single-phase voltage. Also, the maximal reachable fundamental harmonic magnitude under frequency control is 82-90 % of the nominal at lower frequency, for example 25 Hz (as Fig. 7).

Comparison of single-phase motor drive fed by single leg VSI and single leg MxC is describe in the other contribution paper at the Conference. The authors, nowadays, would are testing and investigating such a single-leg MxC system with 2-phase synchronous PMSM motor in the laboratory using dSpace environment.

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