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SOLVING A CERTAIN TWO-ALTERNATIVE PROBLEM IN THE OPTIMAL ORGANIZATION OF AVIATION TRANSPORTATION IN CONDITIONS OF UNCERTAINTY

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Abstract

This paper proposes a solution to a certain two-alternative problem of aviation transportation optimal organization in conditions of uncertainty of the subjective preference functions. Conditional optimization of the objective functional containing the entropy of the individuals' operational effectiveness functions preferences is carried out in the framework of the simplest variational problem. The advantages of the described optimization approach are demonstrated in the generalized terms of the operational effectiveness functions for aviation transportation organization.

Keywords: aviation transportation, operational effectiveness, objective functional optimization, simplest variational problem, entropy
Type of the work: research article, air transport

INTRODUCTION

Entropy models are important for substantiating the evolution of developed situations with uncertainties. Optimality in the organization of aviation transportation needs to be found in conditions of uncertainty of the operational effectiveness functions. This involves rationality in economic behavior, as noted by [1], and requires considerations of subjective factors in accordance with [2]. The combination of the ideas of publications [1] and [2] has been discussed in a previous paper [3]. Uncertainty of preferences was estimated in [2] and [3] using the entropy paradigm of [4–6]. The potential field of application of such an approach is quite wide, ranging from environmental issues [7], the functioning of companies [8], travel guidance optimality [9], as well as multi-factoring and dynamic modeling of different kinds, as in [10–17], to the various problems considered in [18–22]. Some prospective applications of the approach considered herein are proposed to be applied to the issues initiated in publications [23–28]. The objectives of the presented paper are to continue and develop the topics initiated in [3, 12–17, 22], to show the existence of optimality.

PROBLEM STATEMENT

The approach proposed herein takes into consideration the uncertainty of individuals' preferences in regards to the specified two-optional effectiveness functions of the organization of aviation transportation.

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The objective functional is as in [2, 3]:

$$\Phi_\pi = \int_{t_1}^{t_2} \left(-\sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) + \beta \left[\pi_1(t) x(t) \dot{x}(t) + \alpha \pi_2(t) \frac{\dot{x}(t)}{x(t)} \right] \right) + \gamma \left[\sum_{i=1}^{N=2} \pi_i(t) - 1 \right] dt, \quad (1)$$

where $[t_1, t_2]$ is the period of integration; $\pi_i(t)$ are the subjective preferences functions that the individuals distribute according to the options (alternatives): $x(t)\dot{x}(t)$ and $\dot{x}(t)/[x(t)]$ correspondingly; $x(t)$ and $\dot{x}(t)$ are the basic aviation transportation organization optional effectiveness functions, where $\dot{x}(t) = dx/(dt)$; α and β are the corresponding coefficients used for evaluating the distribution of the above-mentioned preferences in accordance with the alternatives or options, α also being applied in order to set the measurement units and dimensions in correspondence and γ being the normalized conditions coefficient.

PROPOSED SOLUTION

Entropy Conditional Optimization

The first member of the objective functional (1) is the entropy of the available options' subjective preferences functions $\pi_i(t)$:

$$H_\pi = -\sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t). \quad (2)$$

Therefore, the optimization of the value (1), in the mathematical sense, could be considered as the conditional extremum of the degree of subjective preferences uncertainty, which is expressed with the entropy formula (2), subject to the constraints of the conditions of the subjective effectiveness function:

$$\beta \left[\pi_i(t) x(t) \dot{x}(t) + \alpha \pi_2(t) \frac{\dot{x}(t)}{x(t)} \right], \quad (3)$$

and the normalizing condition described by:

$$\gamma \left[\sum_{i=1}^{N=2} \pi_i(t) - 1 \right]. \quad (4)$$

Both restrictions (3) and (4) are taken into account and introduced with the corresponding Lagrange uncertainty multipliers (or weight coefficients) β and γ, assuming the subjectively estimated role of the described factors [2, 3].

Conditional optimization of the objective functional (1) deals with the necessary extremum conditions expressed in view of the Euler-Lagrange equation system:

$$\frac{\partial R^*}{\partial \pi_i} - \frac{d}{dt} \frac{\partial R^*}{\partial \dot{\pi}_i} = 0 \quad \text{and} \quad \frac{\partial R^*}{\partial x} - \frac{d}{dt} \frac{\partial R^*}{\partial \dot{x}} = 0, \quad (5)$$

where R^* is the under-integral function (integrand) of the objective functional (1).

Solution to the Simplest Variational Problem

In the considered case:

$$\frac{\partial R^*}{\partial \dot{\pi}_i} \equiv 0, \qquad \frac{d}{dt}\frac{\partial R^*}{\partial \dot{\pi}_i} \equiv 0, \tag{6}$$

the system of equations of (5) works out to:

$$\frac{\partial R^*}{\partial \pi_i} = 0, \qquad \frac{\partial R^*}{\partial x} - \frac{d}{dt}\frac{\partial R^*}{\partial \dot{x}} = 0. \tag{7}$$

In accordance with (7):

$$\frac{\partial R^*}{\partial \pi_1} = -\ln \pi_1 - 1 + \beta x \dot{x} + \gamma = 0, \qquad \frac{\partial R^*}{\partial \pi_2} = -\ln \pi_2 - 1 + \alpha\beta \frac{\dot{x}}{x} + \gamma = 0. \tag{8}$$

From which we derive:

$$\pi_1 = e^{-1+\beta x \dot{x}+\gamma} = e^{\gamma-1} e^{\beta x \dot{x}}, \qquad \pi_2 = e^{-1+\alpha\beta \frac{\dot{x}}{x}+\gamma} = e^{\gamma-1} e^{\alpha\beta \frac{\dot{x}}{x}}. \tag{9}$$

Normalizing the condition yields:

$$\pi_1 + \pi_2 = 1 = e^{\gamma-1} e^{\beta x \dot{x}} + e^{\gamma-1} e^{\alpha\beta \frac{\dot{x}}{x}} = e^{\gamma-1}\left(e^{\beta x \dot{x}} + e^{\alpha\beta \frac{\dot{x}}{x}} \right),$$

$$e^{\gamma-1} = \frac{1}{e^{\beta x \dot{x}} + e^{\alpha\beta \frac{\dot{x}}{x}}}. \tag{10}$$

The dependencies obtained from (1) – (10) are:

$$\pi_1 = \frac{e^{\beta x \dot{x}}}{e^{\beta x \dot{x}} + e^{\alpha\beta \frac{\dot{x}}{x}}}, \qquad \pi_2 = \frac{e^{\alpha\beta \frac{\dot{x}}{x}}}{e^{\beta x \dot{x}} + e^{\alpha\beta \frac{\dot{x}}{x}}}. \tag{11}$$

For the extremal of $x(t)$, at $\beta \neq 0$, we can find:

$$\frac{\partial R^*}{\partial x} = \beta \pi_1 \dot{x} - \frac{\alpha\beta\pi_2 \dot{x}}{x^2}, \qquad \frac{\partial R^*}{\partial \dot{x}} = \beta \pi_1 x - \frac{\alpha\beta\pi_2}{x},$$

$$\frac{d}{dt}\frac{\partial R^*}{\partial \dot{x}} = \beta\left(\dot{\pi}_1 x + \pi_1 \dot{x}\right) + \alpha\beta\left(\frac{\dot{\pi}_2 x - \pi_2 \dot{x}}{x^2}\right),$$

$$\beta\pi_1\dot{x}-\frac{\alpha\beta\pi_2 x}{x^2}-\beta\dot{\pi}_1 x-\beta\pi_1\dot{x}-\alpha\beta\left(\frac{\dot{\pi}_2 x}{x^2}\right)+\alpha\beta\left(\frac{\pi_2\dot{x}}{x^2}\right)=0,$$

$$-\dot{\pi}_1\cancel{x}-\alpha\left(\frac{\dot{\pi}_2\cancel{x}}{x^2}\right)=0,\qquad \dot{\pi}_1=-\alpha\left(\frac{\dot{\pi}_2}{x^2}\right). \tag{12}$$

For the presented problem, setting:

$$\dot{\pi}_i=\frac{d\pi_i}{dt}=\frac{\partial\pi_i}{\partial x}\dot{x}+\frac{\partial\pi_i}{\partial\dot{x}}\ddot{x}. \tag{13}$$

$$\frac{\partial\pi_1}{\partial x}=\frac{\beta\dot{x}e^{\beta x\dot{x}}\left(e^{\beta x\dot{x}}+e^{\alpha\beta\frac{\dot{x}}{x}}\right)-e^{\beta x\dot{x}}\left(\beta\dot{x}e^{\beta x\dot{x}}-\alpha\beta\frac{\dot{x}}{x^2}e^{\alpha\beta\frac{\dot{x}}{x}}\right)}{\left(e^{\beta x\dot{x}}+e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial\pi_1}{\partial x}=\frac{\beta\dot{x}e^{\beta x\dot{x}}\left(e^{\alpha\beta\frac{\dot{x}}{x}}\right)-e^{\beta x\dot{x}}\left(-\alpha\beta\frac{\dot{x}}{x^2}e^{\alpha\beta\frac{\dot{x}}{x}}\right)}{\left(e^{\beta x\dot{x}}+e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial\pi_1}{\partial x}=\beta\dot{x}\pi_1\pi_2\left(1+\frac{\alpha}{x^2}\right). \tag{14}$$

$$\frac{\partial\pi_1}{\partial\dot{x}}=\frac{\beta x e^{\beta x\dot{x}}\left(e^{\beta x\dot{x}}+e^{\alpha\beta\frac{\dot{x}}{x}}\right)-e^{\beta x\dot{x}}\left(\beta x e^{\beta x\dot{x}}+\frac{\alpha\beta}{x}e^{\alpha\beta\frac{\dot{x}}{x}}\right)}{\left(e^{\beta x\dot{x}}+e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial\pi_1}{\partial\dot{x}}=\frac{\beta x e^{\beta x\dot{x}}\left(e^{\alpha\beta\frac{\dot{x}}{x}}\right)-e^{\beta x\dot{x}}\left(-\frac{\alpha\beta}{x}e^{\alpha\beta\frac{\dot{x}}{x}}\right)}{\left(e^{\beta x\dot{x}}+e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial\pi_1}{\partial\dot{x}}=\beta\pi_1\pi_2\left(x-\frac{\alpha}{x}\right). \tag{15}$$

$$\dot{\pi}_1 = \frac{d\pi_1}{dt} = \frac{d\pi_1}{\partial x}\dot{x} + \frac{\partial \pi_1}{\partial \dot{x}}\ddot{x},$$

$$\dot{\pi}_1 = \beta \dot{x}\pi_1\pi_2\left(1+\frac{\alpha}{x^2}\right)\dot{x} + \beta\pi_1\pi_2\left(x-\frac{\alpha}{x}\right)\ddot{x}. \tag{16}$$

$$\frac{\partial \pi_2}{\partial x} = \frac{-\alpha\beta\frac{\dot{x}}{x^2}e^{\alpha\beta\frac{\dot{x}}{x}}\left(e^{\beta x\dot{x}} + e^{\alpha\beta\frac{\dot{x}}{x}}\right) - e^{\alpha\beta\frac{\dot{x}}{x}}\left(\beta\dot{x}e^{\beta x\dot{x}} - \alpha\beta\frac{\dot{x}}{x^2}e^{\alpha\beta\frac{\dot{x}}{x}}\right)}{\left(e^{\beta x\dot{x}} + e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial \pi_2}{\partial x} = \frac{-\alpha\beta\frac{\dot{x}}{x^2}e^{\alpha\beta\frac{\dot{x}}{x}}\left(e^{\beta x\dot{x}}\right) - e^{\alpha\beta\frac{\dot{x}}{x}}\left(\beta\dot{x}e^{\beta x\dot{x}}\right)}{\left(e^{\beta x\dot{x}} + e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial \pi_2}{\partial x} = \beta\dot{x}\pi_1\pi_2\left(-\frac{\alpha}{x^2}-1\right). \tag{17}$$

$$\frac{\partial \pi_2}{\partial \dot{x}} = \frac{\frac{\alpha\beta}{x}e^{\alpha\beta\frac{\dot{x}}{x}}\left(e^{\beta x\dot{x}} + e^{\alpha\beta\frac{\dot{x}}{x}}\right) - e^{\alpha\beta\frac{\dot{x}}{x}}\left(\beta x e^{\beta x\dot{x}} + \frac{\alpha\beta}{x}e^{\alpha\beta\frac{\dot{x}}{x}}\right)}{\left(e^{\beta x\dot{x}} + e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial \pi_2}{\partial \dot{x}} = \frac{\frac{\alpha\beta}{x}e^{\alpha\beta\frac{\dot{x}}{x}}\left(e^{\beta x\dot{x}}\right) - e^{\alpha\beta\frac{\dot{x}}{x}}\left(\beta x e^{\beta x\dot{x}}\right)}{\left(e^{\beta x\dot{x}} + e^{\alpha\beta\frac{\dot{x}}{x}}\right)^2},$$

$$\frac{\partial \pi_2}{\partial \dot{x}} = \beta\pi_1\pi_2\left(\frac{\alpha}{x}-x\right). \tag{18}$$

Substituting (17) and (18) for (13), we can derive:

$$\dot{\pi}_2 = \beta\dot{x}\pi_1\pi_2\left(-\frac{\alpha}{x^2}-1\right)\dot{x} + \beta\pi_1\pi_2\left(\frac{\alpha}{x}-x\right)\ddot{x}. \tag{19}$$

Substituting (16) and (19) for (12), we can derive:

$$\beta \dot{x}\pi_1\pi_2\left(1+\frac{\alpha}{x^2}\right)\dot{x}+\beta\pi_1\pi_2\left(x-\frac{\alpha}{x}\right)\ddot{x}=-\frac{\alpha}{x^2}\left[\beta\dot{x}\pi_1\pi_2\left(-\frac{\alpha}{x^2}-1\right)\dot{x}+\beta\pi_1\pi_2\left(\frac{\alpha}{x}-x\right)\ddot{x}\right]. \quad (20)$$

Canceling for $\pi_1\pi_2 \neq 0$, this yields:

$$\beta \dot{x}\left(1+\frac{\alpha}{x^2}\right)\dot{x}+\beta\left(x-\frac{\alpha}{x}\right)\ddot{x}=-\frac{\alpha}{x^2}\left[\beta\dot{x}\left(-\frac{\alpha}{x^2}-1\right)\dot{x}+\beta\left(\frac{\alpha}{x}-x\right)\ddot{x}\right]. \quad (21)$$

From which we derive:

$$\beta \dot{x}\left(1+\frac{\alpha}{x^2}\right)\dot{x}+\beta\left(x-\frac{\alpha}{x}\right)\ddot{x}\neq 0,$$

$$\frac{\beta \dot{x}\left(1+\frac{\alpha}{x^2}\right)\dot{x}+\beta\left(x-\frac{\alpha}{x}\right)\ddot{x}}{\beta \dot{x}\left(1+\frac{\alpha}{x^2}\right)\dot{x}+\beta\left(x-\frac{\alpha}{x}\right)\ddot{x}}=\frac{\alpha}{x^2}=1. \quad (22)$$

From the expression of (22) it follows:

$$x=\sqrt{\alpha} \quad (23)$$

ANALYSIS

Optional Method

The same result as (23) follows directly from (12) at $\dot{\pi}_2 \neq 0.$ Indeed,

$$\pi_1 = 1-\pi_2, \qquad \dot{\pi}_1=\frac{d(1-\pi_2)}{dt}=\frac{d\pi_2}{dt}=-\dot{\pi}_2, \qquad -\dot{\pi}_2=-\frac{\alpha}{x^2}\dot{\pi}_2, \qquad x=\sqrt{\alpha}. \quad (24)$$

In such case as considered here, (1) – (24):

$$\pi_1=\pi_2=\frac{1}{2}. \quad (25)$$

The result of (25) ensures a maximum value to the entropy (2).

Other Solution

Another solution than the one that has been obtained here has to be found on the condition of:

$$\beta \dot{x}\left(1+\frac{\alpha}{x^2}\right)\dot{x}+\beta\left(x-\frac{\alpha}{x}\right)\ddot{x}=0. \quad (26)$$

Possible Modifications

As a whole, the subjective effectiveness function (3) can be modified in many ways, for example, it might include members with combinations of the basic aviation transportation organization optional effectiveness functions of $x(t)$ and $\dot{x}(t)$. In order to check the extremality of the obtained solutions, the method of variations can be proposed.

Distinguished Certainty

In case of modeling the certainty or uncertainty with respect to the "right" or "wrong" ("good" or "bad") attainable options or alternative distributions in aviation transportation organization, a hybrid model of the combined pseudo-entropy function of the subjective preferences [22] can be proposed:

$$\bar{H}_{\max - \frac{\Delta\pi}{|\Delta\pi|}} = \frac{H_{\max} - H_{\pi}}{H_{\max}} \frac{\Delta\pi}{|\Delta\pi|}, \tag{27}$$

where $H_{\max}$ is the maximum value of the entropy:

$$H_{\max} = \ln N, \tag{28}$$

$\Delta\pi$ is the factor or index of the options or alternatives σ_i preferences functions $\pi(\sigma_i)$ dominance or prevailing:

$$H_{\pi} = -\sum_{i=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i), \tag{29}$$

$$\Delta\pi = \sum_{j=1}^{M} \pi\left(\sigma_j^+\right) - \sum_{k=1}^{L} \pi\left(\sigma_k^-\right), \tag{30}$$

where σ_j^+ are positive and σ_k^- are negative alternatives or options correspondingly; M is the number of the positive alternatives and L is the number of the negative alternatives, respectively:

$$M + L = N. \tag{31}$$

CONCLUSION

We conclude that the objective functional containing the uncertainty measure of the preferences functions of the aviation transportation organization optional effectiveness functions has a solution in the framework of the simplest variational problem setting, which delivers a maximum value to the preference function entropy.

For modeling the certainty or uncertainty with respect to the "right" or "wrong" ("good" or "bad") attainable options or alternative distributions in aviation transportation organization, a hybrid model of the combined pseudo-entropy function of the subjective preferences has been proposed.

In future research, it is proposed to search for an optimal solution with respect to the second-order differential equation.

REFERENCES

[1] E. Silberberg and W. Suen, *The Structure of Economics*. A Mathematical Analysis. New York: McGraw-Hill Higher Education, 2001.

[2] V. Kasianov, *Subjective Entropy of Preferences. Subjective Analysis: Monograph*. Warsaw, Poland: Institute of Aviation Scientific Publications, 2013.

[3] A. Goncharenko, "Optimal price choice through buyers' preferences entropy," in *Proceedings of the International Conference on Advanced Computer Information Technologies* (ACIT-2020), Deggendorf, Germany, pp. 537–540, September 2020.

[4] Jaynes , E. T., "Information theory and statistical mechanics," *Physical Review*, 106(4), pp. 620-630, 1957. https://bayes.wustl.edu/etj/articles/theory.1.pdf

[5] Jaynes E. T., "Information theory and statistical mechanics II," *Physical Review*, 108(2), pp. 171-190, 1957. https://bayes.wustl.edu/etj/articles/theory.2.pdf

[6] Jaynes E. T., "On the rationale of maximum-entropy methods," *Proceedings of the IEEE*, 70, pp. 939-952, 1982.

[7] Reznik, O., Mazievich, T., Shebanits, D., Puzanova, G., and Pyrih, I., "Peculiarities of ecological taxation in Ukraine and the world," *Journal of Legal, Ethical and Regulatory Issues*, 23(1), pp. 1-6, 2020.

[8] Říhová, Z., "Integration of information systems in mergers and acquisition of companies," in *Proceedings of the International Conference on Advanced Computer Information Technologies* (ACIT-2018), Ceske Budejovice, Czech Republic, pp. 262–265, June 2018.

[9] Vichrova, M. K., Hájek, P., Kepka, M., Fiegler, L., Dorner, W., and Juha, M., "Peregrinus Silva Bohemica: A digital travel guide for navigation assistance," in *Proceedings of the International Conference on Advanced Computer Information Technologies* (ACIT-2019), Ceske Budejovice, Czech Republic, pp. 492–495, June 2019.

[10] Dostálek, L. and Dostálková, I., "Omnifactor authentication," in *Proceedings of the International Conference on Advanced Computer Information Technologies* (ACIT-2018), Ceske Budejovice, Czech Republic, pp. 228–231, June 2018.

[11] Dyvak, M., Brych, V., Spivak, I., Honchar, L., and Melnyk, N., "Discrete dynamic model of retail trade market of computer equipment in Ukraine," in *Proceedings of the International Conference on Advanced Computer Information Technologies* (ACIT-2018), Ceske Budejovice, Czech Republic, pp. 50–53, June 2018.

[12] Goncharenko, A. V., "Multi-optional hybrid effectiveness functions optimality doctrine for maintenance purposes," in *Proceedings of the International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering* (TCSET-2018), Lviv-Slavske, Ukraine, pp. 20–24, February 2018.

[13] Goncharenko, A. V., "Expediency of unmanned air vehicles application in the framework of subjective analysis," in *Proceedings of the IEEE 2nd International Conference on Actual Problems of Unmanned Aerial Vehicles Developments* (APUAVD), IEEE, Kyiv, Ukraine, pp. 129–133, October 2013.

[14] Goncharenko, A. V., "Navigational alternatives, their control and subjective entropy of individual preferences," in *Proceedings of the IEEE 3rd International Conference on Methods and Systems of Navigation and Motion Control* (MSNMC), IEEE, Kyiv, Ukraine, pp. 99–103, October 2014.

[15] Goncharenko, A. V., "Applicable aspects of alternative UAV operation," in *Proceedings of the IEEE 3rd International Conference on Actual Problems of Unmanned Aerial Vehicles Developments* (APUAVD), IEEE, Kyiv, Ukraine, pp. 316–319, October 2015.

[16] Goncharenko, A. V., "Several models of artificial intelligence elements for aircraft control," in *Proceedings of the IEEE 4th International Conference on Methods and Systems of Navigation and Motion Control* (MSNMC), IEEE, Kyiv, Ukraine, pp. 224–227, October 2016.

[17] Goncharenko, A. V., "Aeronautical and aerospace material and structural damages to failures: theoretical concepts," *International Journal of Aerospace Engineering*, Article ID 4126085, 7 pages, 2018.

[18] Solomentsev, O., Zaliskyi, M., and Zuiev, O., "Estimation of quality parameters in the radio flight support operational system," *Aviation*, vol. 20, no. 3, pp. 123–128, 2016.

[19] Patel, G. C. M., Chate, G. R., Parappagoudar, M. B., and Gupta K., "Intelligent modelling of hard materials machining," *Springer Briefs in Applied Sciences and Technology*, pp. 73-102, 2020.

[20] Béjar, S. M., Vilches, F. J. T., Gamboa, C. B., and Hurtado, L. S., "Fatigue behavior parametric analysis of dry machined UNS A97075 aluminum alloy," Metals, vol. 10, no. 5, 631, 2020.

[21] Hulek, D. and Novák, M., "Expediency analysis of unmanned aircraft systems," in Proceedings of the 23rd International Conference on Transport Means, Palanga, Lithuania, pp. 959–962, October 2019.

[22] Goncharenko, A. V., “Multi-optional hybridization for UAV maintenance purposes,” in Proceedings of the IEEE 5th International Conference on Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD), IEEE, Kyiv, Ukraine, pp. 48–51, October 2019.

[23] Kasjanov, V. and Szafran, K., 2015, “Some hybrid models of subjective analysis in the theory of active systems,” *Transactions of the Institute of Aviation*, 3(240), pp. 27-31. doi: 10.5604/05096669.1194963.

[24] Pagowski Z. T. and Szafran K., 2014, “*Ground effect* inter-modal fast sea transport,” *International Journal on Marine Navigation and Safety of Sea Transportation*, 8(2), pp. 317-320. doi: 10.12716/1001.08.02.18.

[25] Szafran K., 2014, “Bezpieczenìstwo lotu – zasada maksymalnej entropii” [Flight safety – the principle of maximum entropy] (in Polish), *Bezpieczeństwo na lądzie, morzu I w powietrzu w XXI wieku*, pp. 247-251, ISBN 978-83-61520-02-3.

[26] Szafran, K. and Kramarski, I., 2015, “Safety of navigation on the approaches to the ports of the Republic of Poland on the basis of the radar system on the aerostat platform,” *International Journal on Marine Navigation and Safety of Sea Transportation*, 9(1), pp. 129-134. doi: 10.12716/1001.09.01.16.

[27] Szafran K., 2014, “Bezpieczeństwo operatora pojazdu trakcyjnego – stanowisko prób dynamicznych” [Traction vehicle operator safety – dynamic test station], *Logistyka*, 6, pp. 192-197.

[28] Krzysztofik, I. and Koruba, Z., 2014, “Mathematical model of movement of the observation and tracking head of an unmanned aerial vehicle performing ground target search and tracking,” *Journal of Applied Mathematics*, Special Issue (2014). doi: 10.1155/2014/934250.