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# Critical infrastructure operating area climate-weather change process including extreme weather hazards

**Keywords**

critical infrastructure, operation, prediction, climate-weather change

**Abstract**

The climate-weather change process for a critical infrastructure operating area is considered and its states are introduced. A semi-Markov approach is used to construct a general probabilistic model of this process by defining its basic parameters. Further, the procedure of the climate-weather change process characteristics prediction is proposed.

## 1. Introduction

The climate-weather change process for a critical infrastructure operating area is considered and its states are introduced. A semi-Markov approach is used to construct a general probabilistic model of this process by defining its following parameters: the vector of probabilities of the process staying at the initial climate-weather states, the matrix of probabilities of the process transitions between the climate-weather states, the matrix of distribution functions of the process conditional sojourn times at the climate-weather states. Further, the procedure of the climate-weather change process characteristics prediction is proposed.

It is obvious that the climate-weather change process at a critical infrastructure operating area has an essential and often destructive influence on this critical infrastructure operation and safety (wider: on critical infrastructures of an area). Usually, the climate-weather change process has either an explicit or an implicit strong influence on the critical infrastructures operation and safety. As a rule, some of the extreme weather events, called extreme weather hazards (EWH), define a set of different climate-weather states of the climate-weather change process that cause the changes of the critical infrastructure structure, its operation process and its components safety parameters which have an essential destructive influence on the critical infrastructure safety and its operating environment.

The first step in this influence analysis is to fix the climate-weather parameters that are crucial in this influence on critical infrastructure safety.

The second step is to recognize the existing observations and the possibility of statistical data collection and to collect sufficiently large sets of the fixed data coming from the climate-weather change process realizations during the sufficiently large period of observation time.

The third step is to fix the way of using existing observations and collected statistical data coming from the climate-weather change process realizations in this process identification and evaluating unknown parameters of its probabilistic model.

The fourth step is to predict the climate-weather change process characteristics based on this process identified probabilistic model.

## 2. Modelling climate-weather change process

### 2.1. States of climate-weather change process

To define the climate-weather states in the fixed area, we distinguish $a$, $a \in N$, parameters that describe the climate-weather states in this area and mark the values they can take by $w_1, w_2,..., w_a$. Further, we assume that the possible values of the $i$-th parameter $w_i$, $i = 1,2,\ldots, a$, can belong to the interval $< b_i, d_i)$, $i = 1,2,\ldots, a$. We divide each of the intervals $< b_i, d_i)$, $i = 1,2,\ldots, a$, into $n_i$, $n_i \in N$, disjoint subintervals

$$< b_{i1}, d_{i1}), < b_{i2}, d_{i2}),\ldots, < b_{in_i}, d_{in_i}),$$

such that

$$< b_{i1}, d_{i1}) \cup < b_{i2}, d_{i2}) \cup \ldots \cup < b_{in_i}, d_{in_i})$$
$$= < b_i, d_i), d_{ij_i} = b_{ij_i+1}, j_i$$
$$= 1,2,\ldots,n_i - 1, i = 1,2,\ldots,a.$$

Thus, the vector $(w_1, w_2,..., w_a)$ describing the climate-weather states can take values from the set of the $a$ dimensional space points of the Descartes product

$$< b_1, d_1) \times < b_2, d_2) \times \ldots \times < b_a, d_a)$$

that is composed of the $a$ dimensional space domains of the form

$$< b_{1j_1}, d_{1j_1}) \times < b_{2j_2}, d_{2j_2}) \times \ldots \times < b_{aj_a}, d_{aj_a}),$$
where $j_i = 1,2,\ldots n_i$, $i = 1,2,\ldots, a$.

The domains of the above form are called the climate-weather states of the climate-weather change process and numerated from 1 up to the value $w = n_1 \cdot n_2 \cdot \ldots \cdot n_a$ and mark by $c_1, c_2,\ldots, c_w$.
The interpretation of the states of the climate-weather change process in the case $a = 2$ is given in Figure 4.1. In this case, we have $w = n_1 \cdot n_2$ climate-weather states of the climate-weather change process represented in *Figure 4.1* by the squares marked by $c_1, c_2,\ldots, c_w$.

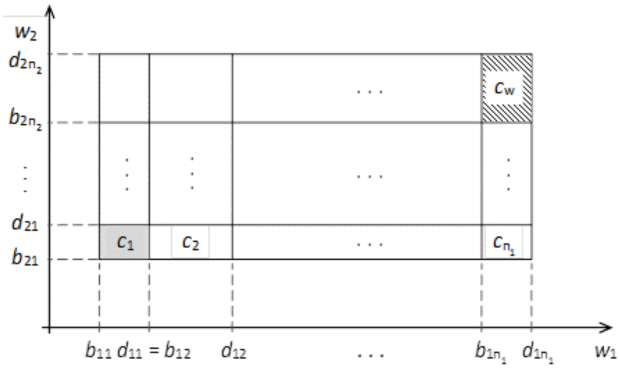



*Figure 1*. Interpretation of the climate-weather change process two dimensional climate-weather states

### 2.2. Extreme weather hazard states of climate-weather states

In subsection 4.2.1, to define the climate-weather states in the fixed area, we distinguished $a$, $a \in N$, parameters that describe them. The values the parameters can take were marked by $w_1, w_2,..., w_a$. Further, it was assumed that the possible values of the $i$-th parameter $w_i$, $i = 1,2,\ldots, a$, can belong to the interval
$< b_i, d_i)$, $i = 1,2,\ldots, a$. Each of the intervals $< b_i, d_i)$, $i = 1,2,\ldots, a$, were divided into $n_i$, $n_i \in N$, disjoint subintervals

$$< b_{i1}, d_{i1}), < b_{i2}, d_{i2}),\ldots, < b_{in_j}, d_{in_j}), i = 1,2,\ldots, a.$$

These intervals can be called the weather parameter $w_i$, $i = 1,2,\ldots, a$, states. Some of those states of the weather parameters can change the critical infrastructure operation process and they also can have dangerous influence on the critical infrastructure safety.
Thus, each of the states of the weather parameter $w_i$, $i = 1,2,\ldots, a$, that have most negative influence on the critical infrastructure operation and safety can be called the 1st category extreme weather hazard state of the weather parameter $w_i$, $i = 1,2,\ldots, a$.
Further, according to subsection 2.1, the climate-weather change process states are defined by the vectors

$$(w_1, w_2,..., w_a)$$

and marked by

$$c_1, c_2,\ldots, c_w, \; w = n_1 \cdot n_2 \cdot \ldots \cdot n_a,$$

then we can call each of the climate-weather change process state $c_j$, $j = 1,2,\dots, w$, of the vector form $(w_1, w_2,..., w_a)$:

- the $a^{th}$ category extreme weather hazard state of the climate-weather change process if all $a$ weather parameters $w_i$, $i = 1,2,\dots, a$, are at the $1^{st}$ category extreme weather hazard state;
- the $(a\text{-}1)^{th}$ category extreme weather hazard state of the climate-weather change process if $a$-1 of weather parameters $w_i$, $i = 1,2,\dots, a$, are at the $1^{st}$ category extreme weather hazard state;
- the $(a\text{-}2)^{th}$ category extreme weather hazard state of the climate-weather change process if $a$-2 of weather parameters $w_i$, $i = 1,2,\dots, a$, are at the $1^{st}$ category extreme weather hazard state;
- …
- the $1^{st}$ category extreme weather hazard state of the climate-weather change process if 1 of weather parameters $w_i$, $i = 1,2,\dots, a$, are at the $1^{st}$ category extreme weather hazard state;
- the $0^{os}$ category extreme weather hazard state of the climate-weather change process if none of weather parameters $w_i$, $i = 1,2,\dots, a$, are at the $1^{st}$ category extreme weather hazard state.

Thus, the $a^{th}$ category extreme weather hazard state of the climate-weather change process is the most dangerous for the critical infrastructure operation and safety.

## 2.3. Semi-Markov model of climate-weather change process

To model the climate-weather change process for the critical infrastructure operating area we assume that the climate-weather in this area is taking $w$, $w \in N$, different climate-weather states $c_1, c_2,\dots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{c_1, c_2,\dots, c_w\}$. Assuming that the climate-weather change process $C(t)$ is a semi-Markov process it can be described by:

– the number of climate-weather states $w$, $w \in N$;

– the vector

$$[q_b(0)]_{1Xw} = [q_1(0), q_2(0),\dots, q_w(0)] \tag{1}$$

of the initial probabilities

$$q_b(0) = P(C(0) = c_b),\ b = 1,2,\dots, w,$$

of the climate-weather change process $C(t)$ staying at particular climate-weather states $c_b$, $b = 1,2,\dots, w$, at the moment $t = 0$;

– the matrix

$$[q_{bl}]_{w\text{x}w} = \begin{bmatrix} q_{11}\ q_{12}\ \cdots q_{1w} \\ q_{21}\ q_{22}\ \cdots q_{2w} \\ \cdots \\ q_{w1}\ q_{w2}\ \cdots q_{ww} \end{bmatrix}$$

(2)

of the probabilities of transitions $q_{bl}$, $b, l = 1,2,\dots, w$, $b \neq l$, of the climate-weather change process $C(t)$ from the climate-weather states $c_b$, $b = 1,2,\dots, w$, to $c_l$, $l = 1,2,\dots, w$, $b \neq l$, where by formal agreement

$$q_{bb} = 0 \text{ for } b = 1,2,\dots, w;$$

– the matrix

$$[C_{bl}(t)]_{w\text{x}w} = \begin{bmatrix} C_{11}(t)\ C_{12}(t) \dots C_{1w}(t) \\ C_{21}(t)\ C_{22}(t) \dots C_{2w}(t) \\ \cdots \\ C_{w1}(t)\ C_{w2}(t) \dots C_{ww}(t) \end{bmatrix}$$

of the conditional distribution functions

$$C_{bl}(t) = P(C_{bl} < t),\ b, l = 1,2,\dots, w,$$

of the conditional sojourn times $C_{bl}$ at the climate-weather states $c_b$, $b = 1,2,\dots, w$, when its next climate-weather state is $c_l$, $l = 1,2,\dots, w$, $b \neq l$, where by formal agreement

$$C_{bb}(t) = 0 \text{ for } b = 1,2,\dots, w.$$

Further, we introduce the matrix

$$[c_{bl}(t)]_{w\text{x}w} = \begin{bmatrix} c_{11}(t)\ c_{12}(t) \dots c_{1w}(t) \\ c_{21}(t)\ c_{22}(t) \dots c_{2w}(t) \\ \cdots \\ c_{w1}(t)\ c_{w2}(t) \dots c_{ww}(t) \end{bmatrix} \tag{4}$$

of the conditional density functions of the climate-weather change process $C(t)$ conditional sojourn times $C_{bl}$ at the climate-weather states corresponding to the conditional distribution functions $C_{bl}(t)$, where

$$c_{bl}(t) = \frac{d}{dt}[c_{bl}(t)] \text{ for } b, l = 1,2,\dots, w,\ b \neq l,$$

and by formal agreement

$c_{bb}(t) = 0$ for $b = 1,2,\ldots, w$.

### 2.4. Conditional sojourn times at climate-weather states

We assume that the typical distributions suitable to describe the climate-weather change process $C(t)$ conditional sojourn times $C_{bl}$, $b$, $l = 1,2,\ldots, w$, $b \neq l$, at the particular climate-weather states are [Kolowrocki, Soszynska-Budny, 2011]:
– the uniform distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{1}{y_{bl} - x_{bl}}, & x_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{5}$$

where

$0 \le x_{bl} < y_{bl} < +\infty;$

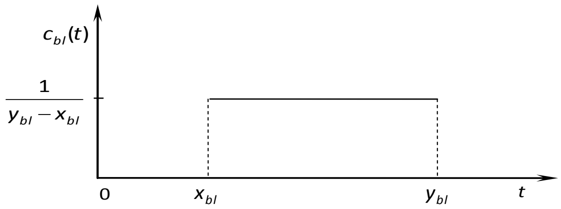



*Figure 2* The graph of the uniform distribution's density function

– the triangular distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{6}$$

where

$0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty, y_{bl} \neq 0;$

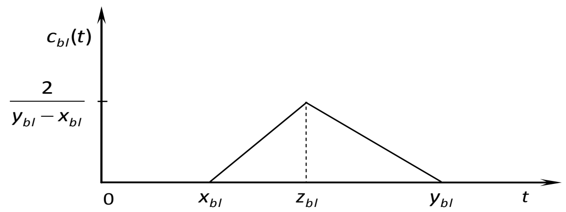



*Figure 3* The graph of the triangular distribution's density function

– the double trapezium distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ - q_{bl}] \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ w_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ - w_{bl}] \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{7}$$

where

$0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty, y_{bl} \neq 0;\ 0 \le q_{bl} < +\infty,$
$0 \le w_{bl} < +\infty,\ 0 \le q_{bl}(z_{bl} - x_{bl}) + w_{bl}(y_{bl} - z_{bl}) \le 2;$

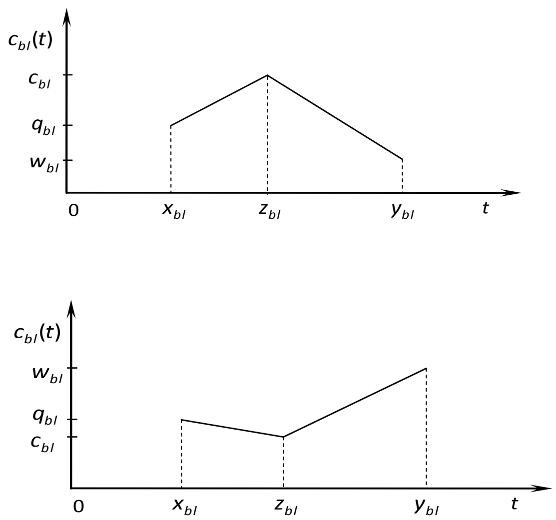



*Figure 4.* The graphs of the double trapezium distribution's density functions

– the quasi-trapezium distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + \dfrac{A_{bl} - q_{bl}}{z_{bl}^1 - x_{bl}}(t - x_{bl}), & x_{bl} \le t \le z_{bl}^1 \\ A_{bl}, & z_{bl}^1 \le t \le z_{bl}^2 \\ w_{bl} + \dfrac{A_{bl} - w_{bl}}{y_{bl} - z_{bl}^2}(y_{bl} - t), & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \tag{8}$$

where

$$A_{bl} = \frac{2 - q_{bl}(z_{bl}^1 - x_{bl}) - w_{bl}(y_{bl} - z_{bl}^2)}{z_{bl}^2 - z_{bl}^1 + y_{bl} - x_{bl}},$$

$0 \le x_{bl} \le z_{bl}^1 \le z_{bl}^2 \le y_{bl} < +\infty, y_{bl} \ne 0,\ 0 \le q_{bl} < +\infty,$
$0 \le w_{bl} < +\infty;$

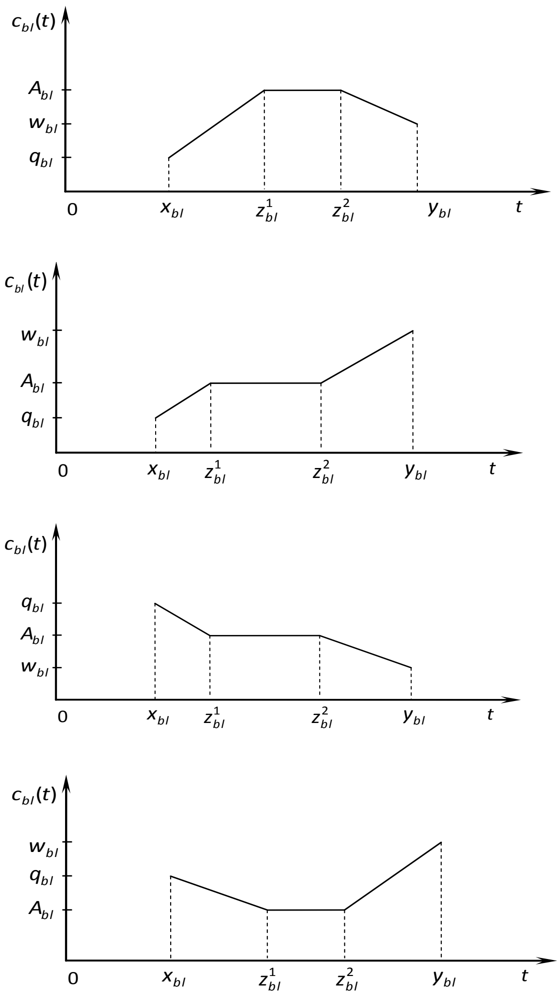



*Figure 5.* The graphs of the quasi-trapezium distribution's density functions

– the exponential distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \alpha_{bl} \exp[-\alpha_{bl}(t - x_{bl})], & t \ge x_{bl}, \end{cases} \tag{9}$$

where

$0 < \alpha_{bl} < +\infty;$

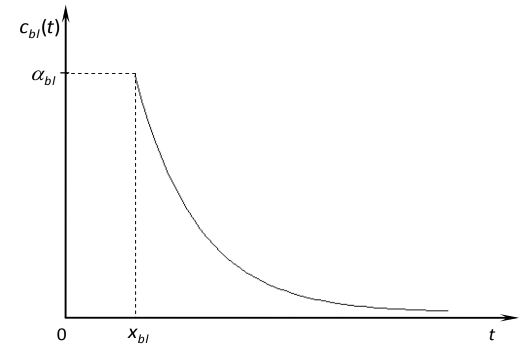



*Figure 6.* The graph of the exponential distribution's density function

– the Weibull distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \alpha_{bl}\beta_{bl}(t - x_{bl})^{\beta bl - 1} \\ \exp[-\alpha_{bl}(t - x_{bl})^{\beta bl}], & t \ge x_{bl}, \end{cases} \tag{10}$$

where

$0 < \alpha_{bl} < +\infty,\ 0 < \beta_{bl} < +\infty;$

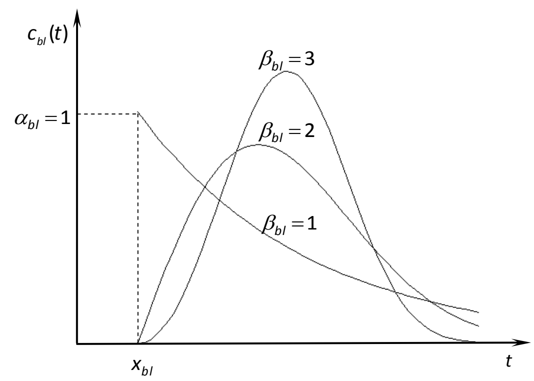


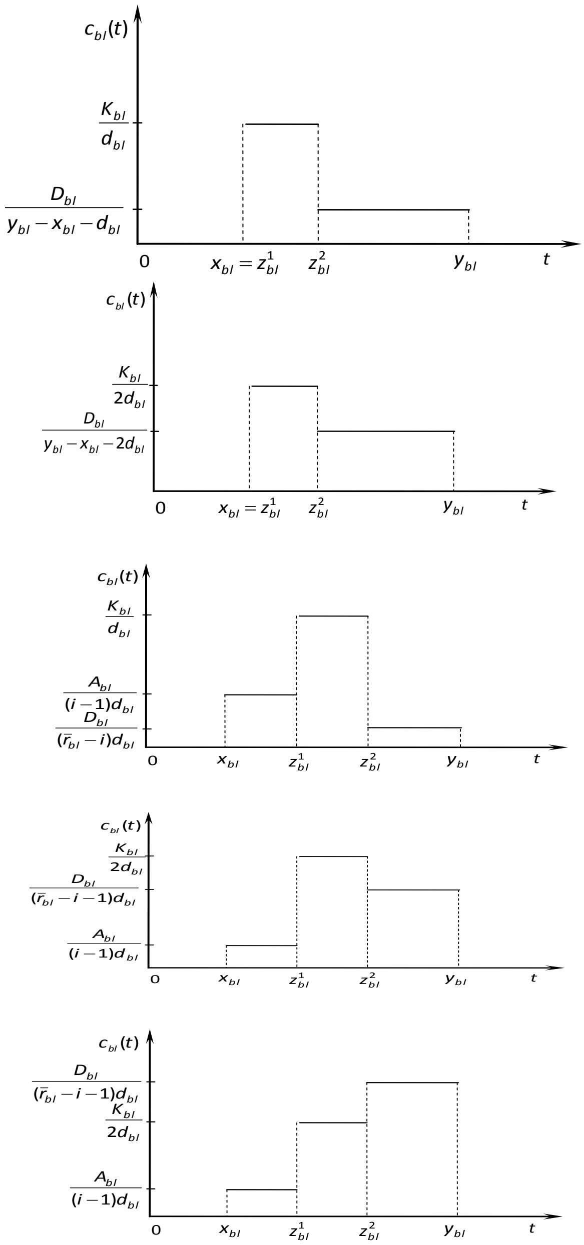

*Figure 7.* The graphs of the Weibull distribution's density functions

– the chimney distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{A_{bl}}{z_{bl}^1 - x_{bl}}, & x_{bl} \le t \le z_{bl}^1 \\ \dfrac{K_{bl}}{z_{bl}^2 - z_{bl}^1}, & z_{bl}^1 \le t \le z_{bl}^2 \\ \dfrac{D_{bl}}{y_{bl} - z_{bl}^2}, & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases} \qquad (11)$$

where

$0 \le x_{bl} \le z_{bl}^1 \le z_{bl}^2 \le y_{bl} < +\infty, y_{bl} \ne 0,\ A_{bl} \ge 0,$
$K_{bl} \ge 0,\ D_{bl} \ge 0,\ A_{bl} + K_{bl} + D_{bl} = 1;$



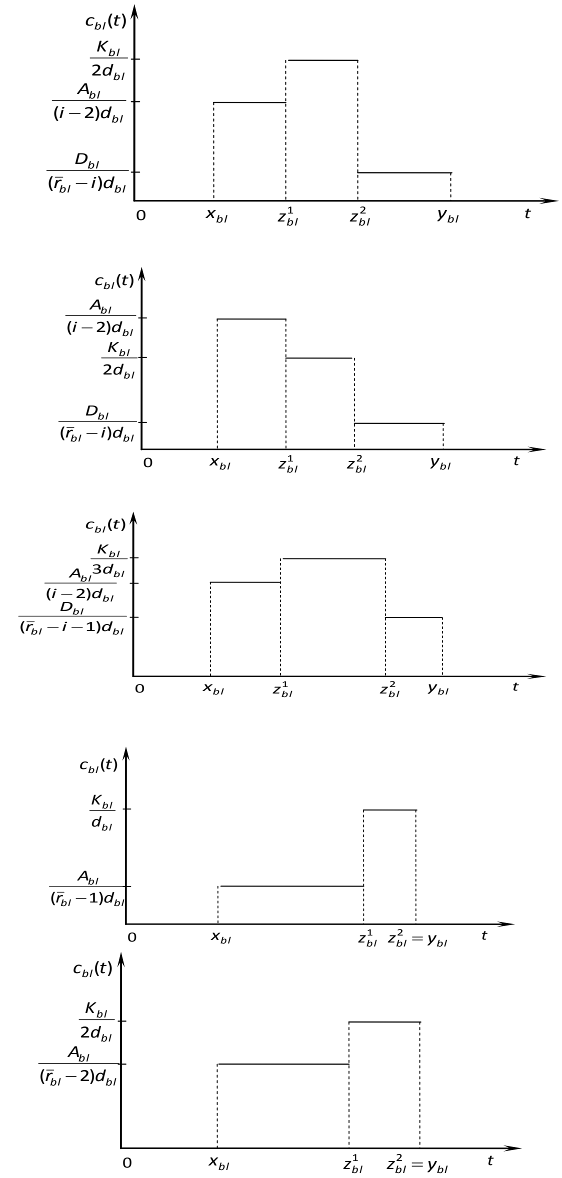




*Figure 8.* The graphs of the chimney distribution's density functions (the meanings of $d_{bl}$ and $\bar{r}_{bl}$ are explained in [EU-Circle Report D2.3-GMU2, 2016])

– the Gamma distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{(t - x_{bl})^{\alpha_{bl}-1} \exp[-(t - x_{bl})/\beta_{bl}]}{\beta_{bl}{}^{\alpha_{bl}} \cdot \Gamma(\alpha_{bl})}, & t \ge x_{bl}, \end{cases}$$

where

$0 < \alpha_{bl} < +\infty,\ 0 < \beta_{bl} < +\infty.$

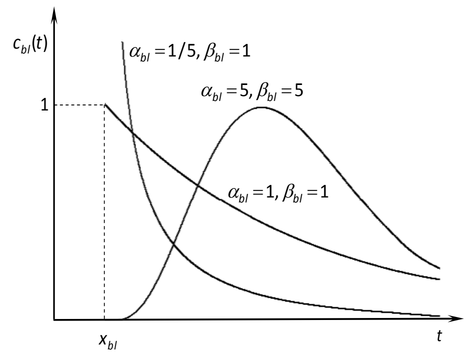



*Figure 9.* The graphs of the Gamma distribution's density functions

## 3. Climate weather change process prediction

Assuming that we have identified the unknown parameters of the climate-weather change process semi-Markov model:
– the initial probabilities $q_b(0)$, $b = 1,2,\ldots,w$, of the climate-weather change process staying at the particular state $c_b$ at the moment $t = 0$;
– the probabilities $q_{bl}$, $b$, $l = 1,2,\ldots,w$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state $c_b$ into the climate-weather state $c_l$;
– the distributions of the climate-weather change process conditional sojourn times $C_{bl}$, $b$, $l = 1,2,\ldots,w$, $b \neq l$, at the particular climate-weather states and their mean values $N_{bl} = E[C_{bl}]$, $b$, $l = 1,2,\ldots,w$;
we can predict this process basic characteristics.
As the mean values of the conditional sojourn times $C_{bl}$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$N_{bl} = E[C_{bl}]$$

$$= \int_0^\infty t dC_{bl}(t) = \int_0^\infty t c_{bl}(t) dt, \ b,l = 1,2,\ldots,w, \quad (13)$$

then for the distinguished distributions (5)-(12), the mean values of the climate-weather change process $C(t)$ conditional sojourn times $C_{bl}$, $b,l = 1,2,\ldots,w$, $b \neq l$, at the particular operation states are respectively given by [EU-CIRCLE Report D2.1-GMU2, 2016]:

- for the uniform distribution

$$N_{bl} = E[C_{bl}] = \frac{x_{bl} + y_{bl}}{2}; \quad (14)$$

- for the triangular distribution

$$N_{bl} = E[C_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3}; \quad (15)$$

- for the double trapezium distribution

$$N_{bl} = E[C_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3}$$
$$+ \frac{w_{bl}(y_{bl})^2 - q_{bl}(x_{bl})^2}{2}$$
$$+ \frac{w_{bl} + q_{bl}}{6}[(x_{bl} z_{bl} - y_{bl} z_{bl}) + \frac{x_{bl} y_{bl}(x_{bl} + y_{bl})}{y_{bl} - x_{bl}}]$$
$$- \frac{(x_{bl})^3 q_{bl} + (y_{bl})^3 w_{bl}}{3(y_{bl} - x_{bl})}; \quad (16)$$

- for the quasi-trapezium distribution

$$N_{bl} = E[C_{bl}] =$$
$$\frac{q_{bl}}{2}[(z_{bl}^1)^2 - (x_{bl})^2]$$
$$- \frac{A_{bl} - q_{bl}}{6}[2(z_{bl}^1)^2 - 5x_{bl} z_{bl}^1 - (x_{bl})^2]$$
$$+ \frac{A_{bl}}{2}[(z_{bl}^2)^2 - (z_{bl}^1)^2] + \frac{w_{bl}}{2}[(y_{bl})^2 - (z_{bl}^2)^2]$$
$$- \frac{w_{bl} - A_{bl}}{6}[2(z_{bl}^2)^2 - 5y_{bl} z_{bl}^2 - (y_{bl})^2]; \quad (17)$$

- for the exponential distribution

$$N_{bl} = E[C_{bl}] = x_{bl} + \frac{1}{\alpha_{bl}}; \quad (18)$$

- for the Weibull distribution

$$N_{bl} = E[C_{bl}] = x_{bl} + \alpha_{bl}^{-\frac{1}{\beta_{bl}}} \Gamma(1 + \frac{1}{\beta_{bl}}), \quad (19)$$

where

$$\Gamma(u) = \int_0^{+\infty} t^{u-1} e^{-t} dt, u > 0,$$

is the gamma function;

- for the chimney distribution

$$N_{bl} = E[c_{bl}] = \frac{1}{2}[A_{bl}(x_{bl} + z_{bl}^1) + C_{bl}(z_{bl}^1 + z_{bl}^2)$$

$$+ D_{bl}(z_{bl}^2 + y_{bl})]; \tag{20}$$

– for the Gamma distribution

$$N_{bl} = E[c_{bl}] = x_{bl} + \alpha_{bl} \cdot \beta_{bl}. \tag{21}$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $C_b$, $b = 1,2,...,w$, of the climate-weather change process $C(t)$ at the climate-weather states $c_b$, $b = 1,2,...,w$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$C_b(t) = \sum_{l=1}^{v} q_{bl} C_{bl}(t), \ b = 1,2,...,w. \tag{22}$$

Hence, the mean values $E[C_b]$ of the climate-weather change process $C(t)$ unconditional sojourn times $C_b$, $b = 1,2,...,w$, at the climate-weather states are given by

$$N_b = E[C_b] = \sum_{l=1}^{v} q_{bl} N_{bl}, \ b = 1,2,...,w, \tag{23}$$

where $N_{bl}$ are defined by the formula (13) in a case of any distribution of sojourn times $C_{bl}$ and by the formulae (14)-(21) in the cases of particular defined respectively by (5)-(12) distributions of these sojourn times.

The limit values of the climate-weather change process $C(t)$ transient probabilities at the particular operation states

$$q_b(t) = P(C(t) = c_b), \ t \in <0,+\infty), \ b = 1,2,...,w,$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$q_b = \lim_{t \to \infty} q_b(t) = \frac{\pi_b N_b}{\sum_{l=1}^{v} \pi_l N_l}, \ b = 1,2,...,w, \tag{24}$$

where $N_b$, $b = 1,2,...,w$, are given by (23), while the steady probabilities $\pi_b$ of the vector $[\pi_b]_{1xw}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][q_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \tag{25}$$

In the case of a periodic climate-weather change process, the limit transient probabilities $q_b$, $b = 1,2,...,w$, at the climate-weather states defined by (24), are the long term proportions of the climate-weather change process $C(t)$ sojourn times at the particular climate-weather states $c_b$, $b = 1,2,...,w$.

Other interesting characteristics of the system climate-weather change process $C(t)$ possible to obtain are its total sojourn times $\hat{C}_b$ at the particular climate-weather states $c_b$, $b = 1,2,...,w$, during the fixed time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the climate-weather change process total sojourn times $\hat{C}_b$ at the particular climate-weather states $c_b$ for sufficiently large time $C$ have approximately normal distributions with the expected value given by

$$\hat{N}[\hat{C}_b] = q_b C, \ b = 1,2,...,w, \tag{26}$$

where $q_b$ are given by (24).

## 4. Conclusions

The proposed in this paper general approach to the climate-weather change process analysis and its influence on the critical infrastructure safety can also be used wider to modelling and predicting the climate influence on any critical assets we are interested in. The probabilistic model of the climate-weather change process for the critical infrastructure operating area built in [EU-CIRCLE Report D2.1-GMU3, 2016] and the various models of the climate-weather change process influence on the critical infrastructure safety presented in [EU-CIRCLE Report D3.3-GMU0, 2016] are suggested to be the basis for further practical considerations in the next reports of the project. This models will be used to construct a general joint climate-weather change process and operation process of the critical infrastructure safety model and finally to construct the General Integrated Model of Critical Infrastructure Safety (GIMCIS) Including Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) - GIMCIS Model 4 [EU-CIRCLE Report D3.3-GMU0, 2016]. Methods of estimation of this this general model unknown parameters will be given in another project report [EU-CIRCLE Report D2.3-GMU2, 2016] as well.

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