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Statistical identification and prediction of the port oil pipeline system's operation process and its reliability and risk evaluation

Keywords

semi-markov processes, system operations process, reliability

Abstract

In the paper a Semi-markov processes are used to construct a general model of complex industrial systems' operation processes. Main characteristics of this model are determined as well. In particular case, for a port oil pipeline transportation system, its operation states are defined, the relationships between them are fixed and particular model of its operation process is constructed and its main characteristics are determined. Further, the joint model of the system operation process and the system reliability is defined and applied to the reliability and risk evaluation of the port oil pipeline transportation system.

1. Introduction

The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics is often very difficult to fix and to analyse. A convenient tool for solving this problem is semi-markov modelling of the systems operation processes proposed in the paper. Therefore, the common usage of the system's reliability evaluation methods and semi-markov modelling the system's exploitation process in order to construct a general system reliability model related to its operation process is proposed in the paper. Moreover, statistical methods of the general model unknown parameters estimation are proposed and applied in pipeline transport.

3. Modelling of system operation process

We assume that the system during its operation process has v different operation states. Thus we can define the system operation process $Z(t)$, $t \in < 0,+\infty >$, as the process with discrete operation states from the set

$$Z = \{z_1, z_2, \ldots, z_v\}.$$

In practice a convenient assumption is that $Z(t)$ is a semi-Markov process [1] with its conditional sojourn times θ_{bl} at the operation state z_b when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l$. In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the system operation process initial states

$$[p_b(0)]_{1xv} = [p_1(0), p_2(0),..., p_v(0)],$$

where

$$p_b(0) = P(Z(0) = z_b) \text{ for } b = 1,2,...,v,$$

- the matrix of probabilities of the system operation process transitions between the operation states

$$[p_{bl}]_{vxv} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \cdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix}, \quad (1)$$

where $p_{bb} = 0$ for $b = 1,2,...,v$,

- the matrix of the system operation process conditional sojourn times θ_{bl} distribution functions

$$[H_{bl}(t)]_{v\times v} = \begin{bmatrix} H_{11}(t)\; H_{12}(t) \ldots H_{1v}(t) \\ H_{21}(t)\; H_{22}(t) \ldots H_{2v}(t) \\ \ldots \\ H_{v1}(t)\; H_{v2}(t) \ldots H_{vv}(t) \end{bmatrix}, \qquad (2)$$

where

$$H_{bl}(t) = P(\theta_{bl} < t) \text{ for } b, l = 1,2,\ldots,v,\ b \neq l,$$

and

$$H_{bb}(t) = 0 \text{ for } b = 1,2,\ldots,v.$$

Under these assumptions, the mean values of the system operation process conditional sojourn times θ_{bl} are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t),\ b, l = 1,2,\ldots,v,\ b \neq l. \qquad (3)$$

By the formula for total probability the unconditional distribution functions of the sojourn times θ_b of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,\ldots,v,$ are given by

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t),\ b = 1,2,\ldots,v. \qquad (4)$$

Hence, the mean values $E[\theta_b]$ of the system operation process unconditional sojourn times θ_b in the particular operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl},\ b = 1,2,\ldots,v, \qquad (5)$$

where M_{bl} are defined by (3).
Moreover, it is well known [1] that the limit values of the system operation process transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),\ b = 1,2,\ldots,v,$$

are given by

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l},\ b = 1,2,\ldots,v, \qquad (6)$$

where M_b, $b = 1,2,\ldots,v,$ are defined by (5), whereas the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \qquad (7)$$

Other interesting characteristics of the operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ in the particular operation states z_b, $b = 1,2,\ldots,v.$ It is well known [5] that the system operation process total sojourn times $\hat{\theta}_b$ in the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distribution with the expected value given by

$$E[\hat{\theta}_b] = p_b \theta,\ b = 1,2,\ldots,v, \qquad (8)$$

where p_b are given by (6).

3. The oil terminal in Dębogórze description

The oil terminal in Dębogórze [9] is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil [9].
The considered system is composed of three terminal parts *A*, *B* and *C*, linked by the piping transportation systems. The scheme of this system is presented in *Figure 1*.

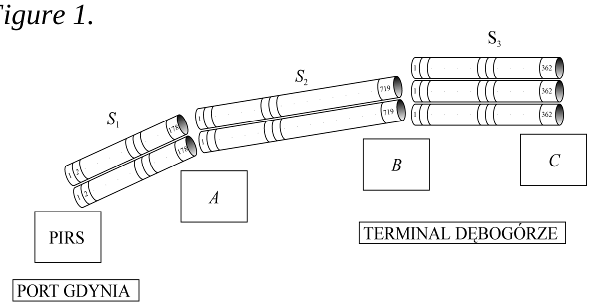

Figure 1. The scheme of port oil transport system.

The unloading of tankers is performed at the pier placed in the Port of Gdynia. The pier is connected with terminal part *A* through the transportation subsystem S_1 built of two piping lines composed of steel pipe segments with diameter of 600 mm. In the part *A* there is a supporting station fortifying tankers pumps and making possible further transport of oil by the subsystem S_2 to the terminal part *B*. The subsystem S_2 is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part *B* is connected with the terminal part *C* by the subsystem S_3. The subsystem S_3 is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part *C* is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the Port of Gdynia and further to the interior of the country.

The Port Oil Transportation system consists of three subsystems S_1, S_2, S_3. (see *Figure 1*)

Subsystem S_1 consist of k_n = 2 two identical pipelines, each composed of l_n = 178 elements. In each pipeline there are:
- 176 pipe segments,
- 2 valves.

Subsystem S_2 consist of k_n = 2 two identical pipelines, each composed of l_n = 719 elements. In each pipeline there are:
- 717 pipe segments,
- 2 valves.

Subsystem S_3 consist of two pipelines of the first type and one second type, each composed of l_n = 362 elements. In each pipeline of the first type there are:
- 360 pipe segments (Ø=350mm),
- 2 valves.

In pipeline of the second type there are:
- 360 pipe segments (Ø=500mm),
- 2 valves.

4. The port oil pipeline transportation system operation process and its preliminary statistical identification

Taking into account the varying in time operation process of the considered system we distinguish the following as its eight operation states:

- an operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines in subsystem S_3,

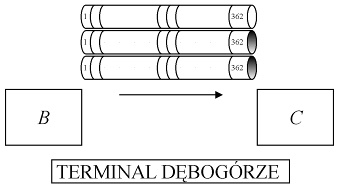

Figure 2. The scheme of port oil transportation system at operation state z_1

- an operation state z_2 – transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines in subsystem S_3,

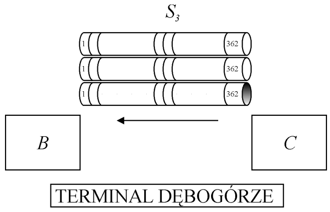

Figure 3. The scheme of port oil transportation system at operation state z_2

- an operation state z_3 – transport of one kind of medium from the terminal part B through part A to the pier using one out of two pipelines in subsystem S_2 and one out of two pipelines in subsystem S_1,

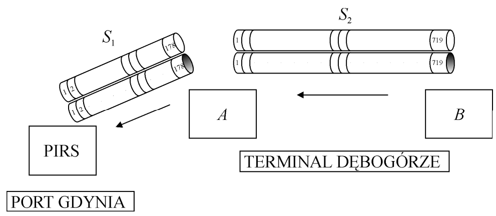

Figure 4. The scheme of port oil transportation system at operation state z_3

- an operation state z_4 – transport of two kinds of medium from the pier through parts A and B to part C using one out of two pipelines in subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines in subsystem S_3,

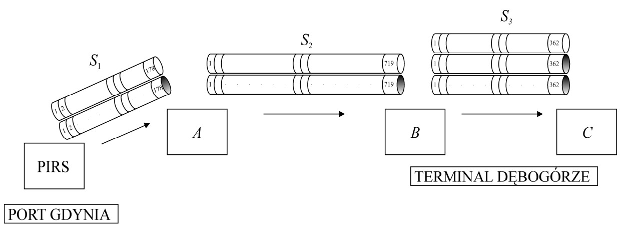

Figure 5. The scheme of port oil transportation system at operation state z_4

- an operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines in subsystem S_1 and one out of two pipelines in subsystem S_2,

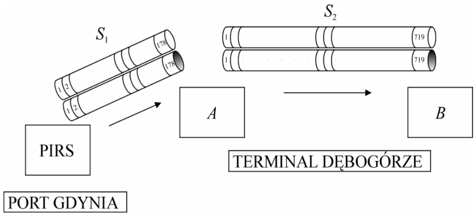

Figure 6. The scheme of port oil transportation system at operation state z_5

- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines in parts S_1 and one out of two pipelines in subsystem S_2,

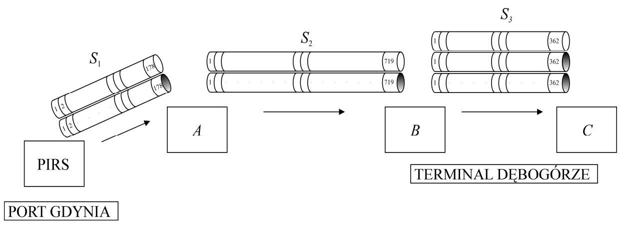

Figure 7. The scheme of port oil transportation system at operation state z_6

- an operation state z_7 – lack of medium transport (system is not working)

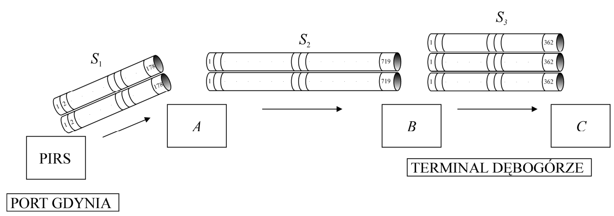

Figure 8. The scheme of port oil transportation system at operation state z_7

- an operation state z_8 – transport of one kind of medium from the terminal part B to C using one out of three pipelines in part S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part S_3.

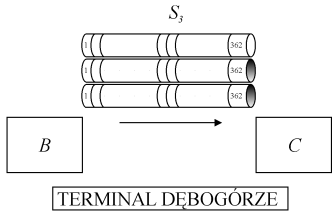

Figure 9. The scheme of port oil transportation system at operation state z_8

In the *Table 1* there is given an example realization of the conditional sojourn times in particular operation states of the oil pipeline system operation process.

Table 1. Realization of conditional sojourn times in operations states during one week.

Conditional sojourn times θ_{bl} in operation state	Realization of conditional sojourn times θ_{bl} in operation state			
θ_{17}	8h	7h	15h	12h
θ_{71}	1h 30m	15h	8h	
θ_{57}	7h 30m	21h 10m		
θ_{75}	0h 40m	41h		
θ_{56}	15h 10m			
θ_{65}	16h			

It is assumed that one week of working of the system is a single realization of its operation process. The conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b,l = 1,2,3,4,5,6,7,8,\ b \neq l,$ of each single realization of the pipeline system operation process are given in separate line. In [4] there are collected realizations of the conditional sojourn times in particular operation states of considered system on the basis of a sample composed of $n = 10$ realizations.
To identify all parameters of the pipeline system operation process the statistical data about this process is needed. The statistical data that has been collected up to now is given in [4]
From data given in [4] the following basic operation process statistical data are fixed:
- the number of the pipeline system operation process states

$\nu = 8$;

- the pipeline system operation process observation/experiment time

Θ = 70 days = 10 weeks;

- the number of the pipeline system operation process realizations

$n(0) =$ 10;

- the realization $n_b(0)$ of the number of the pipeline system operation process transitions in the particular operation states z_b at the initial moment t = 0

$n_1(0) = 7, n_2(0) = 0,\ n_3(0) = 0,\ n_4(0) = 0,$

$n_5(0) = 1,\ n_6(0) = 1,\ n_7(0) = 1,\ n_8(0) = 0,$

where

$n_1(0) + n_2(0) + n_3(0) + n_4(0) + n_5(0) + n_6(0)$

$+ n_7(0) + n_8(0) = 10;$

- the vector of realizations of the numbers of the pipeline system operation process transitions in the particular operation states z_b at the initial moment t = 0

$$[n_b(0)] = [n_1(0), n_2(0), n_3(0), n_4(0),$$

$$n_5(0), n_6(0), n_7(0), n_8(0)]$$

$$= [7,0,0,0,1,1,1,0];$$

- the realization n_{bl} of the numbers of pipeline system operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 70$ days

$n_{11} =$ 0, $n_{12} = 42$, $n_{13} =$ 0, $n_{14} = 0$, $n_{15} = 3$,
$n_{16} = 3$, $n_{17} = 43$, $n_{18} = 1$,
$n_{21} =$ 0, $n_{22} = 0$, $n_{23} =$ 0, $n_{24} = 0$, $n_{25} = 0$,
$n_{26} = 0$, $n_{27} = 0$, $n_{28} = 0$,
$n_{31} =$ 0, $n_{32} = 0$, $n_{33} =$ 0, $n_{34} = 0$, $n_{35} = 0$,
$n_{36} = 0$, $n_{37} = 0$, $n_{38} = 0$,
$n_{41} =$ 0, $n_{42} = 0$, $n_{43} =$ 0, $n_{44} = 0$, $n_{45} = 0$,
$n_{46} = 0$, $n_{47} = 0$, $n_{48} = 0$,
$n_{51} =$ 2, $n_{52} = 0$, $n_{53} =$ 0, $n_{54} = 0$, $n_{55} = 0$,
$n_{56} = 2$, $n_{57} = 11$, $n_{58} = 1$,
$n_{61} =$ 2, $n_{62} = 0$, $n_{63} =$ 0, $n_{64} = 0$, $n_{65} = 3$,
$n_{66} = 0$, $n_{67} = 0$, $n_{68} = 0$,
$n_{71} =$ 45, $n_{72} = 0$, $n_{73} =$ 0, $n_{74} = 0$, $n_{75} = 9$,
$n_{76} = 0$, $n_{77} = 0$, $n_{78} = 1$,
$n_{81} = 2$, $n_{82} = 0$, $n_{83} =$ 0, $n_{84} = 0$, $n_{85} = 0$,
$n_{86} = 0$, $n_{87} = 1$, $n_{88} = 0$;

- the matrix of realizations n_{bl} of the numbers of the pipeline system operation process transitions from the state z_b into the state z_l during the experiment time $\Theta = 70$ days

$$[n_{bl}]$$

$$= \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} & n_{15} & n_{16} & n_{17} & n_{18} \\ n_{21} & n_{22} & n_{23} & n_{24} & n_{25} & n_{26} & n_{27} & n_{28} \\ n_{31} & n_{32} & n_{33} & n_{34} & n_{35} & n_{36} & n_{37} & n_{38} \\ n_{41} & n_{42} & n_{43} & n_{44} & n_{45} & n_{46} & n_{47} & n_{48} \\ n_{51} & n_{52} & n_{53} & n_{54} & n_{55} & n_{56} & n_{57} & n_{58} \\ n_{61} & n_{62} & n_{63} & n_{64} & n_{65} & n_{66} & n_{67} & n_{68} \\ n_{71} & n_{72} & n_{73} & n_{74} & n_{75} & n_{76} & n_{77} & n_{78} \\ n_{81} & n_{82} & n_{83} & n_{84} & n_{85} & n_{86} & n_{87} & n_{88} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 3 & 3 & 43 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 2 & 11 & 1 \\ 2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 45 & 0 & 0 & 0 & 9 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix};$$

- the realization n_b of the total numbers of the pipeline system operation process transitions from the operation state z_b during the experiment time $\Theta = 70$ days (the sums of the numbers of the matrix $[n_{bl}]$)

$n_1 = n_{11} + n_{12} + n_{13} + n_{14} + n_{15} + n_{16} + n_{17} + n_{18}$
= 50,
$n_2 = n_{21} + n_{22} + n_{23} + n_{24} + n_{25} + n_{26} + n_{27} + n_{28}$ = 0,
$n_3 = n_{31} + n_{32} + n_{33} + n_{34} + n_{35} + n_{36} + n_{37} + n_{38}$ = 0,
$n_4 = n_{41} + n_{42} + n_{43} + n_{44} + n_{45} + n_{46} + n_{47} + n_{48}$ = 0,

$n_5 = n_{51} + n_{52} + n_{53} + n_{54} + n_{55} + n_{56} + n_{57} + n_{58} = 16,$
$n_6 = n_{61} + n_{62} + n_{63} + n_{64} + n_{65} + n_{66} + n_{67} + n_{68} = 5,$
$n_7 = n_{71} + n_{72} + n_{73} + n_{74} + n_{75} + n_{76} + n_{77} + n_{78} = 55,$
$n_8 = n_{81} + n_{82} + n_{83} + n_{84} + n_{85} + n_{86} + n_{87} + n_{88} = 3;$

- the matrix of realizations of the total numbers of the pipeline system operation process transitions from the operation state z_b during the experiment time $\Theta = 70$ days

$$[n_b] = [n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8]$$

$$= [50,0,0,0,16,5,55,3].$$

On the basis of the above statistical data it is possible to evaluate

- the vector of realizations

$$[p(0)] = [0.7, 0, 0, 0, 0.1, 0.1, 0.1, 0]$$

of the initial probabilities $p_b(0)$, $b = 1,2,3,4,5,6,7,8,$ (1) [5] of the pipeline system operation process transients in the particular states z_b at the moment t = 0

- the matrix of realizations

$$[p_{bl}]$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0.06 & 0.06 & 0.86 & 0.02 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.125 & 0 & 0 & 0 & 0 & 0.125 & 0.687 & 0.063 \\ 0.4 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0.82 & 0 & 0 & 0 & 0.16 & 0 & 0 & 0.02 \\ 0.67 & 0 & 0 & 0 & 0 & 0 & 0.33 & 0 \end{bmatrix}$$

of the transition probabilities p_{bl}, $b, l = 1,2,3,4,5,6,7,8,$ (1) of the pipeline system operation process from the operation state z_b into the operation state z_l.

5. The port oil pipeline system operation process characteristics evaluation

At the moment because of the luck of sufficient statistical data about the oil terminal operation process it is not possible to estimate its all operational characteristics. Namely, it is not possible to determine the matrix of the conditional distribution functions $[H_{bl}(t)]_{8x8}$ of the lifetimes θ_{bl} for $b, l = 1,2,...,8,$ $b \neq l,$ (2) and further consequently, according to (20) [5], it is also not possible to determine the vector $[H_b(t)]_{1x8}$ of the unconditional distribution functions of the lifetimes θ_b of this operation process at the operation states z_b, $b = 1,2,...,8.$ However, on the basis of the preliminary statistical data coming from experiment it is possible to evaluate approximately the conditional mean values $M_{bl} = E[\theta_{bl}],$ $b, l = 1,2,...,8,$ $b \neq l,$ (3) of lifetimes in the particular operation states. On the basis of the statistical data given in *Tables 1-10* in [4] their approximate evolutions are as follows:

$M_{15} = 720,$ $M_{16} = 420,$ $M_{17} = 698.95,$ $M_{18} = 480,$

$M_{51} = 750,$ $M_{56} = 564,$ $M_{57} = 748.7,$ $M_{58} = 540,$

$M_{61} = 360,$ $M_{65} = 360,$

$M_{71} = 975.3,$ $M_{75} = 872.4,$ $M_{78} = 600,$

$M_{81} = 900,$ $M_{87} = 420.$

Hence, by (5), the unconditional mean sojourn times in the particular operation states are given by:

$$M_1 = E[\theta_1] = p_{15}M_{15} + p_{16}M_{16} + p_{17}M_{17} + p_{18}M_{18}$$

$$= 0.06 \cdot 720 + 0.06 \cdot 420 + 0.86 \cdot 698.95$$

$$+ 0.02 \cdot 480 \cong 679.1,$$

$$M_5 = E[\theta_5] = p_{51}M_{51} + p_{56}M_{56} + p_{57}M_{57} + p_{58}M_{58}$$

$$= 0.125 \cdot 750 + 0.125 \cdot 564 + 0.687 \cdot 748.7$$

$$+ 0.063 \cdot 540 \cong 712.63,$$

$$M_6 = E[\theta_6] = p_{61}M_{61} + p_{65}M_{65}$$

$$= 0.4 \cdot 360 + 0.6 \cdot 360 = 360,$$

$$M_7 = E[\theta_7] = p_{71}M_{71} + p_{75}M_{75} + p_{78}M_{78}$$

$$= 0.82 \cdot 975.3 + 0.16 \cdot 872.4 + 0.02 \cdot 600$$

$$\cong 951.33,$$

$$M_8 = E[\theta_8] = p_{81} M_{81} + p_{87} M_{87}$$

$$= 0.67 \cdot 900 + 0.33 \cdot 420 \cong 741.6.$$

Since from the system of equations below (7)

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8] \\ = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8][p_{bl}]_{8x8} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 = 1, \end{cases}$$

we get

$$\pi_1 = 0.396, \ \pi_2 = 0, \ \pi_3 = 0, \ \pi_4 = 0,$$

$$\pi_5 = 0.116, \ \pi_6 = 0.038, \ \pi_7 = 0.435, \ \pi_8 = 0.015.$$

Then the limit values of the transient probabilities $p_b(t)$ at the operational states z_b, according to (6), are given by

$$p_1 = 0.34, \ p_2 = 0, \ p_3 = 0, \ p_4 = 0,$$

$$p_5 = 0.1, \ p_6 = 0.02, \ p_7 = 0.53, \ p_8 = 0.01. \qquad (9)$$

6. Reliability of systems in variable operation process

We assume that the changes of the system operation process *Z*(*t*) states have an influence on the system components E_i, $i = 1,2,...,n$, reliability and the system reliability structure as well. Thus, we denote the conditional reliability function of the system component E_i while the system is at the operational state z_b, $b = 1,2,...,v$, by

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, ... [R_i(t,z)]^{(b)}],$$

for $t \in <0,\infty)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, where

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b)$$

for $t \in < 0,\infty)$, $i = 1,2,...,n$, $b = 1,2,...,\nu$, $u = 1,2,...,z$,

and the conditional reliability function of the system while the system is at the operational state z_b, $b = 1,2,...,\nu$, by

$$[\boldsymbol{R}_n(t,\cdot)]^{(b)} = [1, [\boldsymbol{R}_n(t,1)]^{(b)}, ... [\boldsymbol{R}_n(t,z)]^{(b)}],$$

for $t \in <0,\infty)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, where

$$[\boldsymbol{R}_n(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b)$$

for $t \in < 0,\infty)$, $b = 1,2,...,\nu$, $n \in N$,

where

$$T^{(b)}(u) = T(T_1^{(b)}(u), T_2^{(b)}(u), ..., T_n^{(b)}(u))$$

for $t \in < 0,\infty)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, $n \in N$,

and

$$[\boldsymbol{R}_n(t,u)]^{(b)}$$

$$= \boldsymbol{R}_n([R_1(t,u)]^{(b)}, [R_2(t,u)]^{(b)}, ..., [R_n(t,u)]^{(b)})$$

for $t \in < 0,\infty)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, $n \in N$.

The reliability function $[R_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the reliability state subset $\{u, u+1, ..., z\}$ [5] is greater than *t*, while the process *Z*(*t*) is at the operation state z_b. Similarly, the reliability function $[\boldsymbol{R}_n(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the reliability state subset $\{u, u+1, ..., z\}$ [5] is greater than *t*, while the process *Z*(*t*) is at the operation state z_b. In the case when the system operation time is large enough, the unconditional reliability function of the system is given by

$$\boldsymbol{R}_n(t,\cdot) = [1, \boldsymbol{R}_n(t,1), ... \boldsymbol{R}_n(t,z)],$$

where

$$\boldsymbol{R}_n(t,u) = P(T(u) > t) \cong \sum_{b=1}^{\nu} p_b \, [\boldsymbol{R}_n(t,u)]^{(b)} \qquad (10)$$

for $t \in < 0,\infty)$, $u = 1,2,...,z$,

where $T(u)$ is the unconditional lifetime of the system in the reliability state subset $\{u, u+1, ..., z\}$

and the mean value of the system lifetime is

$$\mu(u) \cong \sum_{b=1}^{v} p_b \mu_b(u), \quad u = 1,2,...,z, \tag{11}$$

where

$$\mu_b(u) = \int_0^\infty [\boldsymbol{R}_n(t,u)]^{(b)} dt, \tag{12}$$

and p_b are given by (6), and the variance of the system lifetime is

$$\sigma^2(u) = 2\int_0^\infty t \ \boldsymbol{R}_n(t,u)dt - [\mu(u)]^2. \tag{13}$$

7. Reliability and risk evaluation of port oil pipeline transportation system in variable operation process

After discussion with experts, taking into account the safety of the operation of the oil pipeline transportation system, we distinguish the following three reliability states of its components [2]:

- a reliability state 2 – piping operation is fully safe,
- a reliability state 1 – piping operation is less safe and more dangerous because of the possibility of environment pollution,
- a reliability state 0 – piping is destroyed.

Next, using the model considering in section 6, the results of section 5 and the results given in [4] by (9) and (10), we have

$$\boldsymbol{R}_n(t,\cdot) = [1, \boldsymbol{R}_n(t,1), \boldsymbol{R}_n(t,2)],$$

where

$$\begin{aligned}\boldsymbol{R}_n(t,1) &= p_1[\boldsymbol{R}_n(t,1)]^{(1)} + p_2[\boldsymbol{R}_n(t,1)]^{(2)}\\ &+ p_3[\boldsymbol{R}_n(t,1)]^{(3)} + p_4[\boldsymbol{R}_n(t,1)]^{(4)}\\ &+ p_5[\boldsymbol{R}_n(t,1)]^{(5)} + p_6[\boldsymbol{R}_n(t,1)]^{(6)}\\ &+ p_7[\boldsymbol{R}_n(t,1)]^{(7)} + p_8\boldsymbol{R}_n(t,1)]^{(8)}\\ &= 0.34\cdot[\boldsymbol{R}_n(t,1)]^{(1)} + 0\cdot[\boldsymbol{R}_n(t,1)]^{(2)}\\ &+ 0\cdot[\boldsymbol{R}_n(t,1)]^{(3)} + 0\cdot[\boldsymbol{R}_n(t,1)]^{(4)}\\ &+ 0.1\cdot[\boldsymbol{R}_n(t,1)]^{(5)} + 0.02\cdot[\boldsymbol{R}_n(t,1)]^{(6)}\\ &+ 0.53\cdot[\boldsymbol{R}_n(t,1)]^{(7)} + 0.01\cdot[\boldsymbol{R}_n(t,1)]^{(8)}\end{aligned} \tag{14}$$

for $t \geq 0$,

$$\begin{aligned}\boldsymbol{R}_n(t,2) &= p_1[\boldsymbol{R}_n(t,2)]^{(1)} + p_2[\boldsymbol{R}_n(t,2)]^{(2)}\\ &+ p_3[\boldsymbol{R}_n(t,2)]^{(3)} + p_4[\boldsymbol{R}_n(t,2)]^{(4)}\\ &+ p_5[\boldsymbol{R}_n(t,2)]^{(5)} + p_6[\boldsymbol{R}_n(t,2)]^{(6)}\\ &+ p_7[\boldsymbol{R}_n(t,2)]^{(7)} + p_8\boldsymbol{R}_n(t,2)]^{(8)}\\ &= 0.34\cdot[\boldsymbol{R}_n(t,2)]^{(1)} + 0\cdot[\boldsymbol{R}_n(t,2)]^{(2)}\\ &+ 0\cdot[\boldsymbol{R}_n(t,2)]^{(3)} + 0\cdot[\boldsymbol{R}_n(t,2)]^{(4)}\\ &+ 0.1\cdot[\boldsymbol{R}_n(t,2)]^{(5)} + 0.02\cdot[\boldsymbol{R}_n(t,2)]^{(6)}\\ &+ 0.53\cdot[\boldsymbol{R}_n(t,2)]^{(7)} + 0.01\cdot[\boldsymbol{R}_n(t,2)]^{(8)}\end{aligned} \tag{15}$$

for $t \geq 0$,

where $[\boldsymbol{R}_n(t,1)]^{(1)}$, $[\boldsymbol{R}_n(t,1)]^{(2)}$, $[\boldsymbol{R}_n(t,1)]^{(3)}$, $[\boldsymbol{R}_n(t,1)]^{(4)}$, $[\boldsymbol{R}_n(t,1)]^{(5)}$, $[\boldsymbol{R}_n(t,1)]^{(6)}$, $[\boldsymbol{R}_n(t,1)]^{(7)}$, $[\boldsymbol{R}_n(t,1)]^{(8)}$ and $[\boldsymbol{R}_n(t,2)]^{(1)}$, $[\boldsymbol{R}_n(t,2)]^{(2)}$, $[\boldsymbol{R}_n(t,2)]^{(3)}$, $[\boldsymbol{R}_n(t,2)]^{(4)}$, $[\boldsymbol{R}_n(t,2)]^{(5)}$, $[\boldsymbol{R}_n(t,2)]^{(6)}$, $[\boldsymbol{R}_n(t,2)]^{(7)}$, $[\boldsymbol{R}_n(t,2)]^{(8)}$ are the system reliability functions in particular operation states determined by (23), (29), (41), (56), (69), (84), (99), (105) and (24), (30), (42), (57), (70), (85), (100), (106) given in [4].

Since according to the results given in [4], the mean values of the conditional system lifetimes in the reliability state subsets by (12) in years are:

$$\mu_1(1) \cong 0.364, \ \mu_1(2) \cong 0.304,$$

$$\mu_2(1) \cong 0.807, \ \mu_2(2) \cong 0.666,$$

$$\mu_3(1) \cong 0.307, \ \mu_3(2) \cong 0.218,$$

$$\mu_4(1) \cong 0.079, \ \mu_4(2) \cong 0.058,$$

$$\mu_5(1) \cong 0.307, \ \mu_5(2) \cong 0.218,$$

$$\mu_6(1) \cong 0.079, \ \mu_6(2) \cong 0.058,$$

$$\mu_7(1) \cong 0.11, \ \mu_7(2) \cong 0.083,$$

$$\mu_8(1) \cong 0.364, \ \mu_8(2) \cong 0.304,$$

then applying (14), (15), (11) and (13), we get the mean value and the standard deviation of the system unconditional lifetime in the reliability state subsets given by

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1) + p_4\mu_4(1)$$

$$+ p_5\mu_5(1) + p_6\mu_6(1) + p_7\mu_7(1) + p_8\mu_8(1)$$

$$\cong 0.218,$$

$$\sigma(1) \cong 0.228 \text{ year},$$

$$\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2) + p_4\mu_4(2)$$

$$+ p_5\mu_5(2) + p_6\mu_6(2) + p_7\mu_7(2) + p_8\mu_8(2)$$

$$\cong 0.173,$$

$$\sigma(2) \cong 0.185 \text{ year}.$$

If the critical safety state [2] is $r = 1$, then the system risk function, is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}_3(t,1)$$

$$= 1 - [0.34 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(1)} + 0.1 \cdot [\overline{\boldsymbol{R}}(t,1)]^{5)}$$

$$+ 0.02 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(6)} + 0.53 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(7)}$$

$$+ 0.01 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(8)}] \text{ for } t \geq 0.$$

Hence, the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.011 \text{ years}.$$

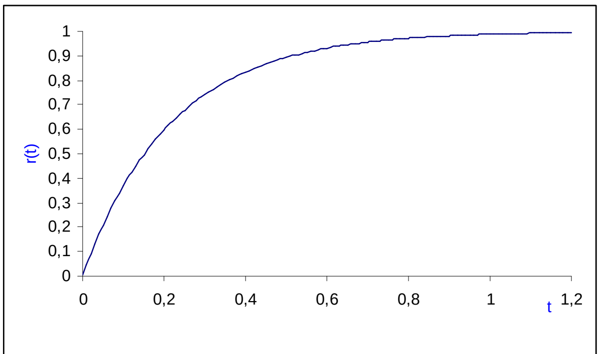

Figure10. The graph of the port oil pipeline system risk function $r(t)$

8. Conclusion

The paper proposes an approach to the solution of practically very important problem of linking the systems' reliability and their operation processes. To involve the interactions between the systems' operation processes and their varying in time reliability structures and components' reliability characteristics a semi-markov model of the systems' operation processes and system conditional reliability functions are used. This approach gives practically important in everyday usage tool for reliability evaluation of the systems with changing reliability structures and components' reliability characteristics during their operation processes.
Application of the proposed method is illustrated in the reliability and risk evaluation of the port oil pipeline transportation system. The reliability input data concerned with the operation process and reliability functions of the components of the port oil transportation system are not precise. They are coming from experts and are concerned with the mean lifetimes of the system components and with the conditional sojourn times of the system in the operation states. To improve the achieved results it is supposed that the statistical data given in [4] will be collected for the next two years and after this period of time the full identification of the pipeline oil transportation operation process will be performed and this process main characteristics will be determined and used in pipeline transportation systems reliability, risk and availability more precise analysis and evaluation.

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