BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 63, No. 4, 2015
DOI: 10.1515/bpasts-2015-0106

A multi-level method of support for management of product flow through supply chains

M. MAGIERA*

AGH University of Science and Technology, Faculty of Management, Chair of Operations Research and Information Technology,
30 A. Mickiewicza Ave., 30-059 Krakow, Poland

Abstract. The paper presents a three-level method used to support the management of product flow through supply chain links (e.g. production lines) and between these links: suppliers and recipients of products of various types.The supply chain includes both producers of components (suppliers) and recipients of these components, which are used for the production of complex goods. The method is dedicated to the development of schedules of product flows through particular production plants (links in supply chains) and between individual plants. Each module of the developed system refers to a separate production plant. The organization of product flow through production lines covers different types of production routes and different configurations of production systems. At the first level of the method, preliminary production schedules are developed for each plant within the supply chain. The second level of the method is dedicated to the development of delivery schedules of components and semi-finished products to these plants. The determined delivery times of components to the individual production plants constitute data for the third level of the method. At this lowest level, detailed schedules of product flow through production lines with the producers of complex goods are developed. Linear mathematical models have been built for each level of the method. In the developed method, optimization take place in the developed method in the scale of the entire supply chain (cost reduction), as well as in the scale of its links (production lines for which manufacturing schedules are built with various criteria taken into consideration). The computational experiments used for verification of the method have been included.

Key words: supply network, supply chain, scheduling, logistic, integer programming, decision making.

1. Introduction

Supply chain management is a concept of managing the relationships with suppliers and recipients as well as clients in order to deliver the highest value services to the client at the lowest costs for the entire chain [1]. It includes integrated processes of planning, procurement, transport and returns from the given supplier to the recipient, who is the supplier for another recipient.

This paper deals with the above defined control over product flow through supply chains. It concerns not only the issues of planning product flow between the links of supply chains, but also through some links in these chains, namely production lines. The objective of the paper is to present a multi-level method of supporting the management of product flow through supply chains developed by the author of this paper. This method is used, among other applications, to coordinate the flows of products through production lines and between them. The method described further in the paper can be a tool to build two sets of product flow schedules. The first of them includes schedules of flows of products through production lines. Fixed and alternative production routes have been taken into account in the development of these schedules. The types of production routes are defined further in the paper. The schedules of flows of products between supply chain links, that is between suppliers and recipients, are also being built. These links include, among others, the plants with production lines. In order to provide a mathematical description of the problem, a number of indexes, sets, parameters and variables have been formulated. The parameters which describe products, suppliers and recipients (including, among others, supply and demand), as well as transportation modes, have been emphasised. The developed structure of data and variables has been used in the constructed linear mathematical models. Separate mathematical models have been assigned to each of these levels of the method. The models formulated for the first level are used for the construction of preliminary product flow schedules through production lines. The second level of the method is used to construct schedules of flows through supply chains. The third level is dedicated to the construction of final schedules of product flow through production lines.

Time criteria have been taken into account in the construction of product flow schedules through production lines (the first level and the third level). The construction of schedules for a simultaneous flow of various types of products through supply networks is based on the cost criterion (the second level). The minimized sum of costs takes into account the costs of purchase of products from the suppliers, costs of transport, penalties for delays in delivery of products, and storage costs for products delivered before the agreed date of order execution. The cost criterion is thus related to the time of execution of orders and includes costs of deliveries off the schedule. The developed method emphasises the relations between schedules (product flow times), which are used to coordinate the flow of products through supply chain links and between these links.

*e-mail: mmagiera@zarz.agh.edu.pl

The conception of the hierarchical approach to the solution of the problem results mostly from the necessity of simultaneous consideration of a large number of parameters and variables in a large number of constraints. Breaking down the global problem into partial tasks is thus conducive for the possibility of solving problems of relatively large scale. The modular approach to the solution of the problem is important here. For each plant (a production line), a separate mathematical model may be selected which takes into consideration the configuration of the machines and the organization of the product flow.

The hierarchical approach has also been applied for development of product flow schedules through production lines: distribution of operations in time and space (allocation to machines) is preceded with allocation of operations to stages, where each stage includes machines operated in parallel. The selection of this concept is conducive for solving problems of relatively large scale, in which the parameters should be taken into account which describe both products and machines. These parameters are used in a large number of constraints formulated for mathematical models used for construction of preliminary product flow schedules through production lines.

The specific nature of the problem has also affected the selection of the hierarchical approach. For example, knowledge of the demand for components, resulting from the preliminary production plant built at the first level of the method, is taken into account in the construction of the schedule for component deliveries (the second level).

The inspiration for the development of the proposed method came from an observation of the functioning of a company producing machine parts. This company has production lines in 3 locations. A major part of the production involves the execution of single urgent orders. The application of many available systems used to support the management of product flow through a small supply network (e.g. from among those stated below) is, first of all, too expensive, and secondly, it does not always take into account the specific nature of production lines. The advantage of the developed method consists in the fact that a separate mathematical model may be matched with each production line (or machine setup): the module of the system. It will take into account, among other things, the configuration of machines and auxiliary devices (e.g. part feeders) and the product flow route type. Application of the developed method in a test company has positively affected the coordination of the product flow, while simultaneously reducing the organization costs of these flows. The results of the calculation experiments are provided in the final part of the paper.

This part of the paper presents notes on other systems used to provide support for the management of product flow through supply chains.

Today, the tele-information technologies which support supply chain management are developing very rapidly. The tools used for this purpose include:

- Advanced planning and scheduling systems in the APS class (*Advanced Planning System*) for synchronisation of procurement, production and distribution plans, taking limitations into account and using information provided by suppliers and distributors [1];
- Systems supporting the management of contacts with clients in the CRM class (*Customer Relationship Management*) for improving relationships with end clients [2].

The mathematical description of the functioning of systems that support the management of product flows requires consideration of a large number of indices, parameters and variables within mathematical relationships. The formulae in the mathematical relationships are used to support the control over product flow through particular production plants (links in the network) and the organization of product flow between production plants. Breaking down the task into problems solved in succession, as well as a modular approach to the problem, contribute to elimination of simultaneous accounting for a large number of parameters and decision variables. The result is that problems of relatively larger size may be solved. For example, the following planning modules are developed for APS class systems: strategic network planning, joint demand planning, coordination of plans, procurement, production and distribution. These modules very often employ integer programming, which is conducive to better quality of solutions. It has been reported, for example, in publications [3] and [4].

APS-class systems are not always ready for specific applications concerning distribution of operations in space and time. It is thus necessary to adjust systems of this class to specific applications: construction of optimization algorithms which take into account the specific nature of production systems. Moreover, purchase of APS-class systems is related to incurring major costs (even though the assumption is to have the incurred expenditures returned as a result of application of these systems). Implementation of these systems takes a relatively long time.

For this reason, new methods are being developed which are used for planning of product flow through production lines and supply chains. These are methods which feature considerably lower costs and shorter implementation times. The method proposed in this paper has exactly such features. This method was designed for specific applications – it takes into consideration the specific nature of production lines (e.g. configuration of machines and auxiliary devices, limited availability of machines, organization of product flow) for which product flow schedules are built. Specific conditions of product flow between these production lines have been taken into account in this method.

One should also take notice that construction of new methods of planning product flow is not necessarily competitive to the appreciated APS-class systems. New methods often include optimization tools for APS-class systems, thus complimenting these advanced systems.

The companies that want to quickly and effectively react to the ever increasing requirements of clients strive to modernise their supply chains. Management over modern supply chains features, among other things, searching for a compromise between the individual interests of specific companies and the global objective. The maximum use of information is important here, as it may help achieve optimum costs and the highest level of support for individual links in the chain.

Flexibility is one of the most important features of the chains, which allows winning a competitive edge. It may be viewed in different planes [5]:

- flexibility of action: the scale of freedom of operation;
- process flexibility: reaction speed, satisfying fast changing demand for various types of products;
- structure flexibility: readiness for action, the possibility of modification of work cells and production lines [6].

A large portion of supply chains are of network nature. It is due to the large number of parallel operating links: companies, various types of storage facilities, and transport companies. Such configured chains are called supply networks.

The three-level method presented in this paper is used, among others, for the scheduling of transport tasks. Mixed integer programming is used on each level of the method. The application of this mathematical tool to task scheduling for supply networks was inspired by [3] and [4]. These works show that mixed integer programming may be used not only for designing supply networks, but also for task scheduling for these networks, which is the subject matter of this paper.

The mathematical model constructed for the second level of the method described in the paper is based on a transport task formulated for transferring products between suppliers and recipients (for example, recalled in the publication [7]). The classic transport task has been expanded here and many various types of products, different transportation firms and their transportation modes have been taken into account, along with many parameters which describe the products and the possibilities of the transportation modes. The model presented in this paper (constructed for the second level) is used for transportation planning between the links in the supply network. SteadieSeifi et al. [8] prepared a review of literature dedicated to transportation planning with various transportation modes. Their paper presents a structural overview of the multimodal transportation literature from 2005. The traditional strategic, tactical, and operational levels of transportation planning are described. The routing problem is related to the stated issues. The paper [9] describes the inventory routing problems, and presents different models and policies for the class of the problems. This paper also contains an extensive overview of papers connected with inventory routing problems.

A review of the literature related to supply network management was prepared by Miemczyk et al. [10]. They have included in their paper a comparison of different concepts used for the support the of management of product flow through supply chains. As regards organization of product flow through one-way flow systems, permutation systems and systems in which the order of product flow between particular groups of network links (suppliers and recipients) may change, have been discussed. A later review of the literature related to supply network management was prepared by Georgi et al. [11] and Scheuermann and Leukel [12]. In their paper they have included an analysis of several international journals relating to supply chain research.

It may seem that the scheduling of technological operations is similar to the scheduling for processors working in parallel [13]. The same criteria for task scheduling are used very often – for example, the minimization of schedule length. The main difference, however, is connected with the configuration of the system. The method for scheduling described in this paper is constructed for multi-stage unidirectional production lines. Scheduling for a single stage can be compared to scheduling for processors working in parallel.

This paper addresses the issues presented above. The presented system for supporting management of the supply chain is used to find a compromise between the interests of individual companies and the common objective of operation of the supply network. It has been achieved with the modular structure and hierarchical approach to the problem of management of product flow by supply networks. In the developed method, optimization takes place in the scale of the entire supply chain (reduction in costs of operation of a supply network), as well as in the scale of its links. The schedules of product flow are built between the links of the network as well as through the particular links (production plants).

2. The description of the problem and the concept of its solution

The method has been developed for supply chains of network nature. A sample structure of a part of such a chain is given in Fig. 1. The following groups of links may be found in this chain: producers of components, producers of complex products, recipients of complex products. The plants, called producers of complex products, feature the possibility of simultaneous production of various types of products.

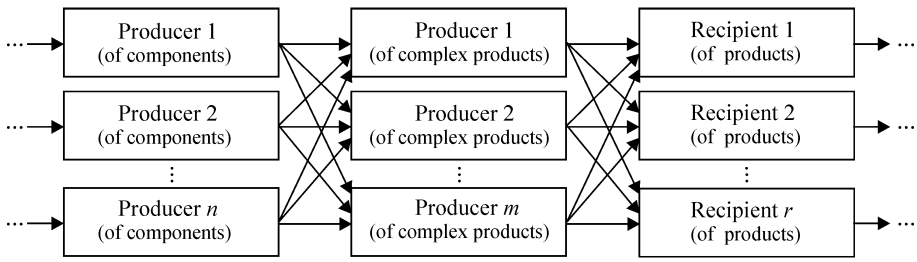

Fig. 1. An example of a fragment of a supply network structure

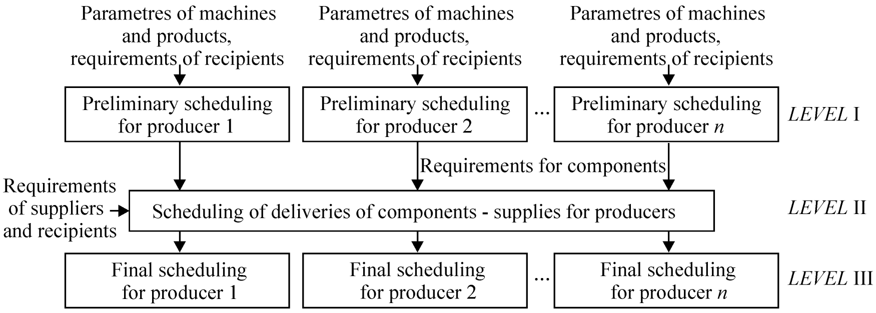

Fig. 2. Block diagram of the multilevel method

The method is used for building schedules of product flows through production lines of the plants (producers of complex products) and scheduling transport operations related to product flows between the links of the network. The block chart of this multi-level method is given in Fig. 2.

At the first level of the method, preliminary production schedules are developed for each plant. It requires taking into account the configuration of the machine resources (the flow system, the cell system, parallel machines, intermediate buffers), directions of product flow (one-way flow, return possibilities), process and time limitations for the production process (e.g. no-wait scheduling). Depending on the size of the problem, one of two approaches to production schedule development is used: one-level approach (monolithic) or multi-level approach (hierarchical). The multi-level approach which consist in breaking down the global problem into partial problems solved in succession, is used for tasks of significant size. The issues of the hierarchical approach to production planning was broadly studied, for example in [14].

The developed method proposes the application of a hierarchical approach to problem solving. The problem of operation scheduling, solved at the first level of the method, has been broken down into two problems solved in succession. First, the problem of equal loading of the stages is solved and then the operations are allocated over time. The block chart of this hierarchical approach is given in Fig. 3. Balancing stage workloads (and not balancing machine workloads) at level I-a gives the possibility of allocation of the operations to the machines only at level I-b. The advantage of this concept is the possibility of building shorter schedules as compared with the approach in which scheduling the operations is preceded with balancing machine workloads.

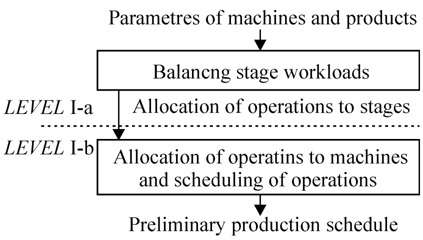

Fig. 3. Block diagram of hierarchical approach for the first level of the method

For each production plant (a module in the system), the method of building the production schedule is assigned, which takes into consideration the configuration of the system and the organization of the product flow. These do not need to be the solutions which are proposed in this paper. For systems in which breaks between the execution of the consecutive operations for the given product (the so-called no-wait scheduling) are forbidden, the methods presented, for example, in [15] and [16] may be applied. For machines configured in a work cell, the method described in [17] may be used. For tasks of relatively large size, using heuristics is recommended, as described, for example, in [18].

The mathematical models built specifically for the method are dedicated to the construction of production schedules for multi-stage flow systems. Every stage constitutes a set of machines working in parallel. Each product flowing through the system is loaded on one machine of the given stage at the most. Some stages may be omitted. An example of a multi-stage production line without intermediate buffers is given in Fig. 4.

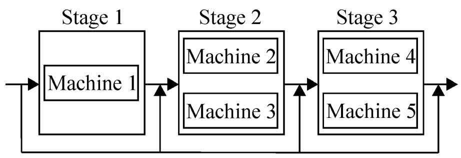

Fig. 4. Diagram of a multi-stage system without intermediate buffers

The mathematical models built specifically for the method refer to the configuration without the intermediate buffers. Two cases are analysed for the systems without intermediate buffers:

- the multi-stage production line with blocking of machines by products awaiting the execution of the next operation – the machines perform the role of buffers;
- the so-called “no-waiting” schedule is built: breaks between the execution of the consecutive tasks result only from transport time between the machines of different stages.

For the mathematical models, two types of production routes have been taken into account:

- a fixed production routes: each operation type is allocated to the machines of the same stage;
- an alternative production routes: each operation type is allocated to at least one machine. These machines may belong to different stages.

Table 1
Symbols for the mathematical models

Model	Level	Description of the mathematical models
M1	I-a	Balancing stage workloads for fixed routes
M2	I-a	Balancing stage workloads for alternative routes
M3	I-b	Preliminary operation scheduling for systems without buffers and with blocking of machines
M4	I-b	Preliminary operation scheduling for systems without buffers and no-waiting scheduling
M5	II	Scheduling tasks of deliveries of components
M6	III	Final scheduling for systems without buffers and with blocking of machines
M7	III	Final scheduling for systems without buffers and no-waiting scheduling

The differences between the defined types of routes may be visualised with the example of assembly lines. Assembly operations consist in (apart from the first operation) adding a component to the components previously assembled. One of the methods of delivering components is to collect them from feeders placed in the vicinity of particular machines, within the working space of the machine. With this configuration of the assembly line, allocation of the type of operation to a machine is related to simultaneous allocation of the pre-defined feeder. For fixed routes, the type of the operation is assigned to exactly one stage: all part feeders which are used in the operation are included in one and the same stage. Application of alternative routes allows to place identical feeders in a larger number of stages (the feeder must be placed in at least one stage). If products of various types have the same type of operation assigned, then operations of the given type may be executed in various stages: in the stages to which this type of operations is assigned (and, at the same time, part feeders which allow execution of these operations). Setting several feeders with the same components is thus allowed. Each one of the products which require collection of a part from such a feeder is supported only by one of these feeders. This means that exactly one route is set for each product irrespective of the route type.

The mathematical models constructed for systems without intermediate buffers are described in the next sections. The mathematical models built for systems with intermediate buffers are described in [18] (by the author of this paper).

The information about demand for components (in particular periods of time) is used at the second level of the method. The schedule of deliveries of components and semi-finished products is developed here. Solution of the problem of scheduling transport operations gives data on time of delivery of individual components for producers of complex products. These data are used at the third level of the method. At the lowest level, the production schedules are defined for all producers of complex products.

A detailed description of the levels of the method is provided in the further sections. The linear mathematical models of discrete optimization problems built for the individual levels of the method are presented there. Table 1 shows the symbols for the developed mathematical models.

3. Level I

At the first level of the method, preliminary production schedules are developed for each plant within the supply chain. The problem of task scheduling, solved at this level of the method, has been broken down into two problems solved in succession (Fig. 3). Table 2 gives a summary of indices, parameters and variables used in the mathematical models constructed for the first level.

One of the parameters described in Table 2 is the number of the time ranges H – the periods for which the information is known as regards availability of individual machines. Taking into consideration too high a number of the periods may result in a major increase in the size of the problem, which may result in a relatively long computational time or lack of the possibility of finding any solution to the problem due to limited possibilities of the discrete optimization packets. A low value of the parameter H may result in the inability to solve the problem when machines should be loaded for a longer time. The procedure for determination of the number of time ranges H is described in [18].

These are the mathematical models M1 and M2 built for the I-a level of the method:

Minimize: $$P_{\max}, \tag{1}$$
subject to:

$$\sum_{k\in K}\sum_{j\in J_k}\frac{p_{jk}z_{vjk}}{m_v}+\sum_{l\in L}\sum_{i\in I:\,(i,v)\in F}\frac{(1-\mu_{il})}{m_v}\le P_{\max},\quad v\in V, \tag{2}$$

$$\sum_{v\in V_j} x_{vj}=1,\quad j\in J \quad \text{– for the M1 model only} \tag{3}$$

$$\sum_{v\in V_j} x_{vj}\ge 1,\quad j\in J \quad \text{– for the M2 model only} \tag{4}$$

$$\sum_{j\in J_c} a_{vj}x_{vj}\le b_v m_v,\quad v\in V, \tag{5}$$

$$z_{vjk}\le x_{vj},\quad v\in V,\quad j\in J_k,\quad k\in K, \tag{6}$$

$$\sum_{v\in V} z_{vjk}=1,\quad j\in J_k,\quad k\in K, \tag{7}$$

$$\sum_{v\in V} v\, z_{vjk}\le \sum_{v\in V} v\, z_{vrk},\quad k\in K,\quad (j,r)\in R_k, \tag{8}$$

$$x_{vj},\quad z_{vjk}\in\{0,1\},\quad v\in V,\quad j\in J,\quad k\in K. \tag{9}$$

Table 2
Summary of indices, parameters and variables used in the mathematical models: M1–M4

Indices:		
i	–	machine; $i \in I = \{1, \ldots, M\}$;
j	–	type of operation; $j \in J = \{1, \ldots, N\}$;
k	–	product; $k \in K = \{1, \ldots, W\}$;
l	–	time interval (period); $l \in L = \{1, \ldots, H\}$;
v	–	stage; $v \in V = \{1, \ldots, A\}$.
Parameters:		
a_{vj}	–	working space of machine in stage v required for execution of operation j;
b_v	–	total working space of the machine placed in the stage v;
g_{vk}	–	transport time for product k from the machine in which last task has been completed to machine in the stage v, taking into account the spatial orientation of the product and fixing it in the grip;
m_v	–	number of the machines in stage v;
p_{jk}	–	processing time for operation j of product k;
μ_{il}	–	1, if machine i is available during period l, otherwise $\mu_{il} = 0$;
α	–	any integral number larger than the number of the analysed time intervals (periods);
F	–	the set of arranged pairs (i, v), such that the machine i belongs to the stage v;
I_j	–	the set of machines capable of performing operation j;
J_k	–	the set of operations required for product k, $J_k \subset J$;
J_c	–	the set of operations which require using the feeder for the components, $J_c \subset J$;
R_k	–	the set of pairs of operations (j, r) executed in succession for product k;
V_j	–	the set of the stages in which the machine are capable of execution of operation j;
λ	–	maximum break in the execution of the operation sequence for a single product;
Decision variables:		
• for the level I-a:		
$P_{\max}$	–	maximum stage workload;
x_{vj}	–	1, if operation j is assigned to stage v, otherwise $x_{vj} = 0$;
z_{vjk}	–	1, if operation j for product k is assigned to stage v, otherwise $z_{vjk} = 0$.
• for the level I-b:		
q_{ikl}	–	1, if during period l operation of product k is executed on machine i, otherwise $q_{ikl} = 0$;
w_{ikl}	–	1, if during period l machine i is loaded by product k, awaiting the execution of the next operation (the machine performs the role of the buffer), otherwise $w_{ikl} = 0$ – only for the M3 model.

The mathematical models M1 and M2 are designed to minimize the loading in the individual stages (1). These models are prepared for discrete optimization packages – unlike similar models [25] with constraints constructed for the heuristic method. The constraints built for the M1and M2 models guarantee: (2) – determination of the loading of the most loaded stage with limited availability of the machines taken into account – time intervals of machine inaccessibility are taken into consideration in the determination of stage load: these machines are regarded in these time intervals as if they were loaded (second summation term in the left-hand of (2)); (3) – ensuring fixed production routes – the task of a given type must be executed in the same stage (for the M1 model only); (4) –ensuring alternative production routes – the tasks of a given type may be executed in different stages (for the M2 model only); (5) – taking into consideration limited working space of the individual stations; (6) – the allocation of the operations assigned to the particular product to these stages which are assigned (according to (3), (4)) the possibility of execution of the operation of this type; (7) – distribution of all the operations (assigned to the products) between the stages; (8) – maintaining the order of execution of the operations according to the given plans (sequences) with a unidirectional product flow; (9) – binary of all the decision-making variables.

The results of the problem solved at the I-a level constitute the input data for the problem of operations scheduling solved at the I-b level. These data include parameters t_{vk} – time of loading stage v by product k, determined according to the equation (10):

$$t_{vk} = \sum_{j \in J_k} p_{jk} z_{vjk}, \quad v \in V, \quad k \in K. \tag{10}$$

These are the mathematical models M3 and M4 for scheduling of operations, built for the I-b level of the method:

Minimize:
$$\sum_{i \in I} \sum_{k \in K} \sum_{l \in L} l q_{ikl}, \tag{11}$$

subject to:

$$\sum_{i \in I:\, (i,v) \in F} \sum_{l \in L} q_{ikl} = t_{vk}, \qquad v \in V, \qquad k \in K, \tag{12}$$

$$\sum_{k \in K} q_{ikl} \le \mu_{il}, \qquad i \in I, \qquad l \in L, \tag{13}$$

$$\begin{gathered} l q_{ikl} - f q_{ikf} \le t_{vk} - 1 + \alpha \left(1 - q_{ikf}\right), \\ (i,v) \in F, \qquad k \in K, \qquad l, f \in L, \qquad l > f, \end{gathered} \tag{14}$$

$$\sum_{i\in I:\,(i,v)\in F}\sum_{l\in L}\frac{lq_{ikl}}{t_{vk}}-\sum_{\tau\in I:(\tau,\varepsilon)\in F}\sum_{l\in L}\frac{lq_{\tau kl}}{t_{\varepsilon k}} -\frac{t_{vk}+t_{\varepsilon k}}{2}\geq g_{vk}, \quad k\in K,\quad v,\varepsilon\in V,\quad t_{vk},\quad t_{\varepsilon k}>0, \tag{15}$$

$$q_{ikl}+q_{\tau kf}\leq 1, \quad k\in K,\quad (\tau,v),(i,v)\in F,\quad i\neq\tau, \tag{16}$$

$$q_{ikl}\in\{0,\,1\},\quad i\in I,\quad k\in K,\quad l\in L. \tag{17}$$

The constraints formulated for the M3 model (with blocking of machines only):

$$w_{ikl}\leq\sum_{f\in L}q_{ikf},\quad i\in I,\quad k\in K,\quad l\in L, \tag{18}$$

$$\sum_{i\in I:(i,v)\in F}\sum_{l\in L}\frac{lq_{ikl}}{t_{vk}}-\sum_{\tau\in I:(\tau,\varepsilon)\in F}\sum_{l\in L}\frac{lq_{\tau kl}}{t_{\varepsilon k}} -\frac{t_{vk}+t_{\varepsilon k}}{2}-g_{vk}=\sum_{\tau\in I:(\tau,\varepsilon)\in F}\sum_{l\in L}w_{\tau kl}, \quad k\in K,\quad v\in V\backslash\{1\},\quad \varepsilon\in V,\quad \varepsilon<v, \quad t_{vk},t_{\varepsilon k}>0,\quad \sum_{\psi=\varepsilon}^{v}t_{\psi k}=t_{\varepsilon k}+t_{vk}, \tag{19}$$

$$lw_{\tau kl}+\alpha\left(1-y_{\tau kl}\right) \geq\sum_{v\in I:(v,\varepsilon)\in F}\sum_{f\in L}\frac{fq_{vkf}}{t_{\varepsilon k}}+\frac{t_{\varepsilon k}+1}{2}, \quad (\tau,\varepsilon)\in F,\quad k\in K,\quad t_{\varepsilon k}>0, \quad \sum_{\rho\in V:\varepsilon\leq\rho}t_{\rho k}>t_{\varepsilon k},\quad l\in L, \tag{20}$$

$$lw_{\tau kl}\leq\sum_{i\in I:(i,v)\in F}\sum_{f\in L}\frac{fq_{ikf}}{t_{vk}}-\frac{t_{vk}+1}{2} -g_{vk}+\alpha\left(1-w_{\tau kl}\right),\quad (\tau,\varepsilon)\in F, \quad v\in V,\quad v>\varepsilon,\quad k\in K,\quad t_{vk},t_{\varepsilon k}>0, \quad \sum_{\rho\in V:\varepsilon\leq\rho\leq v}t_{\rho k}=t_{\varepsilon k}+t_{vk},\quad l\in L, \tag{21}$$

$$q_{ikl}+w_{ikl}\leq 1,\quad i\in I,\quad k\in K,\quad l\in L, \tag{22}$$

$$w_{ikl}\in\{0,\,1\},\quad i\in I,\quad k\in K,\quad l\in L. \tag{23}$$

Moreover, to the M3 model, the constraint (24) may be added, whereas for the model M4, $\lambda=0$ should be assumed: the maximum break in the execution of the operations

$$l\sum_{i\in I:\,(i,v)\in F}q_{ikl}-f\sum_{\tau\in I:\,(\tau,\varepsilon)\in F}q_{\tau kf}\leq g_{vk} +t_{vk}+t_{\varepsilon k}-1+\alpha\left(1-\sum_{\tau\in I:\,(\tau,\varepsilon)\in F}q_{\tau kf}\right)+\lambda, \quad \varepsilon,v\in V,\quad v>\varepsilon,\quad k\in K,\quad t_{\varepsilon k},t_{vk}>0, \quad \sum_{\rho\in V:\varepsilon\leq\rho\leq v}t_{\rho k}=t_{\varepsilon k}+t_{vk},\quad l,f\in L. \tag{24}$$

Minimization of the schedule length is approximated using the sum (11) – for the mathematical models M3 and M4. Minimization of this sum ensures also relatively short times of completion of production of each product. The specific constraints built for the mathematical models M3 and M4 guarantee: (12) – distribution of all the operations between the machines and their execution in the given time; (13) – ensuring the execution of at the most one operation on the machine at a given time if this machine is made available for the execution of the operation in the analysed period; (14) – ensuring indivisibility of the execution of the operations; (15) – maintaining the order of the execution of the tasks in the unidirectional flow system, taking into account the transport time of the products between the machines from different stages.

The description of consecutive constraints is preceded with the following explanations related to building mathematical relations.

The constraints formulated for the mathematical models are presented after reduction of similar mathematical expressions. For example, the constraint (15) was constructed as follows (maintaining the order of the execution of the tasks in the unidirectional flow system):

$$\underbrace{\sum_{i\in I:\,(i,v)\in F}\sum_{l\in L}\frac{lq_{ikl}}{t_{vk}}-\frac{t_{vk}}{2}+\left(\frac{1}{2}\right)}_{\text{the start time of the next operation (in the stage } v\text{) for the product } k} -\underbrace{\left(\sum_{\tau\in I:(\tau,\varepsilon)\in F}\sum_{l\in L}\frac{lq_{\tau kl}}{t_{\varepsilon k}}+\frac{t_{\varepsilon k}}{2}-\frac{1}{2}\right)}_{\text{the end time of the previos operation (in the stage } \varepsilon\text{) for the product } k}-1\geq\underbrace{g_{vk}}_{\text{the transport time}},$$

$$k\in K,\quad \varepsilon,v\in V,\quad t_{\varepsilon k},t_{vk}>0.$$

The above constraint describes the flow of products $k\in K$ between machines within the stages ε and v. The time known for the execution of operations for the product k in the particular stages: t_{vk}, $t_{\varepsilon k}$ has been used to determine the time for starting the next operation for the product k (in the stage v) and the end time of the execution of the operation for the product k (in the stage ε). The difference in these times is to ensure not only the proper order of execution of the operations for the given product k (stage load v precedes execution of the operations in the stage ε), but also to guarantee the time for transporting the product between the machines in different stages; the parameter g_{vk} is used for this purpose.

The next constraints built for the mathematical models M3 and M4 guarantee: (16) – ensuring loading of one machine at the most by the product flowing through the given stage; (17) – binary of the decision-making variables.

The next group of constraints is formulated for the systems without intermediate buffers and with the possibility of blocking of machines by products (for the M3 model only).

The constraints built for this production systems guarantee: (18) – ensuring the possibility of blocking by the product of only these machines to which the relevant tasks have been assigned; (19) – determining the time of blocking the machine by the product awaiting the execution of the following tasks: the product is waiting in the machine in which the last task has been completed – as with the described constraint (15), the start time for the next operation for the product k (t_{vk}) is included in the stage v along with the time of completion of the previous operation in the stage ε ($t_{\varepsilon k}$), where $\varepsilon < v$ – after transporting the product to the stage v (with the transport time g_{vk} being taken into account), the product remains in the buffer (within the stage v) from the time of its delivery to the buffer until the time of beginning the first operation in the stage v; to verify whether the product directly loads two successive stages (ε, v), the sum $t_{\varepsilon k} + t_{vk}$ is determined and compared with the sum $t_{\psi k}$ defined in the constraint (19), and equality of these sums means direct flow of the product from the stage ε to the stage v; (20) and (21) – determining the time ranges in which the machine performs the role of the buffer (more specifically: (20) – the blocking of machine directly after the operation – the machine τ included in the stage ε is blocked by the product k after completion of execution of the process operations until the time when it is transported to the machine of the next stage. This constraint was constructed as follows:

$$\underbrace{lw_{\tau kl}}_{\substack{\text{the machine } \tau \\ \text{in the stage } \varepsilon \text{ is blocking} \\ \text{in the period } l}} + \underbrace{\alpha\left(1 - y_{\tau kl}\right)}_{\substack{\text{or the machine } \tau \text{ isn't} \\ \text{blocking in the period } l \\ \text{by the product } k}}$$

$$\geq \underbrace{\sum_{v \in I:(v,\varepsilon)\in F} \sum_{f \in L} \frac{f q_{vkf}}{t_{\varepsilon k}} + \frac{t_{\varepsilon k}+1}{2}}_{\substack{\text{the end time of the last opeation} \\ \text{in the stage } \varepsilon \text{ for the product } k}},$$

$$(\tau, \varepsilon) \in F, \quad k \in K, \quad t_{\varepsilon k} > 0,$$

$$\underbrace{\sum_{\rho \in V:\varepsilon \leq \rho} t_{\rho k} > t_{\varepsilon k}}_{\substack{\text{for the fow of the product } k \\ \text{by the stage } v \text{ and the next}}}, \quad l \in L.$$

The next constraints guarantee: (21) – blocking of the machine precedes the transport of the product to the next stage – the range of time was determined in the previous constraint (20) in which the machine performs the role of the buffer, whereas the constraint (21) is used to determine the end of the buffer role by the machine τ included in the stage ε – thus, on the right side of the inequality (21) the start time for execution of the operation in the next stage v and duration of the transport of the product to this stage is taken into account: blocking of the machine in the stage ε is terminated directly before the execution of the stated operations; (22) – elimination of the machine performing the role of the buffer during the execution of the production tasks; (23) – binary of the decision variables which determine the individual machines performing the role of the buffer.

The last constraint (24) is formulated to take account of the maximum break in the execution of the operation sequence for a single product. In the case of no-waiting scheduling (the M4 model), waiting for these breaks is not allowed, the parameter $\lambda = 0$ (Table 2). The constraint (24) includes mathematical relations between the time ranges f, $l \in L$, in which the machine is loaded, used for the execution of the operations for the products which flow through two stages. When the product first loads the machine in the stage ε (the machine τ) and then in the stage v (the machine i) (that is when the equality: defined in the final part of the constraint (24) the sum $t_{\rho k} = t_{\varepsilon k} + t_{vk}$) the particular time ranges between execution of the consecutive operations in the machines are booked for execution of transport activities and the allowed a break in the execution of the operations, and the duration of this break cannot exceed λ.

For the schedules built on the basis of described mathematical models, the length of the schedule may be determined according to (25)

$$C_{\max} = \max_{i \in I, k \in K, l \in L} \left(l q_{ikl}\right). \tag{25}$$

4. Level II

Knowledge of the demand for components, resulting from the preliminary production plant built at the first level of the method, is taken into account in the construction of the schedule for component deliveries. The preliminary production schedules which are built for the particular supply chain links are known.

For this purpose, the problem has been formulated for the organization of a product flow between two groups of links in the network: component producers (i.e. suppliers) and recipients of these components (producers of complex products). The symbols used in the mathematical model formulated for this problem are given in Table 3. This table contains the information obtained from the solution of the problem formulated for the first level. This includes the elements of the set P. After determination of $C_{\max}$ for the given configuration of the machines on the basis of (25), sets may be built composed of triples (i, k, l), marked P (in which producer i has products k available for transport during period l). The construction of the set T, which includes the triples (i, j, l), takes into account not only the schedules determined at the first level, but also the information about availability of transportation modes.

The M5 model, formulated for the second level of the method:
Minimize:

$$\begin{aligned} &\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} c^1_{ik} x_{ijkl} - \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} b_{ijk} y_{ijk} \\ &+ \sum_{(i,j,l)\in T} c^2_{ijl} z_{ijl} + \sum_{(j,k)\in A} \sum_{l \in L} \left(c^3_{jk} q_{jkl} + c^4_{jk} w_{jkl}\right). \end{aligned} \tag{26}$$

Table 3
Summary of indices, parameters and variables used in the M5 mathematical model

Indices:		
i	–	supplier; $i \in I$;
j	–	recipient; $j \in J$;
k	–	product (part of component); $k \in K$;
l	–	time interval (period); $l \in L$.
Parameters:		
a_{ijk}	–	minimum number of products k, sold by supplier i to recipient j, which grants rights to discount;
b_{ijk}	–	amount of the discount granted to recipient j by supplier i due to one-time sale of products k in the number of at least a_{ijk};
c^1_{ik}	–	price of product k, sold by supplier i (without discount);
c^2_{ijl}	–	cost of one-time use of a transportation mode between supplier i and recipient j (without discount) executed within the time l;
c^3_{jk}	–	penalty for each period of delay in delivery of product k to supplier j;
c^4_{jk}	–	cost of storing the prematurely provided product k with supplier j, incurred within the unit of time;
m^1_k	–	weight of product k along with its package;
m^2_s	–	load capacity of transportation mode for transport during period$l$;
p_{jkl}	–	volume of demand in plant j for the products k during the period l;
s_{ikl}	–	number of products k available in period l from supplier i (supply);
v^1_k	–	weight of product k along with its package;
v^2_l	–	load capacity of transportation mode for transport during period$l$;
A	–	the set of pairs (j, k), in which recipient j has demand for products k;
K	–	the set of pairs (i, k), in which supplier i produces products k;
P	–	the set of three elements (i, k, l), in which producer i has products k available for transport during period l;
R	–	the set of three elements (j, k, l), in which recipient j has demand for products k in period l;
T	–	the set of three elements (i, j, l), where transport of products between supplier i and recipient j is possible during the period l;
U	–	the set of three elements (i, j, k), in which producer i, supplying products k to supplier j, applies discounts related to ordering the appropriate volume of these products k;
α	–	any integral number larger than the number of all units of the considered type products k.
Variables:		
q_{jkl}	–	shortage of products k with supplier j during period l;
w_{jkl}	–	number of products k in surplus with supplier j during period l;
x_{ijkl}	–	number of units of the products k transported during period l between supplier i and recipient j;
y_{ijk}	–	1, if the number of products k ordered for one transport between supplier i and recipient j is at least a_{ijk}, otherwise $y_{ijk} = 0$;
z_{ijl}	–	1, if during period l transport is executed between supplier i and recipient j, otherwise $z_{ijl} = 0$.

Subject to:

$$\sum_{i\in I}\sum_{l\in L:(i,j,l)\in T} x_{ijkl} = \sum_{l\in L:(j,k,l)\in R} p_{jkl}, \quad j\in J, \quad k\in K, \tag{27}$$

$$\sum_{j\in J} x_{ijkl} \le \sum_{\tau\in L:\,\tau\le l\wedge(i,k,\tau)\in P} s_{ik\tau}, \quad i\in I, \quad k\in K, \quad l\in L, \tag{28}$$

$$\sum_{l\in L:\,(j,k,l)\in R} x_{ijkl} \ge a_{ijk}y_{ijk}, \qquad (i,j,k)\in U, \tag{29}$$

$$\sum_{k\in K} x_{ijkl} \le \alpha z_{ijl}, \qquad (i,j,l)\in T, \tag{30}$$

$$\sum_{k\in K:(i,k,l)\in P} v^1_k x_{ijkl} \le v^2_l, \qquad (i,j,l)\in T, \tag{31}$$

$$\sum_{k\in K:(i,k,l)\in P} m^1_k x_{ijkl} \le m^2_l, \qquad (i,j,l)\in T, \tag{32}$$

$$\sum_{\tau\in L:\tau\le l}\sum_{i\in I} x_{ijk\tau} - \sum_{\tau\in l:\tau\le l\wedge(j,k,\tau)\in R} p_{jk\tau} \le w_{jkl}, \quad j\in J, \quad k\in K, \quad l\in L, \tag{33}$$

$$\sum_{\tau\in L:\tau\le l\wedge(j,k,\tau)\in R} p_{jk\tau} - \sum_{\tau\in l:\tau\le l}\sum_{i\in I} x_{ijk\tau} \le q_{jkl}, \quad j\in J, \quad k\in K, \quad l\in L, \tag{34}$$

$$q_{jkl}, w_{jkl}, x_{ijkl} \ge 0, \ \text{integer}, \quad i\in I, \quad j\in J, \quad k\in K, \quad l\in L, \tag{35}$$

$$y_{ijk}, z_{ijl} \in \{0, 1\}, \quad i\in I, \quad j\in J, \quad k\in K, \quad l\in L. \tag{36}$$

The minimized sum (26) represents the costs of: purchase of products (including discounts for purchases of a pre-defined number of pieces of these products), transport, related to delayed delivery of the products – costs of storing and penalties for any contractual period of delay in the delivery of the product [19].

The constraints built for the M5 mathematical model ensure: (27) – delivery of the required number of particular products to every single recipient; (28) – including availability of particular products in the specific periods with the given suppliers; (29) – granting discounts for ordering the relevant number of pieces of the products; (30) – determining demand for the use of transportation mode between the suppliers and the recipients in specific periods of time; (31) – verification of the loading space for each transportation mode; (32) – verification of the load capacity for transportation mode; (33) – determining surpluses of products with the recipients in particular periods of time; (34) – determining shortages of components with the recipients in particular periods of time; (35) and (36) – the relevant types of variables.

5. Level III

Solution of the problem of scheduling transport operations at the second level of the method provides data on time of delivery of individual components for producers of complex products. The detailed schedule of production for each producer of complex products developed at the third level of the method takes into account these data. On the basis of the known times of deliveries of components, the value of the parameter r_k may be known: the time of readiness for execution of the operations for the product k. This parameter is taken into account in developing production schedules, separately for each plant.

In the mathematical models for solving the problems formulated for the third level of the method, the mathematical relationships have been employed which are included in the mathematical models built for the level I-b. The M6 model (for the systems without the intermediate buffers and with the possibility of blocking of machines) includes all mathematical relationships assigned to the M3 model and, additionally, the constraint (37). The M7 model (for the systems without buffers and no-waiting scheduling) includes the function of the objective and the conditions formulated for the M4 model and the constraint (37). The condition (37) ensures execution of the operations for the product k only when appropriate semi-finished products are delivered. The parameters and variables taken into account in this condition are consistent with the list of markings in Table 2.

$$lq_{ikl} \geq r_k, \qquad i \in I, \qquad k \in K, \qquad l \in L. \qquad (37)$$

The lengths of schedules for individual plants (the links in the network) may be determined similarly to the situation with building preliminary schedules at the first level of the method, according to the relationship (25).

6. Computational experiments

The presented mathematical models have been verified by means of computational experiments. One of the experiments, limited to flows through production plants, is described at the beginning of this section. The assumption has been adopted that all components of products are delivered within the agreed time limit, that is the value of the parameter $r_k = 0$ for $k \in K$, determined according to (37). The results of other ones are described in the final part of the section.

The following example is used not only to show the scheduling for unidirectional production lines (links of supply chains), but also to visualise the differences between the systems with intermediate buffers and without buffers (different organization of the configuration of the machine park) and the differences between fixed and alternative production routes (different organization of product flow).

Three production plants which are producers of complex products are given. Production in Plant no 1 is based around production lines with intermediate buffers. Production lines in Plant no 2 are not fitted with buffers, and there is a possibility of blocking of machines by products awaiting the next operations. The configuration of production lines in Plant no 3 does not include buffers either, and the breaks between process operations are planned for transport. Each one of these plants is fitted with a 3-stage production line. The configuration of machines for these production lines is given in Fig. 4.

A set of stages has thus the form of: $V = \{v_1, v_2, v_3\}$. Production machines $i \in I = \{m_1, m_2, m_3, m_4, m_5\}$ are spaced in the stages. Their belonging to the stages is known due to the set of lines F, defined in Table 2. This set comes in the form: $F = \{(m_1, v_1),\ (m_2, v_2),\ (m_3, v_2),\ (m_4, v_3),\ (m_5, v_3)\}$. There are intermediate buffers between the stages, with identical capacities: $d_2 = d_3 = 3$ – for Plant no 1 only. Tasks have to be performed for 5 different types of products $k \in K = \{k_1, k_2, k_3, k_4, k_5\}$. Tasks of the type $j \in J = \{o_1, o_2, o_3, o_4, o_5, o_6\}$ are assigned to the products. The assignment of the tasks to specific products and the sequences for the execution of these tasks are known due to the following graphs:

for the product k_1: $L \to o_4 \to o_3 \to o_1 \to o_2 \to U$
for the product k_2: $L \to o_5 \to o_4 \to o_2 \to o_6 \to o_1 \to U$
for the product k_3: $L \to o_5 \to o_3 \to o_2 \to o_1 \to U$
for the product k_4: $L \to o_3 \to o_4 \to o_2 \to o_6 \to U$
for the product k_5: $L \to o_5 \to o_2 \to o_1 \to o_6 \to U$,
where L/U denotes loading/unloading operations.

The parameters of the products and of the machines are given in the matrices form: $[p_{jk}]$ – the times for the execution of operation j, $[a_{vj}]$, $[b_v]$ – connected with working space, described in Table 2

$$[p_{jk}] = \begin{bmatrix} 1 & 2 & 2 & 0 & 2 \\ 1 & 2 & 2 & 2 & 2 \\ 2 & 0 & 1 & 2 & 0 \\ 2 & 2 & 0 & 2 & 0 \\ 0 & 3 & 2 & 0 & 3 \\ 0 & 3 & 0 & 3 & 2 \end{bmatrix},$$

$$[a_{vj}] = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \end{bmatrix},$$

$$[b_v] = \begin{bmatrix} 8 \\ 8 \\ 6 \end{bmatrix}.$$

Table 4
Comparison of the solutions for the level I-a – for the fixed routes and for the alternative routes and the parameters: t_{vk}, g_{vk}

The fixed routes (the M1 model)

Stage	Operation assignments	Product assignments k_1	k_2	k_3	k_4	k_5	Workload
Stage v_1	o_5		$o_5(3)$	$o_5(2)$		$o_5(3)$	8 (1 machine)
		$t_{11}=0$	$t_{12}=3$	$t_{13}=2$	$t_{14}=0$	$t_{15}=36$	
Stage v_2	o_3	$o_3(2)$		$o_3(1)$	$o_3(2)$		11 (2 machines)
	o_4	$o_4(2)$	$o_4(2)$		$o_4(2)$		
		$t_{21}=4$	$t_{22}=2$	$t_{23}=1$	$t_{24}=4$	$t_{23}=0$	
Stage v_3	o_1	$o_1(1)$	$o_1(2)$	$o_1(2)$		$o_1(2)$	24 (2 machines)
	o_2	$o_2(1)$	$o_2(2)$	$o_2(2)$	$o_2(2)$	$o_2(2)$	
	o_6		$o_6(3)$		$o_6(3)$	$o_6(2)$	
		$t_{31}=2$	$t_{32}=7$	$t_{33}=4$	$t_{34}=5$	$t_{35}=6$	

$$[t_{vk}] = \begin{bmatrix} 0 & 3 & 2 & 0 & 3 \\ 4 & 2 & 1 & 4 & 0 \\ 2 & 7 & 4 & 5 & 6 \end{bmatrix}, \quad [g_{vk}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The alternative routes (the M2 model)

Stage	Operation assignments	Product assignments k_1	k_2	k_3	k_4	k_5	Workload
Stage v_1	o_3	$o_3(2)$			$o_3(2)$		8 (1 machine)
	o_4	$o_4(2)$			$o_4(2)$		
		$t_{11}=4$	$t_{12}=0$	$t_{13}=0$	$t_{14}=4$	$t_{15}=0$	
Stage v_2	o_1	$o_1(1)$				$o_1(2)$	16 (2 machines)
	o_2				$o_2(2)$	$o_2(2)$	
	o_3			$o_3(1)$			
	o_5		$o_5(3)$	$o_5(2)$		$o_5(3)$	
		$t_{21}=1$	$t_{22}=3$	$t_{23}=3$	$t_{24}=2$	$t_{25}=7$	
Stage v_3	o_1		$o_1(2)$	$o_1(2)$			19 (2 machines)
	o_2	$o_2(1)$	$o_2(2)$	$o_2(2)$			
	o_4		$o_4(2)$				
	o_6		$o_6(3)$		$o_6(3)$	$o_6(2)$	
		$t_{31}=1$	$t_{32}=9$	$t_{33}=4$	$t_{34}=3$	$t_{35}=2$	

$$[t_{vk}] = \begin{bmatrix} 4 & 0 & 0 & 4 & 0 \\ 1 & 3 & 3 & 2 & 7 \\ 1 & 9 & 4 & 3 & 2 \end{bmatrix}, \quad [g_{vk}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Different times for the execution of the operations of the same type for different products result from the fact that the times which are related to the preparation of the product for the execution of the operation, e.g. the time necessary for the orientation of the product, are added here. $p_{jk} = 0$ means that no j type operations are executed for the product k. $a_{vj} = 0$ means that the operation j does not require the allocation of the part feeder. The set of operations which require using the feeder for the components $J_c = \{o_1, o_2, o_3, o_4, o_6\}$.

The technical possibilities of the individual machines assigned to the given stages are known from the sets of the stages V_j (Table 2), to which the machines capable of the execution of the j type operations belong: $V_{o1} = \{v_1, v_3\}$; $V_{o2} = \{v_2, v_3\}$; $V_{o3} = \{v_2\}$; $V_{o4} = \{v_1, v_2, v_3\}$; $V_{o5} = \{v_1, v_2\}$; $V_{o6} = \{v_3\}$.

The availability of machines is not limited in this example. Each one of the production plants receives orders for production of the presented five types. Fixed and alternative production routes should be taken into account in the construction of production schedules. Schedules should be built for individual production lines assigned to individual plants. These schedules should be as short as possible.

The task of load-balancing for stages is solved in the first stage. The M1 model is used for fixed production routes, with the M2 model applied for alternative routes. In order to solve the task, the linear mathematical dependencies have been encoded in the AMPL mathematical programming language [20]. The GNU Linear Programming Kit (GLPK) software has been used for the computational experiments.

The determined allocation types of operations and products to the individual machines are presented in Table 4. The input data for the task of operation scheduling, solved in another level, are the solution of the task formulated at level I-a. The lower part of the table gives load times for the individual stages t_{vk}, which constitute sums of duration of operations assigned to the specific stages given in brackets. This table presents also other data for the operation scheduling task. These include g_{vk} times defined in Table 2.

The results presented in the lower part of Table 4 are later used as the data to solve the operation scheduling task. In the case of production lines with intermediate buffers (Plant no 1), the mathematical model described in [18], which ensures not only minimization of scheduling but also the lowest possible use of intermediate buffers, has been employed. For the production lines without buffers, where machines may be blocked, the M6 model has been used. The M7 model has been used for no-waiting scheduling. The schedules of simultaneous product flow through production lines characterising each of the plants, are given in Fig. 5. The solutions are summarised here for both types of production routes. Construction of these schedules was effected based on the value of the determined variables, mostly on the basis of the value of variables q_{ikl}, which specify the load of the machine i by the product k during the period l (defined in Table 2).

As we can see from Fig. 5, shorter schedules from the scheduling for fixed production routes have been determined for each configuration of a production line with alternative routes. This results from less varied machine loads, which is due to the allowed possibility of setting a larger number of part feeders as compared with a production line with fixed routes. In the case of production lines with fixed routes, the shortest schedule has been determined for a system with intermediate buffers (Fig. 5a). This computational experiment presents various possibilities of the simulation, which are used to present the relationship of the organization of a product flow with a determined length of the schedule. For example, for the assumed data and alternative production routes, schedules of the same length may be determined in the case of a system with buffers (Fig. 5b) as well as a system without buffers (Fig. 5d and Fig. 5f). Thus, the costs of using the buffers may be neglected, which is important in the case of configuration of production lines.

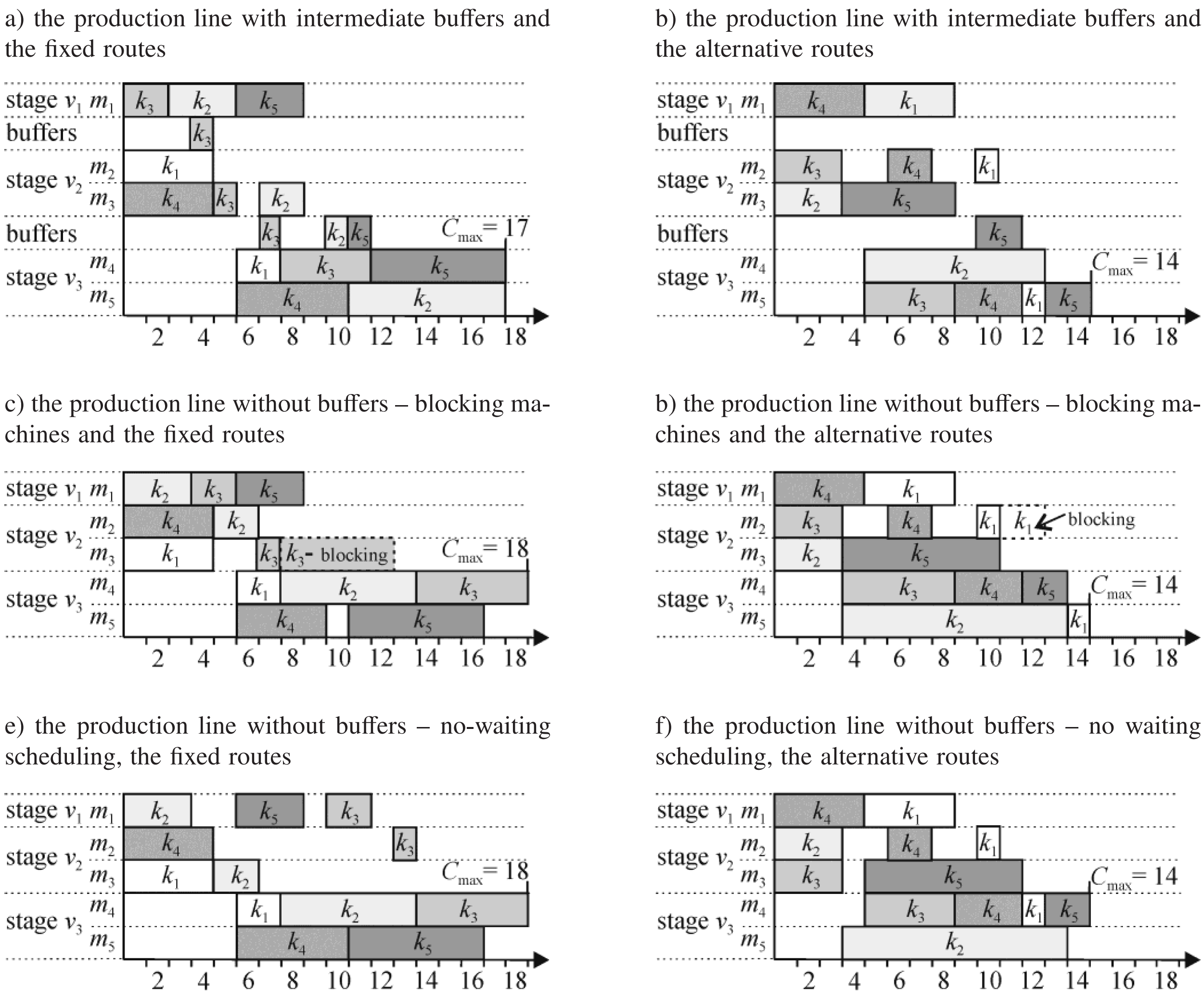

Fig. 5. Schedules for different types of production lines

However, generalised conclusions may be only drawn after completion of a larger number of computational experiments.The developed system supporting management of product flow through supply chains has been verified in computational experiments. All the mathematical models built for the system have been tested. In the data for the test samples, the number of suppliers (producers of components) was 3 to 6, whereas the number of recipients (producers of complex products) was 6 to 8 (producers of machines). Due to the modular nature of the system and the related diversity of production organization and various configurations of machine resources, individual modules of the system are assessed. All the developed mathematical models allow optimum solutions following the obtained criteria. A solution to a global problem may obviously deviate to some extent from the optimum due to the hierarchical nature of the presented method.

The computational experiments allowed not only testing the developed system, but also making some comparisons in examining the effect of system configuration and the types of routes on the incurred costs and the length of the production schedule. The experiments covered 5 groups of test problems (for the producers of parts of machines). For each of the groups, 30 examples were solved. The parameters of these groups and the results of the experiments are given in Table 5. The GUROBI optimizer was used [28].

Two configurations of the system were compared: with intermediate buffers and without them. For each of them, fixed and alternative production routes were taken into account. To assess the comparisons (Table 5), two indicators were used. One of them is the coefficient χ, which is used to compare the schedule length calculated according to (25), defined in the relationship (38). The other coefficient θ, described by Eq. (39) is designed for comparisons of costs incurred due to untimely delivery of products

$$\chi_{\mathrm{BM}} = \frac{C_{\max}^{\mathrm{BM}} - C_{\max}^{\mathrm{B}}}{C_{\max}^{\mathrm{B}}} \cdot 100\%,$$
$$\chi_{\mathrm{NW}} = \frac{C_{\max}^{\mathrm{NW}} - C_{\max}^{\mathrm{B}}}{C_{\max}^{\mathrm{B}}} \cdot 100\%, \tag{38}$$

where $C_{\max}^{\mathrm{B}}$, $C_{\max}^{\mathrm{BM}}$, $C_{\max}^{\mathrm{NW}}$ – the lengths of the schedules for the systems: with buffers (B), without buffers and with blocking of machines (BM) and no-waiting scheduling (NW)

Table 5
Parameters of groups of tasks and average values of results

Group	Parameters of groups of tasks					Average values of indexes [%]							
						For the fixed routes				For the alternative routes			
	A	M	W	K	B	χ_{BM}	χ_{NW}	θ_{BM}	θ_{NW}	χ_{BM}	χ_{NW}	θ_{BM}	θ_{NW}
1	2	4	8	5	3	9.4	10.6	6.5	8.3	8.2	8.9	5.4	7.0
2	2	6	12	7	3	8.8	9.7	6.1	7.9	7.9	8.8	5.3	6.8
3	3	6	14	9	6	8.8	9.1	5.3	7.6	7.6	8.4	5.0	6.5
4	8	16	10	9	8.4	8.9	5.4	7.5	7.1	7.8	4.6	6.2	
5	4	10	18	12	14	8.3	8.7	5.3	7.4	7.0	7.6	4.6	6.0

Numbers of: A – stages, M – machines, W – types of tasks, K – types of products, B – sum of capacity of intermediate buffers (only for systems with buffers).

$$\theta_{\mathrm{BM}} = \frac{\kappa^{\mathrm{BM}} - \kappa^{\mathrm{B}}}{\kappa^{\mathrm{B}}} \cdot 100\%,$$
$$\theta_{\mathrm{NW}} = \frac{\kappa^{\mathrm{NW}} - \kappa^{\mathrm{B}}}{\kappa^{\mathrm{B}}} \cdot 100\%, \tag{39}$$

where κ^{B}, κ^{BM}, κ^{NW} – the costs incurred due to untimely delivery of products, calculated for the systems: with buffers (B), without buffers and with blocking of machines (BM) and no-waiting scheduling (NW).

The solutions of the test problems showed the effect of system configuration and the type of routes on the length of the schedule and the incurred costs. Elimination of the intermediate buffers resulted in an increase in the schedule length by about 7 to 9.4% – for the systems without buffers and 7.6 to 10.6% – for no-waiting scheduling. The configuration without the intermediate buffers resulted in the increase of the costs by about 4.6 to 6.5% – for the systems with blocking of machines and 6.0 to 8.3% – for no-waiting scheduling. In the case of fixed production routes, the lengths of the schedules increased by about 7 to 12% as compared with the alternative routes.

The developed mathematical models allocated for scheduling have been compared with the known algorithms. They were compared with, among others, the heuristics: RITM (Route Idle Time Minimization), RITM-NS (Route Idle Minimization – No Store) [5]. The RITM heuristic algorithm is used for scheduling in multi-stage flow systems with operational buffers of limited capacity. Application of the mathematical model used for scheduling for systems with intermediate buffers allowed obtaining schedules shorter by about 4.5 to 7.2% than in the case of the RITM heuristics. The RITM-NS algorithm is a special case of the RITM heuristics, which is used for scheduling in systems without intermediate buffers, with the possibility of blocking the machines by the products awaiting the next operations allowed. As compared with this algorithm, the application of the developed model for the test examples allowed determination of schedules shorter by about 5.3 to 7.6%.

The solutions of the test examples were also compared with the solutions obtained based on the Johnson algorithm [22]. The algorithm adjusted to the systems with two stages was used for groups 1 and 2 of the test problems (Table 5). The deviation in schedule length did not exceed 2.9%. For the third group of the test problems, an algorithm adjusted to the systems with three stages was used. In this case, the deviation of the schedule length was not larger than 3.8%.

When comparing the mathematical models developed for the prepared system with the known algorithms, attention has to be paid to the advantages related to the possibilities of the modular system. In the presented mathematical models used for scheduling, the operations were isolated which requires the use of a components feeder (e.g. consisting in additional integration of components into the previously assembled components). Taking into consideration the limited availability of machine resources also helps to obtain a better reflection of the actual production processes in the mathematical models.

7. Conclusions

The advantage of the method at hand is an attempt at reconciling the interests of the whole supply network with the efforts of the individual production plants, which constitute the links in this network. The costs of operation of the supply network are minimized, along with minimization of the costs related to delayed execution of operations by each of production plants. Thus, each production plant strives to meet not only the quantity-related conditions, but also the time-related requirements.

The breakdown of the global problem into partial problems solved in succession allowed solution of tasks of a relatively significant size, described with a major number of indexes, parameters and variables used in mathematical relationships. The application of the hierarchical approach slightly extends the schedules for the individual production plants (depending on the input parameters).

Modularity in the support of the management of a supply chain of network nature creates the possibility of rescheduling. Updated production schedules may be built only for these plants for which a change in the scheduling of operations is required. Another benefit of the modular system structure comes in ensuring flexibility of the structure, process and action as defined in the introduction. This feature characterises the method being developed and offers two significant benefits. One of them is the possibility of solving problems of a large scale, which often cannot be solved based on a single-level approach (monolithic). Another significant benefit is the possibility of accounting for various configurations of machines and types of routes. Limited availability of machines may also be added as an advantage of this method.

The described mathematical models may obviously be modified and expanded. Changes may regard individual modules (plants). Modifications may concern the structure of the machine resources (production lines, work cells, part feeders, using parallel machines), organisation of product flow (one-way flow, return flow, the possibility of bypassing stages) or the optimization criterion. The reflection of the planned machine outages in the production schedules (repairs, maintenance, refitting) may be expanded with knowledge of unplanned outages, if the probability of this equipment being capable of undertaking production within the estimated period of their operational use is known. Modifications may include adding the task: selection of suppliers. This task may be preceded by the presented method. It is applicable to multi-criteria methods of selection of suppliers, for example the methods described in [19].

Application of discrete optimization allowed obtaining optimum solutions for the tasks solved at the individual levels of the method. However, it increased the computational time. Using the described mathematical relationships within a heuristic algorithm, e.g. a relaxation algorithm, would significantly shorten computational times for the individual problems, although at the expense of quality of the solutions. The observed development of computer technology and software favour the development of methods based on discrete optimization, such as the presented concept of supporting the management of product flow through supply chains. The discrete optimization packages feature higher computational power and are continuously developed and improved. This allows solving problems of increasing sizes and significant shortening of computational time.

The final comments are about the possible directions of further studies. More and more studies are now dedicated to simultaneous accounting for many aspects in the very quickly developing supply chain engineering, especially as regards designing the chains (selection of suppliers, selection of transportation modes) along with the simultaneous planning of production batch volume for selected network links [23]. Studies are also being

REFERENCES

[1] H. Stadtler, “Supply chain management and advanced planning – basics, overview and challenges”, *Eur. J. Operational Research* 163 (3), 575–588 (2005).

[2] F. Buttle, *Customer Relationship Management. Concepts and technologies*, Elsevier, Oxford, 2009.

[3] T. Sawik, *Scheduling in Supply Chains Using Mixed Integer Programming*, John Wiley & Sons, New Jersey, 2011.

[4] T. Sawik, “An integer programming approach to scheduling in a contaminated area”, *Omega: Int. J. Management Science* 38 (3–4), 179–191 (2010).

[5] T. Sawik, *Production Planning and Scheduling in Flexible Assembly Systems*, Springer-Verlag, Berlin, 1999.

[6] T. Sawik, “A mixed integer program for cyclic scheduling of flexible assembly lines”, *Bull. Pol. Ac.: Tech.* 62 (1), 121–128 (2014).

[7] H. Stadtler, C. Kilger, and H. Meyr, *Supply Chain Management and Advanced Planning, Concepts, Software an Case Studies*, Springer – Verlag, Berlin, 2015.

[8] M. SteadieSeifi, N.P. Dellaert, W. Nuijten, T. Van Woensel, and R. Raoufi. “Multimodal freight transportation planning: a literature review”, *Eur. J. Operational Research* 233 (1), 1–15 (2014).

[9] L. Bertazzi and M.G. Speranza, “Inventory routing problems: an introduction” *EURO J Transportation and Logistics* 1 (4), 307–326 (2012).

[10] J. Miemczyk, T.E. Johnsen, and M. Macquet, “Sustainable purchasing and supply management: a structured literature review of definitions and measures at the dyad, chain and network levels”, *Supply Chain Management: Int. J.* 17 (5), 478–496 (2012).

[11] Ch. Georgi, I.L. Darkow, and H. Kotzab, “Foundations of logistics and supply chain research: a bibliometric analysis of four international journals”, *Int. J. Logistics Research and Applications: a Leading J. Supply Chain Management* 16 (6), 522–533 (2013).

[12] A. Scheuermann and J. Leukel, “Task ontology for supply chain planning – a literature review”, *Int. J. Computer Integrated Manufacturing* 27 (8), 719–732 (2014).

[13] M.S. Barketau, M.Y. Kovalyov, J. Węglarz, and M. Machowiak, “Scheduling arbitrary number of malleable tasks on multiprocessor systems”, *Bull. Pol. Ac.: Tech.* 62 (2), 255–261 (2014).

[14] T.C. Miller, *Hierarchical Operations in Supply Chain Planning*, Springer, London, 2002.

[15] A. Che, M. Chabrol, M. Gurgand, and Y. Wang, “Scheduling multiple robots in no-wait re-entrant robotic flowshop”, *Int. J. Production Economics* 135 (1), 199–208 (2012).

[16] K.W. Pang, “A genetic algorithm based heuristic for two machine no-wait flow shop scheduling problems with class setup times that minimizes maximum lateness”, *Int. J. Production Economics* 141 (1), 127–136 (2013).

[17] T.C Chiang and H.J. Lin, “A simple and effective evolutionary algorithm for multiobjective flexible job shop scheduling”, *Int. J. Production Economics* 141 (1), 87–98 (2013).

[18] M. Magiera, “A relaxation heuristic for scheduling flowshops with intermediate buffers”, *Bull. Pol. Ac.: Tech.* 61 (4), 929–942 (2013).

[19] M. Magiera, “Methods of supplier selection for producers of electric and electronic equipment”, *Przegląd Elektrotechniczny* (*Electrical Review*) 89 (4), 66–70 (2013).

[20] R. Fourer, D. Gay, and B. Kernighan, *AMPL, A Modelling Language for Mathematical Programming*, Duxbury Press, Pacific Grove, 2003.

[21] http://www.gurobi.com (Gurobi Optimizer 6.0, January 2015).

[22] S.M. Johnson, “Optimal two- and three stage production schedules with setup times included”, *Naval Research Logistics Q.* 1, 61–68 (1954).

[23] A.H. Azadnia and M.Z.M. Saman. “Sustainable supplier selection and order lot-sizing an integrated multi-objective decision-making process”, *Int. J. Production Research* 53 (2), 383–408 (2015).

[24] S. Pashaei and J. Olhager. “Product architecture and supply chain design: a systematic review and research agenda”, *Supply Chain Management: Int. J.* 20 (1), 98–112 (2015).