Reduction of magnetic field from a power line using a passive loop conductor

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The study presents in a tutorial manner methods of the calculation of magnetic fields in vicinity of overhead electric power lines without and with mitigation loops. Exact and simplified methods of the determination of the magnetic field of a straight overhead conductor based on the Fourier transform technique are presented. The decomposition of the magnetic fields in two components: magnetic field obtained in free space from the Biot-Savart law and the magnetic field produced by earth current is discussed. It is shown that in practical cases the effects from earth currents can be neglected as compared with effects from line currents.

Moreover the mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located near the power line horizontal or non-parallel to the earth surface.

KEYWORDS: magnetic field, overhead power line, earth return, passive loop.

1. Introduction

Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Nowadays of special concern is the possibility of detrimental environmental effects arising from the electrical and magnetic fields formed adjacent to the overhead transmission lines. These fields may affect both operation of near electric and electronic devices and appliances and also various living organisms.

The topic of mitigating the magnetic fields produced by overhead power lines is gaining more significance in recent years. In this context, efforts are continuously being done in order to maximize the utilization of the available line corridors without exceeding the tolerable limits of the lines' magnetic fields.

There are several possibilities to mitigate the field from existing or new overhead lines, e.g.: increasing the height of conductors, phase rearrangement, compaction, splitting of phases, underground cables, gas-insulated lines, passive shields (ferromagnetic and conductive), etc.

During the last decades, the use of conductive shields (passive non-compensated or series capacitor compensated loops) to mitigate extremely low frequency magnetic fields generated from power lines has been proposed [1 – 15].

Their behavior principle is based on the electromagnetic induction law: time varying, primary magnetic fields, generated by AC sources, induce electromotive forces driving loops currents – additional field sources, which modify and reduce the primary magnetic field.

In the study exact and simplified methods of the determination of the magnetic field of a straight overhead conductor based on the Fourier transform technique are presented. It is shown that in practical cases the effects from earth currents can be neglected as compared with effects from line currents. Simplified magnetic field computation techniques assume that the current carrying power line conductors are straight horizontal wires of infinite length located in free space and the magnetic field under the power line can be obtained from the Biot-Savart law.

Moreover the mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located near the power line horizontal or non-parallel to the earth surface.

2. Magnetic field of a straight overhead conductor

2.1. Exact method (Fourier transform technique)

An infinitely long conductor is placed at height h_k above the earth surface, Fig. 1, and carries the current, which flows in direction of the x-axis. The current varies with the time as $\exp(j\omega t)$ where ω is the radian frequency. The x, y plane is considered to be the earth surface. It is assumed that the earth is an isotropic, homogeneous medium of finite conductivity γ. The magnetic permeability of the soil and of the air is μ_0. The displacement currents in both regions: the air and the earth are neglected.

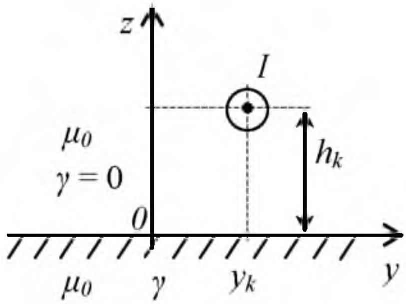

Fig. 1. An infinitely long current carrying conductor above the earth surface

The vector potential of the electromagnetic field has the x – component only denoted $A_x(y, z)$ which satisfies the following equations:

– the Poisson equation in the air:

$$\frac{\partial^2 A_x(y,z)}{\partial y^2}+\frac{\partial^2 A_x(y,z)}{\partial z^2}=-\mu_0 I\delta(y-y_k)\delta(z-h_k) \tag{1}$$

– the Helmholtz equation in the earth:

$$\frac{\partial^2 A_x(y,z)}{\partial y^2}+\frac{\partial^2 A_x(y,z)}{\partial z^2}=k^2 A_x(y,z) \tag{2}$$

where: δ– Dirac delta function, $k^2=j\omega\mu_0\gamma$.

The vector potential $\vec{A}$ in the air can be obtained if the Fourier transform is used and the boundary conditions in the system considered expressing the continuity of the normal component of the magnetic flux density and the tangential components of the magnetic as well as electric intensities are taken into account.

Hence the x – component of the vector potential in the air can be written in the form:

$$A_x(y,z)=\frac{\mu_0 I}{2\pi}\int_0^\infty\left[\frac{e^{-u|z-h_k|}}{u}+\frac{(u-\alpha)e^{-u(z+h_k)}}{u(u+\alpha)}\right]\cos[(y-y_k)u]du \tag{3}$$

where:

$$\alpha=\sqrt{u^2+k^2} \tag{4}$$

It should be noted that the formula (3) is the same that obtained by Carson [17].

The magnetic flux density $\vec{B}$ can be obtained from the equation:

$$\vec{B}=rot\vec{A} \tag{5}$$

In the case considered:

$$\vec{B}=\vec{e}_y\frac{\partial}{\partial z}A(y,z)-\vec{e}_z\frac{\partial}{\partial y}A(y,z) \tag{6}$$

where $\vec{e}_x,\vec{e}_z$ are the unit vectors in the direction x and z respectively.

From eqns (6) and (3) two components of the magnetic flux density in the air $(z\geq 0)$ become:

$$B_y(y,z)=-\frac{\mu_0 I}{2\pi}\int_0^\infty\left[e^{-u|z-h_k|}+\frac{(u-\alpha)e^{-u(z+h_k)}}{u+\alpha}\right]\cos[(y-y_k)u]du \tag{7}$$

$$B_z(y,z)=\frac{\mu_0 I}{2\pi}\int_0^\infty\left[e^{-u|z-h_k|}+\frac{(u-\alpha)e^{-u(z+h_k)}}{u+\alpha}\right]\sin[(y-y_k)u]du \tag{8}$$

Taking into account the formulas (3.893.1) and (3.893.2) in [18]:

$$\int_0^\infty e^{-px}\cos(qx)dx=\frac{p}{p^2+q^2} \tag{9}$$

$$\int_0^{\infty} e^{-px}\sin(qx)dx = \frac{q}{p^2+q^2} \tag{10}$$

each component of the magnetic flux density can be split in two terms:

$$B_y(y,z) = B_{yu}(y,z) + B_{ye}(y,z) \tag{11}$$

$$B_z(y,z) = B_{zu}(y,z) + B_{ze}(y,z) \tag{12}$$

The first term $B_{yu}(y, z)$ given by the relationship

$$B_{yu}(y,z) = -\frac{\mu_0 I}{2\pi}\frac{z-h_k}{(z-h_k)^2+(y-y_k)^2} \tag{13}$$

can be interpreted as the y – component of the magnetic flux density produced by current carrying infinitely long conductor in a free space environment (air), which can be directly obtained from the Biot-Savart law.

The second term $B_{ye}(y, z)$ denotes the y – component of the magnetic flux density produced in the air by currents flowing in the earth, which can be extracted from (7) and it can be written in algebraic form:

$$B_{ye}(y,z) = \frac{\mu_0 I}{2\pi}\left[\begin{array}{l} \dfrac{z+h_k}{(z+h_k)^2+(y-y_k)^2} + \\ -2|k|\int_0^{\infty} nbe^{-(z+h_k)|k|n}\cos[(y-y_k)|k|n]dn \\ -j2|k|\int_0^{\infty} n(n-a)e^{-(z+h_k)|k|n}\cos[(y-y_k)|k|n]dn \end{array}\right] \tag{14}$$

Similarly, in the relation (12), $B_{zu}(y, z)$ denotes the z – component of the magnetic flux density produced by current carrying infinitely long conductor in homogeneous medium (air):

$$B_{zu}(y,z) = \frac{\mu_0 I}{2\pi}\frac{y-y_k}{(z-h_k)^2+(y-y_k)^2} \tag{15}$$

Whereas $B_{ye}(y, z)$ denotes the z – component of the magnetic flux density produced in the air by currents flowing in the earth, which can be calculated from (8), and it can be written in algebraic form:

$$B_{ze}(y,z) = \frac{\mu_0 I}{2\pi}\left[\begin{array}{l} -\dfrac{(y-y_k)}{(z+h_k)^2+(y-y_k)^2} + \\ +2|k|\int_0^{\infty} nbe^{-(z+h_k)|k|n}\sin[(y-y_k)|k|n]dn + \\ +j2|k|\int_0^{\infty} n(n-a)e^{-(z+h_k)|k|n}\sin[(y-y_k)|k|n]dn \end{array}\right] \tag{16}$$

The formulas (14) and (16) are obtained from (7) and (8), if the relationships (17 – 20) are used:

$$u = |k| n \tag{17}$$

$$\sqrt{n^2 + j} = a + jb \tag{18}$$

where:

$$a = \sqrt{\frac{\sqrt{n^4 + 1} + n^2}{2}} \tag{19}$$

$$b = \sqrt{\frac{\sqrt{n^4 + 1} - n^2}{2}} \tag{20}$$

The magnitude of the magnetic flux density:

$$B(y,z) = \sqrt{\left(B_y(y,z)\right)^2 + \left(B_z(y,z)\right)^2} \tag{21}$$

Denoting:

$$B_u(y,z) = \sqrt{\left(B_{yu}(y,z)\right)^2 + \left(B_{zu}(y,z)\right)^2} \tag{22}$$

$$B_e(y,z) = \sqrt{\left(B_{ye}(y,z)\right)^2 + \left(B_{ze}(y,z)\right)^2} \tag{23}$$

the proportion of the magnitudes obtained from relationships (22) and (23) to the total magnitude of the magnetic flux density described by eqn. (21) can be evaluated. For the system shown in Fig.1 the calculations have been curried out for the unit current I = 1 A (y_k = 0) in the observation point y = 0, z = 0. Different values of the earth conductivity: γ = 10^{-1}; 10^{-2}; 10^{-3} S/m and heights of the conductor: h_k = 5,5; 10 m have been considered. The MathCad 2000 has been used. The results of the calculations are shown in the tables 1 and 2.

It follows from the results shown in the tables, that in practical cases the influence of the earth currents on the magnetic field under power lines is insignificant and can be neglected.

Table 1. Proportion of the magnitudes $B_u(0, 0)$ and $B_e(0, 0)$ in the magnitude of the total magnetic flux density $B(0, 0)$ and the ratio $B_e(0, 0)$ / $B_u(0, 0)$ on the ground level for h = 5,5 m

γ S/m	B	B_u	B_e	B_e / B_u
10^{-1}	100 %	98,40 %	2,17 %	0,022
10^{-2}	100 %	99,48 %	0,71 %	0,007
10^{-3}	100 %	99,83 %	0,23%	0,002

On the other hand the magnetic field produced at very large distances from the current carrying conductor can be affected by currents induced in the earth. Fig. 2 shows the ratio B_e/B_u (rms) as a function of the lateral distance y. It follows from the Figure that the earth currents are of importance only at such large distances that the field has already decreased in magnitude below typical ambient conditions.

Table 2. Proportion of the magnitudes $B_u(0, 0)$ and $B_e(0, 0)$ in the magnitude of the total magnetic flux density $B(0, 0)$ and the ratio $B_e(0, 0) / B_u(0, 0)$ on the ground level for h = 10 m

γ S/m	B	B_u	B_e	B_e / B_u
10^{-1}	100 %	97,16 %	3,78 %	0,039
10^{-2}	100 %	99,07 %	1,28 %	0,013
10^{-3}	100 %	99,70 %	0,41 %	0,004

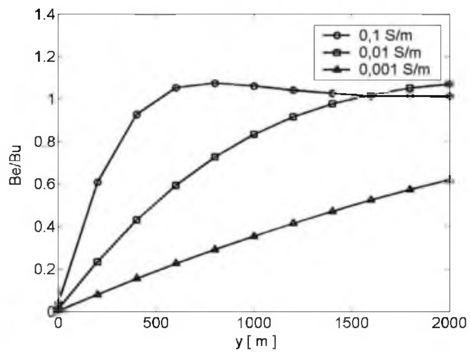

Fig. 2. The ratio $B_e(y,0)/B_u(y,0)$ on the ground level as a function of y for h_k = 10 m and for different values of earth conductivity

2.2. Complex ground return plane approach

As an alternative for the accurate method of the evaluation of the influence of currents induced in the earth on total magnetic field produced by an overhead power line the concept of the complex ground return plane can be applied [19 - 20]. The formal proof of the complex ground return plane approach for calculation of transmission line impedances has been presented in [19]. The formulas derived provide simple and remarkably accurate substitutes to Carson's equations for self and mutual impedances over the whole range of frequencies for which Carson's equations are valid.

It follows from the calculations presented in sec. 2.1 and in [16], that the effects of the currents induced in the earth on the total magnetic field produced under the power line are negligible as compared with the effects due to the currents flowing in the overhead conductors. If the radial distance from the conductor to the point in question is much less then the penetration depth, the Dubaton's formulas for B_y and B_z reduce to formulas (13) and (15) [16], what means that the magnetic field can be calculated using the Biot-Savart law only. Since typical values of δ_s at 50 (60) Hz are 350 – 1600 meters (for the earth resistivity lying in limits 50 – 1000 Ωm), the approximate formulas are valid for observation points on the right-of-way.

2.3. Unmitigated power line magnetic field

The total x, y and z components of the magnetic flux density in the vicinity of the overhead 3-phase power line with phase conductors (n) and earth wires (m) can be obtained by superposition, according to the equations (24) and (25):

$$B_x = \sum_{i=1}^{n+m} B_{xi} \tag{24}$$

$$B_y = \sum_{i=1}^{n+m} B_{yi} \tag{25}$$

It should be pointed out, that when the line currents are changing sinusoidally with time and have a relative phase displacement with respect to each other (as in the case of the three phase line), the produced in the observation point magnetic field vector varies with time not only in the magnitude but also in its orientation. In general, the tip of the magnetic flux density describes the path of an ellipse. Values and orientations of the two semi-axes are determined by the usual search for extremal points of the equation of the ellipse [4].

The maximum value of the elliptic magnetic flux density can be calculated from the relation:

$$B_{\max} = \sqrt{(B_x \cos\beta)^2 + (B_y \cos(\alpha+\beta))^2} \tag{26}$$

where:

$$\alpha = arc(\underline{B}_y) - arc(\underline{B}_x) \tag{27}$$

$$\beta = -\frac{1}{2}\arctan\frac{\sin 2\alpha}{\cos 2\alpha + (\frac{B_x}{B_y})^2} \tag{28}$$

Under normal operating condition the phase currents (I_j , j = 1, 2, 3) of the power lines are dependent on the load flow and can be treated as known. The currents induced in the earth conductors in the case of two earth conductors μ and ν have to be calculated from the relations:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1\mu} & Z_{2\mu} & Z_{3\mu} \\ Z_{1\nu} & Z_{2\nu} & Z_{3\nu} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} + \begin{bmatrix} Z_{\mu} & Z_{\mu\nu} \\ Z_{\nu\mu} & Z_{\nu} \end{bmatrix} \begin{bmatrix} I_{\mu} \\ I_{\nu} \end{bmatrix} \tag{29}$$

The solution of the eqn.(29) takes the form:

$$\begin{bmatrix} I_{\mu} \\ I_{\nu} \end{bmatrix} = \begin{bmatrix} Z_{\mu} & Z_{\mu\nu} \\ Z_{\nu\mu} & Z_{\nu} \end{bmatrix}^{-1} \begin{bmatrix} Z_{1\mu} & Z_{2\mu} & Z_{3\mu} \\ Z_{1\nu} & Z_{2\nu} & Z_{3\nu} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{30}$$

which e.g. for the current I_{μ} gives:

$$I_{\mu} = \frac{\sum_{j=1}^{3n} I_j (Z_{\mu\nu} Z_{j\nu} - Z_{\nu} Z_{j\mu})}{Z_{\mu} Z_{\nu} - Z_{\mu\nu}^{2}} \tag{31}$$

where Z_{μ}, Z_{ν} are the unit-length self impedances, $Z_{\mu\nu}$ is the unit-length mutual impedance between the earth-wires, $Z_{j\mu}$ and $Z_{j\nu}$ are the unit-length mutual impedances between phase conductor and the earth wire μ and ν, respectively. It should be noted that in the calculation of these impedances, the effects of induced currents in the earth on the electromagnetic field of the power line have to be taken into account.

2.4. Current induced in a mitigation loop

The knowledge of the magnetic field density produced by a power line is necessary in calculation of the magnetic flux passing through conductive loop located under the line. The induced in the loop an electromotive force drives the loop-current through the loop impedance. The loop-current creates under the power line a magnetic field, which opposes the one from the line.

Consider the magnetic flux passing through a surface of the horizontal, rectangular loop located underneath an infinitely long overhead conductor with a current, as in Fig. 3.

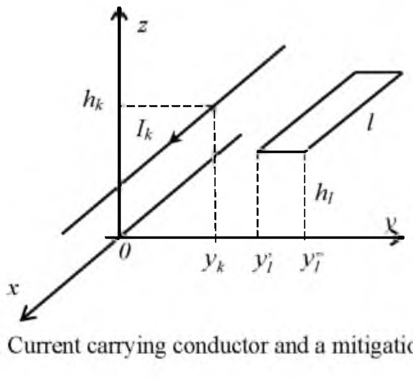

Fig. 3. Current carrying conductor and a mitigation loop

Magnetic flux can be calculated from the formula:

$$\Phi = \int_S \vec{B} \cdot \vec{dS} \tag{32}$$

where $\vec{dS}$ is vectorial surface element.
Taking into account that:

$$\vec{dS} = \vec{e}_z dS_z \tag{33}$$

and $dS_z = ldy$, the magnetic flux takes the form:

$$\Phi = l \int_{y_l'}^{y_l''} B_z(y, z = h_l) dy \tag{34}$$

Inserting relationships (15) and (16) at $z = h_l$ into this integral, the magnetic flux becomes:

$$\Phi = \frac{\mu_0 I_k l}{2\pi} \int_{y_l'}^{y_l''} \left[\begin{array}{l} \dfrac{(y - y_k)}{(h_k - h_l)^2 + (y - y_k)^2} - \dfrac{(y - y_k)}{(h_k + h_l)^2 + (y - y_k)^2} + \\ + 2|k| \displaystyle\int_0^\infty n b e^{-(h_k + h_l)|k| n} \sin\left[(y - y_k)|k| n\right] dn + \\ + j 2|k| \displaystyle\int_0^\infty n(n - a) e^{-(h_k + h_l)|k| n} \sin\left[(y - y_k)|k| n\right] dn \end{array} \right] dy \tag{35}$$

It is permissible to interchange the integrals in this expression, which, when integrated between the limits y'_l and y''_l, gives:

$$\Phi = \frac{\mu_0 I_k l}{\pi} \left[\begin{array}{l} \dfrac{1}{4} \ln \dfrac{[(h_k - h_l)^2 + (y_l'' - y_k)^2][(h_k + h_l)^2 + (y_l' - y_k)^2]}{[(h_k - h_l)^2 + (y_l' - y_k)^2][(h_k + h_l)^2 + (y_l'' - y_k)^2]} + \\ + \displaystyle\int_0^\infty b e^{-(h_k + h_l)|k| n} [\cos[(y_l' - y_k)|k| n] - \cos[(y_l'' - y_k)|k| n]] dn + \\ + j \displaystyle\int_0^\infty (n - a) e^{-(h_k + h_l)|k| n} [\cos[(y_l' - y_k)|k| n] - \cos[(y_l'' - y_k)|k| n]] dn \end{array} \right] \tag{36}$$

The electromotive force induced in the loop can be obtained from the relation:

$$\mathrm{E} = -j\omega\Phi \tag{37}$$

Thus:

$$
E = -\frac{j\omega\mu_0 I_k l}{\pi}\left[\begin{array}{l} \frac{1}{4}\ln\frac{[(h_k-h_l)^2+\left(y_l^{''}-y_k\right)^2][(h_k+h_l)^2+\left(y_l^{'}-y_k\right)^2]}{[(h_k-h_l)^2+\left(y_l^{'}-y_k\right)^2][(h_k+h_l)^2+\left(y_l^{''}-y_k\right)^2]}+ \\ +\int_0^\infty be^{-(h_k+h_l)|k|n}\left[\cos\left[(y_l^{'}-y_k)|k|n\right]-\cos\left[(y_l^{''}-y_k)|k|n\right]\right]dn+ \\ +j\int_0^\infty (n-a)e^{-(h_k+h_l)|k|n}\left[\cos\left[(y_l^{'}-y_k)|k|n\right]-\cos\left[(y_l^{''}-y_k)|k|n\right]\right]dn \end{array}\right] \tag{38}
$$

The electromotive force drives the loop-current through the impedance of the loop:

$$
I_{loop} = \frac{E}{Z_s} \tag{39}
$$

where Z_s is the self-impedance of the loop.

Thus:

$$
I_{loop} = -\frac{j\omega\mu_0 I_k l}{\pi Z_S}\left[\begin{array}{l} \frac{1}{4}\ln\frac{[(h_k-h_l)^2+\left(y_l^{''}-y_k\right)^2][(h_k+h_l)^2+\left(y_l^{'}-y_k\right)^2]}{[(h_k-h_l)^2+\left(y_l^{'}-y_k\right)^2][(h_k+h_l)^2+\left(y_l^{''}-y_k\right)^2]}+ \\ +\int_0^\infty be^{-(h_k+h_l)|k|n}\left[\cos\left[(y_l^{'}-y_k)|k|n\right]-\cos\left[(y_l^{''}-y_k)|k|n\right]\right]dn+ \\ +j\int_0^\infty (n-a)e^{-(h_k+h_l)|k|n}\left[\cos\left[(y_l^{'}-y_k)|k|n\right]-\cos\left[(y_l^{''}-y_k)|k|n\right]\right]dn \end{array}\right] \tag{40}
$$

The Dubanton equation for the unit-length self impedance of the conductor k with the radius r_k is [19, 20]:

$$
Z_s = j\omega\frac{\mu_0}{2\pi}\ln\frac{2(h_k+p)}{r_k} \tag{41}
$$

where:

$$
p = \frac{1}{\sqrt{j\omega\mu_0\gamma}} = \frac{\delta_s}{2}(1-j) \tag{42}
$$

and

$$
\delta_s = \frac{1}{\sqrt{\pi f\mu_0\gamma}} \tag{43}
$$

is the penetration depth.

If the effects of the currents induced in the earth on the total magnetic field produced under the power line are negligible as compared with the effects due to the currents flowing in the overhead conductors, the magnetic flux passing the loop can be determined from eqn. (34) with B_{zu} obtained from eqn.(15) at $z = h_l$ so that:

$$\Phi_u = \frac{\mu_0 I_k l}{4\pi} \ln \frac{(h_l - h_k)^2 + (y_l'' - y_k)^2}{(h_l - h_k)^2 + (y_l' - y_k)^2} \tag{44}$$

The electromotive force can be next obtained from eqn.(37) in the simplified form:

$$\mathrm{E}_u = -\frac{j\omega\mu_0 I_k l}{4\pi} \ln \frac{(h_l - h_k)^2 + (y_l'' - y_k)^2}{(h_l - h_k)^2 + (y_l' - y_k)^2} \tag{45}$$

and the approximate value of the loop-current is then:

$$I_{u\ loop} = -\frac{j\omega\mu_0 I_k l}{4\pi Z_s} \ln \frac{(h_l - h_k)^2 + (y_l'' - y_k)^2}{(h_l - h_k)^2 + (y_l' - y_k)^2} \tag{46}$$

The loop current creates under the current carrying overhead conductor a magnetic field, which opposes the one from the conductor.

It should be noted, that the loop impedance Z_s can be changed by insertion of appropriate series capacitance and its phase angle can be adjusted additionally by choice of an appropriate conductor (resistance) [4].

2.5. Mitigated power line magnetic field

The mitigated magnetic field under the power line is the vectorial sum of the local original magnetic field associated with currents in the phase conductors and the earth conductors and of the local auxiliary field caused by the loop current.

The total x and y components of the magnetic flux density in the vicinity of the overhead 3-phase power line with phase conductors (n) and earth wires (m) and auxiliary conductors (k) forming the loop can be obtained by superposition, according to the equations (24) and (25):

$$B_x = \sum_{i=1}^{n+m+k} B_{xi} \tag{47}$$

$$B_y = \sum_{i=1}^{n+m+k} B_{yi} \tag{48}$$

It should be noted, that the magnetic flux density produced by the *k–th* auxiliary current has to be obtained according to the eqn. (49) and (50) or according to eqn. (50) and (51) for the exact and approximate method respectively.

$$B_{yk}(y,z)=\frac{\mu_0 I_{loop}}{2\pi}\left[\begin{array}{l} -\dfrac{z-h_k}{(z-h_k)^2+(y-y_k)^2}+\dfrac{z+h_k}{(z+h)^2+(y-y_k)^2}+ \\ -2|k|\int\limits_0^\infty nbe^{-(z+h_k)|k|n}\cos[(y-y_k)|k|n]dn \\ -j2|k|\int\limits_0^\infty n(n-a)e^{-(z+h_k)|k|n}\cos[(y-y_k)|k|n]dn \end{array}\right] \tag{49}$$

$$B_{zk}(y,z)=\frac{\mu_0 I_{loop}}{2\pi}\left[\begin{array}{l} \dfrac{y-y_k}{(z-h_k)^2+(y-y_k)^2}-\dfrac{(y-y_k)}{(z+h_k)^2+(y-y_k)^2}+ \\ +2|k|\int\limits_0^\infty nbe^{-(z+h_k)|k|n}\sin[(y-y_k)|k|n]dn+ \\ +j2|k|\int\limits_0^\infty n(n-a)e^{-(z+h_k)|k|n}\sin[(y-y_k)|k|n]dn \end{array}\right] \tag{50}$$

$$B_{yk\,u}(y,z)=-\frac{\mu_0 I_{u\,loop}}{2\pi}\frac{z-h_k}{(z-h_k)^2+(y-y_k)^2} \tag{51}$$

$$B_{zk\,u}(y,z)=\frac{\mu_0 I_{u\,loop}}{2\pi}\frac{y-y_k}{(z-h_k)^2+(y-y_k)^2} \tag{52}$$

Exemplary calculations of the magnetic field distribution under a power line have been carried out according to the method derived. The geometry of a 750 kV transmission line with mitigation loop located beneath outside phases is shown in Fig. 4.

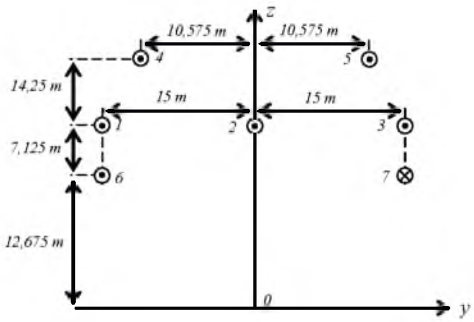

Fig. 4. The assumed tower configuration, adopted from [4]

Assuming balanced three phase currents in phase conductors 1, 2 and 3 $I_1=1500e^{-j120^\circ}A, I_2=1500e^{-j0^\circ}A, I_3=1500e^{j120^\circ}A$, span length l = 470 m, f = 60 Hz, first the currents in the earth conductors 4, 5 have been calculated according to eqn.(30), and the unmitigated magnetic field is obtained from eqns.(24 - 25). The magnetic flux linked with the loop consisting of closed conductors 6 and 7 has been determined from the eqn. (44) and the electromotive force induced in the loop has been evaluated from eqn. (45). Taking into account that the loop with series capacitor 2.10 mF has the impedance $Z_s = 0.2e^{-j30^\circ}\,\Omega$ [4], the current produced within the mitigation loop is $I_{loop} = 692e^{-j90^\circ}\,A$. Finally, the resultant mitigated magnetic flux density has been calculated from eqns.(47 – 48). Lateral profile at height 1.0 m of the magnetic flux density module under the power line without and with the mitigation loop is presented graphically in Fig. 5. Evident reduction of the magnetic field can be observed.

It should be noted, that the results of the magnetic flux density calculations are nearly identically for the exact and simplified method.

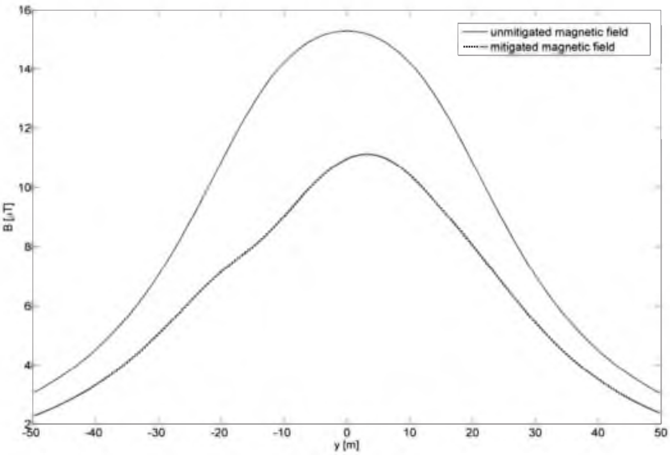

Fig. 5. Lateral profile of the magnetic field under the power line

It should be however pointed out, that the comparison of the above results with results obtained in [4] is not possible due to incorrect calculation of the total magnetic flux penetrating the loop in the cited publication.

2.6. Mitigation loop located non-parallel to the earth surface

Consider the system shown in Fig. 6. Both longitudinal conductors of the rectangular mitigation loop are located at different heights parallel to the current carrying conductor. The angle between the plane of the loop and the earth surface is α. The location of the loop is geometrically defined by the positions of the longitudinal conductors $(y_l^{'}, h_l^{'})$ and $(y_l^{''}, h_l^{''})$.

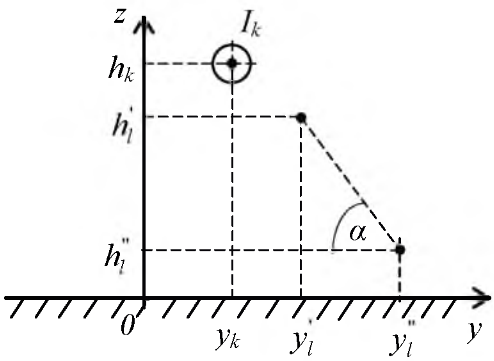

Fig. 6. An infinitely long current carrying conductor and a mitigation loop above the earth surface in evaluation of magnetic flux passing through the loop

The magnetic flux density components in the air due to the current I_k are given by the formulas (13), (14), (15) and (16). Magnetic flux passing through a surface of the loop as in Fig. 6 can be calculated from the formula (32).

The vectorial surface element $\overrightarrow{dS}$ is now

$$\overrightarrow{dS} = \vec{e}_y dS_z + \vec{e}_z dS_y \tag{53}$$

where

$$dS_z = ldz \tag{54}$$

$$dS_y = ldy \tag{55}$$

and l denotes the length of the longitudinal conductor.

The scalar product in eqn.(32) takes the form

$$\vec{B} \cdot \overrightarrow{dS} = B_y ldz + B_z ldy \tag{56}$$

Denoting

$$y = y_l' + zctg\alpha \tag{57}$$

in the formulas (13) and (14) we can calculate the magnetic flux passing through the surface S_z

$$\Phi_y = \frac{\mu_0 I_k l}{2\pi}\int_{h_l'}^{h_l''} \left[\begin{array}{l} -\dfrac{z-h_k}{(z-h_k)^2+(y_l'+zctg\alpha-y_k)^2}+\dfrac{z+h_k}{(z+h_k)^2+(y_l'+zctg\alpha-y_k)^2}+ \\ -2|k|\int_0^\infty nbe^{-(z+h_k)|k|n}\cos[(y_l'+zctg\alpha-y_k)|k|n]dn \\ -j2|k|\int_0^\infty n(n-a)e^{-(z+h_k)|k|n}\cos[(y_l'+zctg\alpha-y_k)|k|n]dn \end{array}\right] dz \tag{58}$$

Similarly, denoting

$$z = z_l^{'} + ytg\alpha \tag{59}$$

in the formulas (15) and (16), the magnetic flux passing through the surface S_y becomes

$$\Phi_z = \frac{\mu_0 I_k l}{2\pi}\int_{y_l^{'}}^{y_l^{''}}\left[\begin{array}{l}\dfrac{y-y_k}{\left(h_l^{'}+ytg\alpha-h_k\right)^2+\left(y-y_k\right)^2}-\dfrac{\left(y-y_k\right)}{\left(h_l^{'}+ytg\alpha+h_k\right)^2+\left(y-y_k\right)^2}+\\ +2|k|\int_0^{\infty} nbe^{-\left(h_l^{'}+ytg\alpha+h_k\right)|k|n}\sin\left[\left(y-y_k\right)|k|n\right]dn+\\ +j2|k|\int_0^{\infty} n(n-a)e^{-\left(h_l^{'}+ytg\alpha+h_k\right)|k|n}\sin\left[\left(y-y_k\right)|k|n\right]dn\end{array}\right]dy \tag{60}$$

The resultant magnetic flux penetrating the mitigation loop as in Fig. 6 is the sum of the fluxes defined by eqns (6) and (8).

It is easy to show that if the angle $\alpha = 0$ (the horizontal location of the loop) the magnetic flux penetrating the loop is given by the (35).

If the effects of the currents induced in the earth on the total magnetic field produced under the power line are negligible as compared with the effects due to the currents flowing in the overhead conductors, the magnetic flux passing the loop can be determined from the following equation

$$\Phi_u = \frac{\mu_0 I_k l}{2\pi}\left\{\int_{h_l^{'}}^{h_l^{''}} -\frac{z-h_k}{\left(z-h_k\right)^2+\left(y_l^{'}+zctg\alpha-y_k\right)^2}dz + \int_{y_l^{'}}^{y_l^{''}}\frac{y-y_k}{\left(h_l^{'}+ytg\alpha-h_k\right)^2+\left(y-y_k\right)^2}dy\right\} \tag{61}$$

The electromotive force induced in the loop can be obtained from the relation (37) and the loop-current from the eqn.(39), respectively.

Another possibility to calculate the magnetic flux using the vector potential will be presented in the sequel.

The x-component of the vector potential in the air given by eqn.(3) can be now written in the form:

$$A_x(y,z) = \frac{\mu_0 I_k}{\pi}\left\{\begin{array}{l}\dfrac{1}{2}\displaystyle\int_0^{\infty}\frac{e^{-u|z-h_k|}-e^{-u(z+h_k)}}{u}\cos\left[\left(y-y_k\right)u\right]du+\\ +\displaystyle\int_0^{\infty}\frac{e^{-u(z+h_k)}}{u+\sqrt{u^2+k^2}}\cos\left[\left(y-y_k\right)u\right]du\end{array}\right\} \tag{62}$$

Taking into account the formula [18]:

$$\int_0^{\infty}\frac{e^{-\alpha u}-e^{-\beta u}}{u}\cos(au)du = \frac{1}{2}\ln\frac{a^2+\beta^2}{a^2+\alpha^2} \tag{63}$$

where: $\alpha > 0, \mathrm{Re}(\alpha) \geq 0, \mathrm{Re}(\beta) > 0$ we get:

$$A_x(y,z) = \frac{\mu_0 I_k}{\pi}\left\{\frac{1}{4}\ln\frac{(z+h_k)^2+(y-y_k)^2}{(z-h_k)^2+(y-y_k)^2} + \int_0^\infty \frac{e^{-u(z+h_k)}}{u+\sqrt{u^2+k^2}}\cos[(y-y_k)u]du\right\} \quad (64)$$

If the formulas (17 – 20) are used the vector potential becomes:

$$A_x(y,z) = \frac{\mu_0 I_k}{\pi}\left\{\begin{array}{l}\frac{1}{4}\ln\frac{(z+h_k)^2+(y-y_k)^2}{(z-h_k)^2+(y-y_k)^2} + \\ + j\int_0^\infty (n-a-jb)e^{-(z+h_k)|k|n}\cos[(y-y_k)|k|n]dn\}\end{array}\right\} \quad (65)$$

Magnetic flux penetrating through the loop (Fig.6) is obtained by applying the equation:

$$\Phi = \oint_l \vec{A}\cdot\vec{dl} = lA_x(y=y_l', z=h_l') - lA_x(y=y_l'', z=h_l'') \quad (66)$$

where l denotes the length of the longitudinal loop-conductor.

Insertion of the eqn.(13) into the formula (14) gives the magnetic flux in the form:

$$\begin{aligned}\Phi = \frac{\mu_0 I_k l}{\pi}\{&\frac{1}{4}\ln\frac{[(h_l'+h_k)^2+(y_l'-y_k)^2][(h_l''-h_k)^2+(y_l''-y_k)^2]}{[(h_l'-h_k)^2+(y_l'-y_k)^2][(h_l''+h_k)^2+(y_l''-y_k)^2]} + \\ &+ \int_0^\infty b[e^{-(h_l'+h_k)|k|n}\cos[(y_l'-y_k)|k|n] - e^{-(h_l''+h_k)|k|n}\cos[(y_l''-y_k)|k|n]]dn + \\ &+ j\int_0^\infty (n-a)[e^{-(h_l'+h_k)|k|n}\cos[(y_l'-y_k)|k|n] - e^{-(h_l''+h_k)|k|n}\cos[(y_l''-y_k)|k|n]]dn\}\end{aligned} \quad (67)$$

It is easy to show that when $h_l' = h_l'' = h_l$ (the horizontal location of the loop) the magnetic flux penetrating the loop is given by the eqn.(36).

3. Conclusions

The design of installation generating low-frequency magnetic and electric fields requires access to effective analytical and computational tools. The paper presents procedures of determining the magnetic flux density of the field produced under power lines.

Exact and approximate methods are developed for analyzing magnetic fields in the vicinity of overhead power lines without and with mitigation passive loops. The exact method of field calculation is based on the Fourier transform technique and takes into account the earth currents. The approximate method assumes that effects of earth currents onto magnetic field are negligible. In both methods phase-currents have prescribed values, based on which all of the remaining currents (in earth conductors and in loop-conductors) are computed.

Moreover the mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located near the power line horizontal or non-parallel to the earth surface.

The results derived can be used as the foundation for almost every study on the magnetic fields for conditions that are almost always satisfied for power engineering applications.

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